Variational principles for hydrodynamics of strongly coupled plasmas

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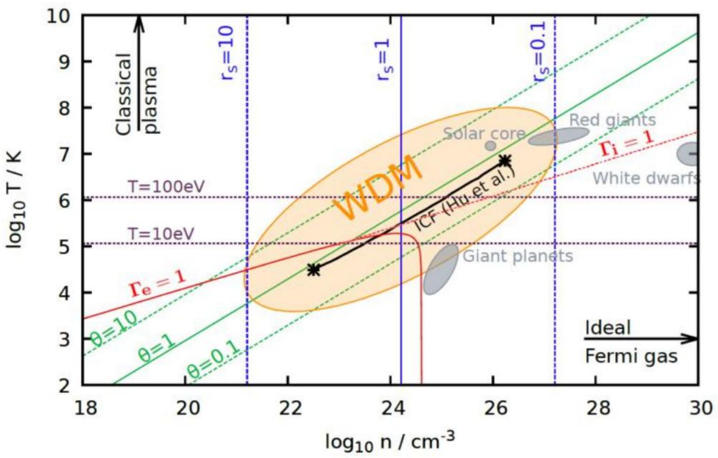
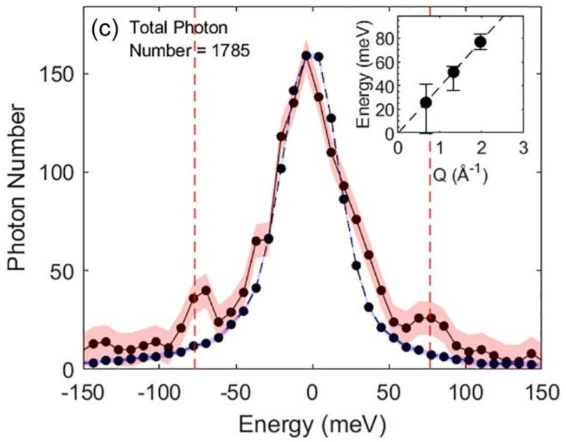
pdf of the talk at
https://www.itap.uni-kiel.de//theo-physik/bonitz/talks.html

Motivation

$$\Gamma_i = \frac{(Ze)^2}{4\pi\varepsilon_0 a} \Big/ k_B T \geq 1$$



M. Bonitz *et al.,* Phys. Plasmas **31**, 110501 (2024)
S. X. Hu *et al.,* Phys. Plasmas **22**, 056304 (2015)



T. G. White *et al.,* Phys. Rev. Research **6**, L022029 (2024)

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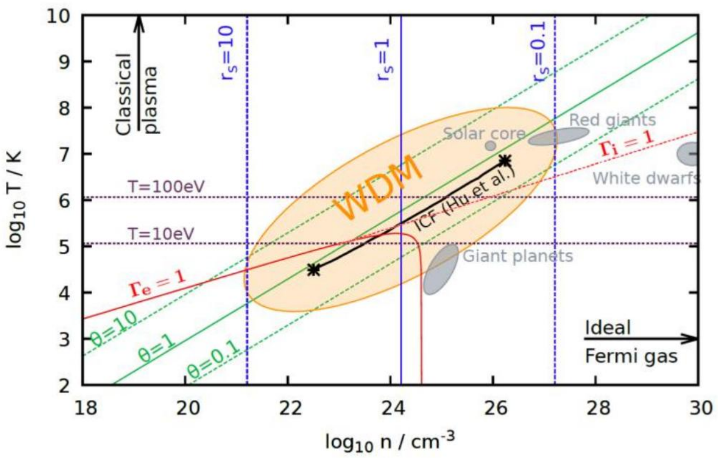

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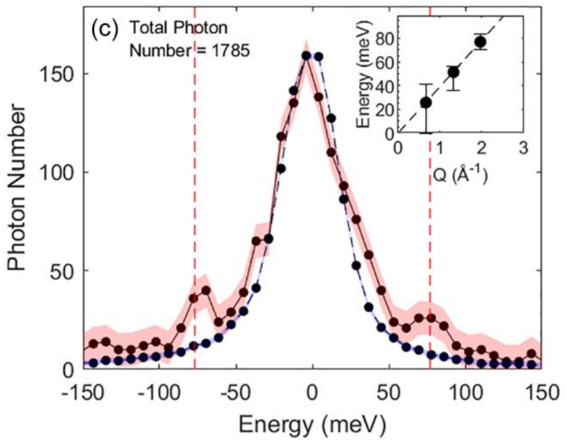

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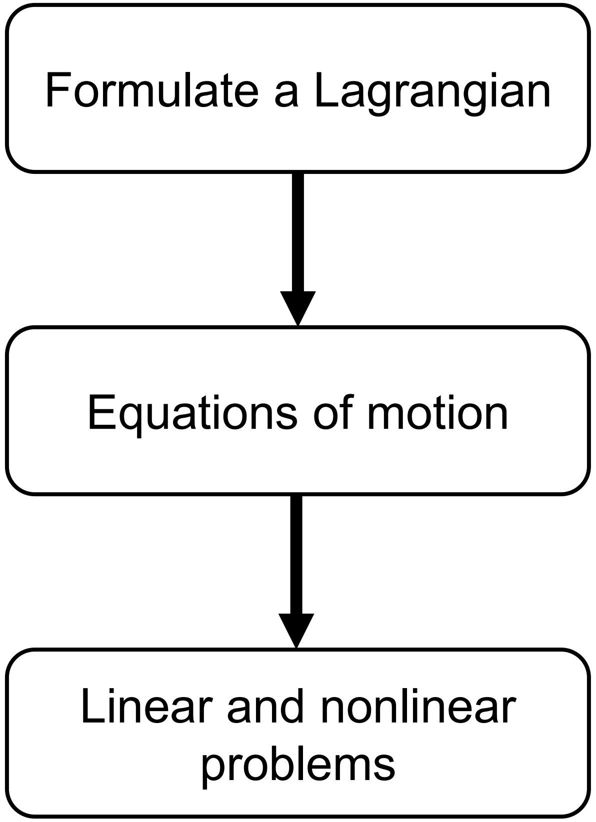
Hydrodynamics:

- simple and physically clear,
- approximate but computationally efficient.

Strong coupling requires the effects of correlations!

Basics of the approach



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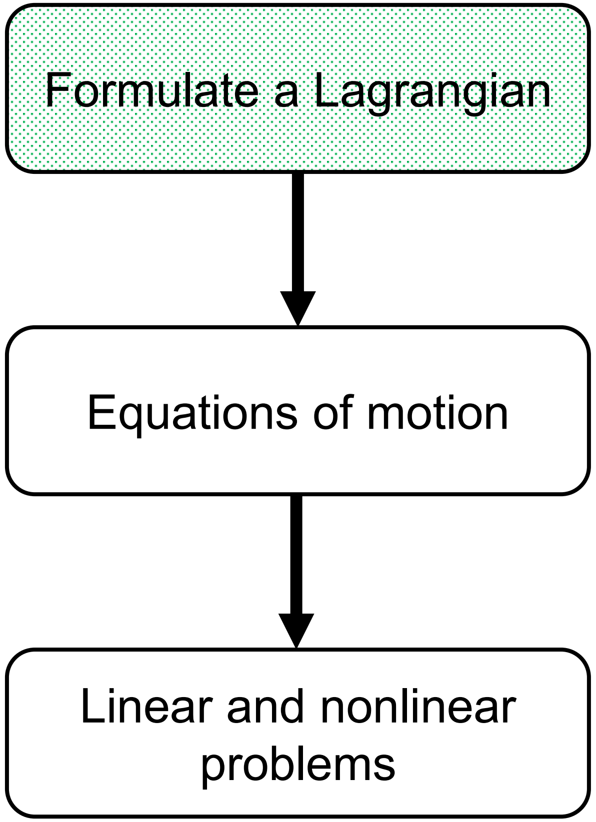
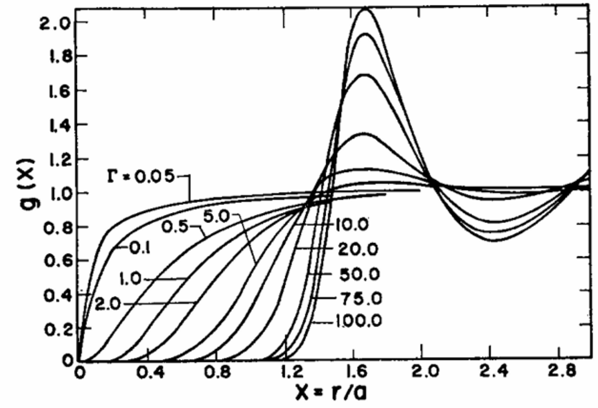


Strong coupling hydrodynamics:

$$L = K - U \Rightarrow \int \frac{m}{2}\left(\frac{\partial\langle\vec{x}\rangle}{\partial t}\right)^2 n\,\mathrm{d}\vec{a} - \int \left(\frac{3}{2}k_B T\right) n\,\mathrm{d}\vec{a}$$

$$-\frac{1}{2}\iint \phi\left(|\langle\vec{x}\rangle - \langle\vec{x}\rangle'|\right) n n' g(n, T, |\vec{a} - \vec{a}'|)\,\mathrm{d}\vec{a}\mathrm{d}\vec{a}'$$



S. G. Brush *et al.*, J. Chem. Phys. **45**, 2102 (1966)

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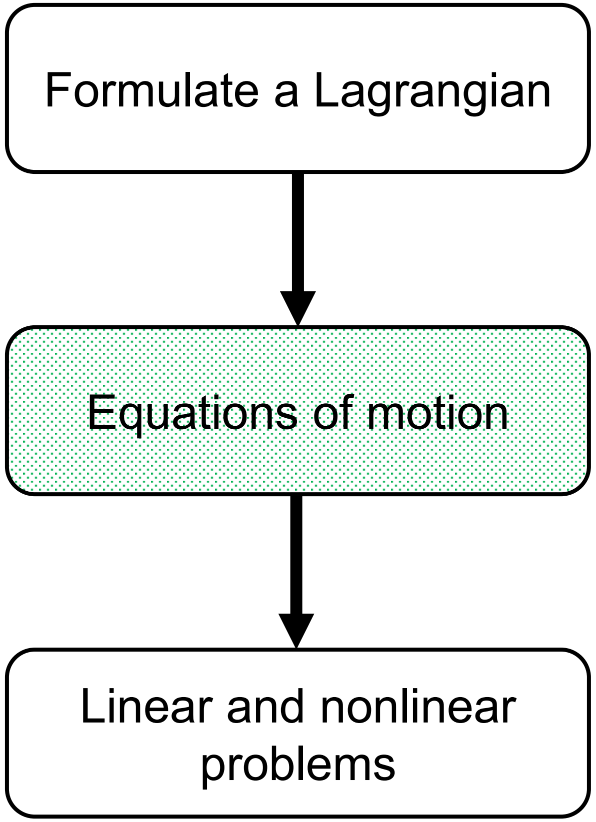
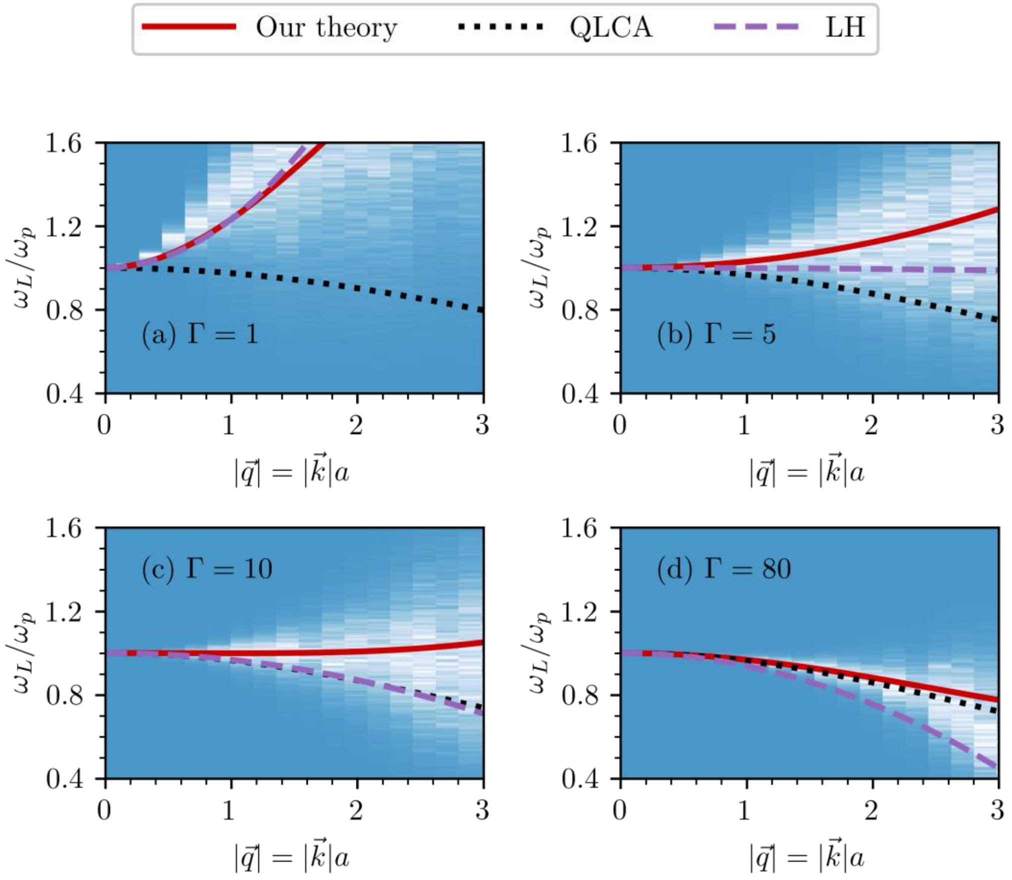


$$mn\left(\frac{\partial \vec{v}}{\partial t} + \left(\vec{v}\cdot\vec{\nabla}\right)\vec{v}\right) = -\vec{\nabla}p + \vec{f}_{\text{nonlocal}}$$

$$p = -\left.\frac{\partial T}{\partial(1/n)}\right|_s \left(\frac{3}{2}k_B + \frac{1}{2}\int n_{\vec{a}'}\phi \left.\frac{\partial g}{\partial T}\right|_{n,|\vec{a}-\vec{a}'|} \mathrm{d}\vec{a}'\right)$$

$$\vec{f}_{\text{nonlocal}} = -n\int n_{\vec{a}'}\left(\frac{g+g^T}{2}\right)\vec{\nabla}\phi\,\mathrm{d}\vec{a}'$$

Dispersion laws of the OCP

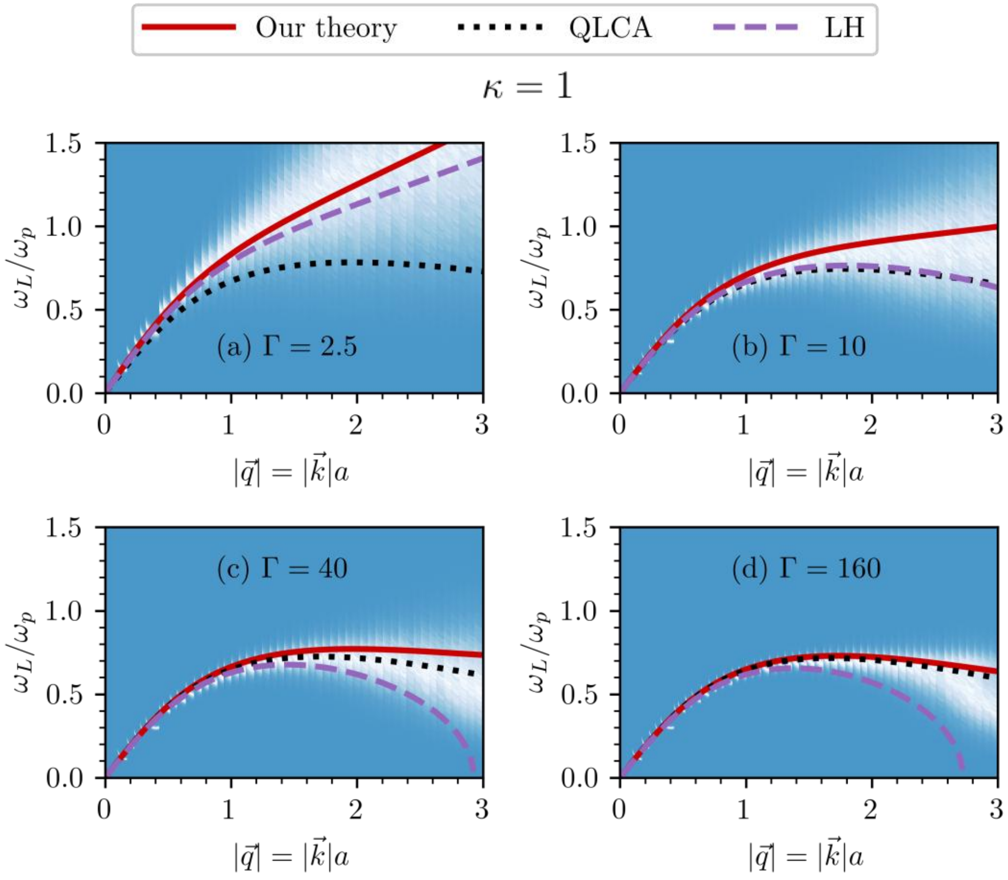


D. Krimans and S. Putterman, "Variational principles for the hydrodynamics of the classical one-component plasma," Phys. Fluids **36**, 037131 (2024),
MD data: I. Korolov *et al.,* Contrib. Plasma Phys. **55,** 421-427 (2015).

$$\frac{\phi(r)}{k_B T} = \frac{\Gamma}{x}, \quad x = r/a$$

- Works for a wide range of coupling and $\lambda \approx a$.
- Correct $|\vec{k}| \to 0^+$ limit with transition at $\Gamma \approx 9.5$.
  (H. Kählert, Phys. Plasmas **31**, 092109 (2024))
- Computationally efficient.
- Better than typical theories:
  - linearized hydrodynamics (LH),
    (P. Vieillefosse *et al*., Phys. Rev. A **12**, 1106 (1975))
  - quasilocalized charge approximation (QLCA).
    (K. I. Golden *et al*., Phys. Plasmas **7**, 14 (2000))

Dispersion laws of the YOCP



D. Krimans and H. Kählert, "Variational hydrodynamics of the classical Yukawa one-component plasma," arXiv:2506.23006 (2025)

$$\frac{\phi(r)}{k_B T} = \frac{\Gamma}{x} e^{-\kappa x}, \quad x = r/a, \quad \kappa = a/\lambda$$

- Works for a wide range of coupling and screening, and $\lambda \approx a$.
- Computationally efficient.
- When looking at the entire range of coupling parameters, better than typical theories:
  - linearized hydrodynamics (LH),
    (G. Salin, Phys. Plasmas **14**, 082316 (2007))
  - quasilocalized charge approximation (QLCA).
    (K. I. Golden *et al.*, Phys. Plasmas **7**, 14 (2000))

Conclusions and future directions

Our variational approach to hydrodynamics:

- physically clear Lagrangian,
- explicitly includes nonlocal correlation effects,
- verified in the linear regime for OCP and YOCP.

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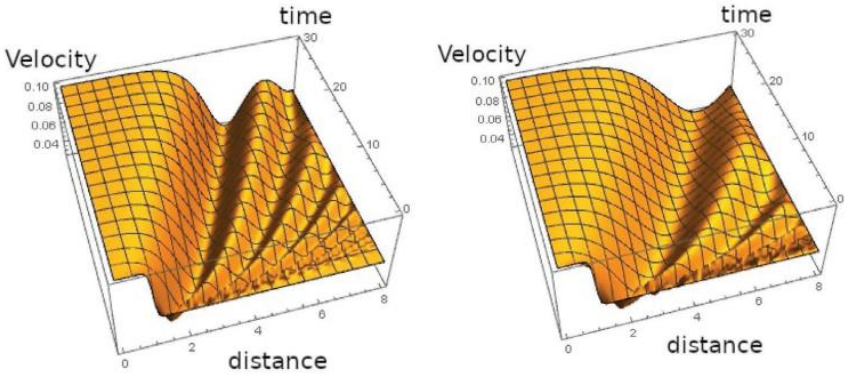
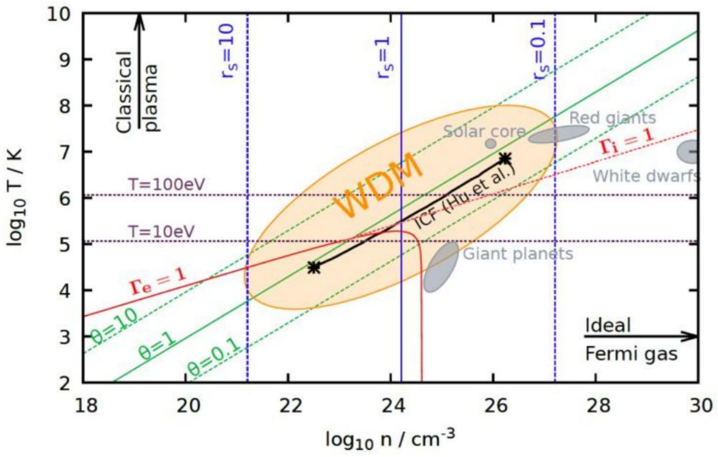


F. Graziani *et al.*, Contrib. Plasma Phys. **62** (2022)

Next:

- nonlinear problems,
- including quantum electrons.



$$\Theta = \frac{T}{T_F} \lesssim 1$$

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Thank you!