ANNALS OF PHYSICS **258**, 320–359 (1997)
ARTICLE NO. PH975703

Non-Markovian Boltzmann Equation

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Received December 27, 1996

A quantum kinetic equation for strongly interacting particles (generalized binary collision approximation, ladder or T-matrix approximation) is derived in the framework of the density operator technique. In contrast to conventional kinetic theory, which is valid on large time scales as compared to the collision (correlation) time only, our approach retains the full time dependencies, especially also on short time scales. This means retardation and memory effects resulting from the dynamics of binary correlations and initial correlations are included. Furthermore, the resulting kinetic equation conserves total energy (the sum of kinetic and potential energy). The second aspect of generalization is the inclusion of many-body effects, such as self-energy, i.e., renormalization of single-particle energies and damping. To this end we introduce an improved closure relation to the Bogolyubov–Born–Green–Kirkwood–Yvon hierarchy. Furthermore, in order to express the collision integrals in terms of familiar scattering quantities (Møller operator, T-matrix), we generalize the methods of quantum scattering theory by the inclusion of medium effects. To illustrate the effects of memory and damping, the results of numerical simulations are presented.

1. INTRODUCTION

The quantum kinetic properties of a many-particle system are determined in most cases by the single-particle density matrix or the Wigner distribution function. The development of the kinetic theory, aiming at the determination of these quantities, began in 1872 when L. Boltzmann published the equation of motion for the distribution function [1]. The quantum features of the colliding particles were included later on in modern quantum-mechanical language. Other "Boltzmann-like" kinetic equations are the well-known Landau and Lenard–Balescu equations, the latter taking into account the dynamic screening in a plasma.

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0003-4916/97 $25.00

The Boltzmann equation (BE) is one of the most fundamental equations of many-particle theory. This equation describes the irreversible relaxation of a many-particle system from an arbitrary initial state towards equilibrium. Furthermore, the Boltzmann equation is the basic equation for the theory of transport processes in macroscopic systems. Regardless of its fundamental character, the BE is, nevertheless, an approximation only. There are phenomena, such as bound states, energy conservation, and short-time dynamics which cannot be described correctly by the BE. In order to discuss all these problems, it is necessary to reconsider the derivation of the BE from the basic equations of quantum statistics, i.e., from the von Neumann equation,

$$ih \frac{\partial \varrho_N}{\partial t} = [H_N, \varrho_N], \tag{1}$$

where ϱ_N and H_N denote the density operator and the Hamiltonian of an N-particle system. This problem was first solved by Bogolyubov [2] who used the idea that Boltzmann's assumption of molecular chaos can be used as an asymptotic condition for the solution of the von Neumann equation. Then, the conventional Boltzmann equation follows from Eq. (1) if, in addition, time retardation effects are neglected completely (details will be discussed below). From this it follows:

(i) there exists a restriction with respect to the time; i.e., the BE is valid only for times t with $t \gg \tau_{\text{corr}}$. Therefore, the BE is not appropriate for short-time kinetics. In particular, it cannot describe initial correlations.

(ii) The BE conserves only the mean kinetic energy $\langle T \rangle = \langle p^2/2m \rangle$.

This means, $\langle T \rangle$ is conserved during the relaxation process from an arbitrary initial distribution into a stationary state. Of course, this is an unphysical behavior. Instead, total energy (the sum of kinetic and potential energy) has to be conserved. As a result, a temporal relaxation of $\langle T \rangle$ and of the mean potential energy $\langle V \rangle$ to their thermodynamic values has to be expected, and the properties of the final state will be determined by correlations. Therefore, the BE cannot yield the correct asymptotic states. For these reasons, a generalization of the Boltzmann equation is necessary.

We have to mention that generalized kinetic equations for the one-particle distribution that include initial correlations and memory effects have been derived already in the early 1960s by Zwanzig [3], Prigogine and Resibois [4], and Fujita [5]. In particular, Prigogine and Resibois demonstrated that initial correlations are being damped and the memory duration is limited, due to interaction in the system, and, therefore, in the long-time limit, the usual Markovian kinetic equation is recovered. The question of total energy conservation has been investigated for a long time, too [6]. In the works of Bärwinkel [7] and Klimontovich and Ebeling [8] it was shown that energy conservation and short-time dynamics are closely connected; see also [9–15]. On the other hand, these early works considered only rather general aspects of the theory and did not derive explicit kinetic equations,

such as the non-Markovian Boltzmann equation [16]. Furthermore, they did not include specific quantum effects.

In the 1970s and early 1980s the interplay between retardation and self-energy in three-particle collisions, including the formation and breakup of bound states, was investigated by Paltsev [12], McLennan [13], and Klimontovich and Kremp [14]. Furthermore, memory effects and initial correlations in two-time kinetic equations were studied in the context of nuclear collisions by Danielowicz [17].

Recently, progress in short-pulse laser technology [18], has revived the interest in non-Markovian kinetic equations for the Wigner distribution. For the theoretical description of ultrafast relaxation of optically excited carriers in semiconductors, generalized kinetic equations are essential. Here, numerous theoretical and numerical investigations have been performed, primarily regarding electron–phonon scattering, e.g., [19–22]. Recently, also, studies of memory effects for the essentilly more involved carrier–carrier scattering problem have been published [23–28], but they were all limited to the Born approximation, i.e., to weak interactions. On the other hand, there is high interest in the case of strong interactions, too, in particular in nuclear collisions, in fluids, dense plasmas, but also in solids at densities sufficiently below the Mott point. In this case, bound state complexes (nuclei, atoms, molecules, excitons, bi-excitons, etc.) exist, which may alter the properties of the system dramatically.

In this paper, we will consider several important aspects of generalized kinetic equations for strongly coupled systems from the point of view of quantum statistics. In particular, we derive the non-Markovian Boltzmann equation, which allows us to describe effects of strong interaction on short time scales. Among others, we discuss the connection between the non-Markovian character of the Boltzmann equation and conservation laws. Another problem we will be concerned with is taking into account initial correlations at a finite time t_0, their decay, and their influence on the properties of the system.

A very powerful tool to derive generalized kinetic equations for the Wigner function is the real-time Green's functions technique, which leads, e.g., to the well-known Kadanoff–Baym equations. These equations are very general and have important properties, such as the exact conservation of total energy and the fully self-consistent account of damping effects. There exist several papers, where numerical solutions of the Kadanoff–Baym equations were given [17, 29, 30, 26, 25]. However, due to the two-time structure, a numerical analysis of the Kadanoff–Baym equations is very complicated and still restricted to the Born approximation. Simplifications are possible if the transition to kinetic equations for the Wigner function is made. Such generalized non-Markovian kinetic equations have been derived from the Kadanoff–Baym equations, e.g., in [31–34]. Nevertheless, there exist several problems in these derivations. The first is related to the initial correlations. The Kadanoff–Baym equations were originally derived under the assumption of an asymptotic condition for $t_0 \to -\infty$, assuming an uncorrelated initial state [6]. There have been several attempts to generalize this derivation to include arbitrary correlations which may exist in the system at a finite initial time t_0 [5, 35,

17, 36]. The most satisfactory solutions appear to be those of Daniewlowicz [17], who generalized the Wick theorem to the case of initial correlations, and a recent alternate derivation which was based on the solution of the Martin–Schwinger hierarchy [36]. These results coincide in the equal-time limit with the density operator results. Another problem is related to the reconstruction of the correlation functions $g^<(t, t'), g^>(t, t')$ in terms of Wigner distributions $f(t)$; here we mention the successful attempt by Lipavský *et al.* [37]. Still there remain some questions concerning the choice of the retarded/advanced propagators in this ansatz and, in particular, the problem of energy conservation for different choices of propagators.

Based on their own experience with the real-time Green's function theory, e.g. [38, 39, 31], the authors think it is very important to study also an alternative approach which is based on the Bogolyubov–Born–Green–Kirkwood–Yvon (BBGKY)-hierarchy. The latter results are a valuable test case for the former in the equal time limit. Moreover, the density operator results are physically intuitive. It is one of the aims of this paper to show that the technique of density operators yields the same results for the Wigner function as the Green's function technique. In particular, we arrive, without any assumption, at the generalized Kadanoff–Baym ansatz of Lipavský *et al.* Of course, there are, also drawbacks in the density operator approach. So, it is difficult to incorporate various many-particle effects, such as screening and self-energy in a simple manner. But the authors think it is worthwhile to investigate this formalism and to solve these problems for the reasons mentioned above; see also [40, 41].

This paper is organized as follows. Starting from the von Neumann equation, in Section II, we show that there is total energy conservation in the untruncated BBGKY-hierarchy. Futhermore, energy conservation holds, for any approximation to the hierarchy, which allows for arbitrary permutations among the n particles ($n \geqslant 3$) involved in an n-particle density operator [42]. We then introduce a new hierarchy closure on the level of ternary correlations, which corresponds to a generalization of the binary collision approximation. This closure allows us to define effective two-particle Hamiltonians and to derive the corresponding equations of motion for the single and two-particle density operators. We include Pauli blocking effects (phase space occupation) according to Boercker and Dufty [43] (see also [39, 31]) and, moreover, self-energies in binary collision approximation in order to account for quasi-particle energies and damping.

In Section III, we construct a complete solution for the two-particle density operator. To this end we introduce appropriate propagators, generalized T-matrices, and Møller-operators. We end up with an equation for the two-particle correlation operator $g_{12}(t)$ which is, for short times, essentially determined by the initial correlations. Only after these correlations are being damped out, the Bogolyubov-regime is reached which is characterized by "weakening of initial correlations." The correlation operator $g_{12}(t)$ is given by a nonlocal function of time which leads to memory effects that influence the behavior of the system. Furthermore, we perform a gradient expansion of the correlation operator which simplifies the problem significantly, and, at the same time allows us to highlight the main physical effects

more clearly. In Section IV, we construct the collision integral for the equation of motion of the single-particle density operator (kinetic equation), which includes Pauli blocking and self-energy and accounts for initial correlations and memory effects. Due to self-energy effects, initial correlations are damped, and the memory has a "finite depth." Due to the complex structure of the collision integral, we consider two useful approximations. In Section IV, we consider the second Born approximation of the T-matrix, leading to the Landau equation. In Section V, the kinetic equation is treated in the first-order gradient expansion. In the Markov limit, this yields the "usual" Boltzmann equation and, furthermore, important corrections that just guarantee (total) energy conservation and the correct macroscopic properties of an interacting many-particle system. In Section VI, we present numerical solutions of the non-Markovian Landau equation to illustrate the memory effects in strongly coupled systems. Finally, Section VII is devoted to a concluding discussion of the results.

II. THE BOGOLYUBOV HIERARCHY; GENERALIZED BINARY COLLISION APPROXIMATION; ENERGY CONSERVATION

A. *Bogolyubov hierarchy and total energy conservation*

We consider a quantum many-body system with short-range binary interactions $V(r_i - r_j)$. We permit the interaction to be of arbitrary strength, so that the mean value of the potential energy $\langle V \rangle$ may be of the same order or even larger than that of the kinetic one $\langle T \rangle$. Moreover, in case of an attractive potential, bound states may be included. To simplify the notation below, we consider a one-component system and the spatially homogeneous case only. Generalizations to several species and to inhomogeneous systems are straightforward.

In the quantum statistical description, the state of such a system of N particles is completely determined by the density operator ρ_N. However, more suitable for practical purposes are the reduced s-particle density operators $F_{1\cdots s}$, defined by

$$F_{1\cdots s} = \mathcal{V}^s \operatorname*{Tr}_{s+1\cdots N} \varrho_N; \qquad \operatorname*{Tr}_{1\cdots s} F_{1\cdots s} = \mathcal{V}^s. \tag{2}$$

In terms of the reduced density operators, the average of an s-particle observable A is given by

$$< A > = \frac{n^s}{s!} \operatorname*{Tr}_{1\cdots s} F_{1\cdots s} A_{1\cdots s}. \tag{3}$$

Here $\mathcal{V}$ and n denote the volume of the system and the density, respectively. The equations of motion for the reduced density operators (Bogolyubov or BBGKY-hierarchy) follow from the von Neumann equation (1) by taking the partial trace

$$ih\frac{\partial F_{1\cdots s}}{\partial t}-[H_{1\cdots s},F_{1\cdots s}]$$
$$=n\operatorname*{Tr}_{s+1}\sum_{i=1}^{s}[V_{i,s+1},F_{1\cdots s+1}],\qquad s=1\cdots N-1. \tag{4}$$

The l.h.s. of each hierarchy equation has the same form as the von Neumann equation, but the r.h.s. includes the coupling to the next higher order of $s+1$ particle quantities. The full hierarchy is equivalent to the von Neumann equation. Interesting features of the complete hierarchy (4) are

(i) time reversibility, and

(ii) conservation of particle number and (total) energy including the potential one; this latter conservation is in contrast to the "usual" Boltzmann equation which conserves only the kinetic energy.

The energy conservation is readily demonstrated. It follows from the first two hierarchy equations. We first consider the mean value of the kinetic energy using the first hierarchy equation,

$$\frac{h}{i}\frac{d}{dt}\langle T\rangle=\frac{n}{2}\operatorname*{Tr}_{12}\left(\frac{p_1^2+p_2^2}{2m}[V_{12},F_{12}]\right)$$
$$=-\frac{n}{2}\operatorname*{Tr}_{12}(V_{12}[H_{12},F_{12}]). \tag{5}$$

Here we took into account that, due to the invariance of the trace,

$$\operatorname*{Tr}_{12}(V_{12}[V_{12},F_{12}])=0.$$

From the second hierarchy equation we obtain

$$\operatorname*{Tr}_{12}(V_{12}[H_{12},F_{12}])$$
$$=-\operatorname*{Tr}_{12}\left(V_{12}\frac{h}{i}\frac{dF_{12}}{dt}\right)-\frac{n}{2}\operatorname*{Tr}_{12}(V_{12}[V_{13}+V_{23},F_{123}]). \tag{6}$$

Using the definition of the potential energy,

$$\langle V\rangle=\frac{n^2}{2}\operatorname*{Tr}_{12}(V_{12}F_{12}), \tag{7}$$

we may write the central equation

$$\frac{d}{dt}(\langle T\rangle+\langle V\rangle)=\frac{n^3}{2}\operatorname*{Tr}_{123}(V_{12}[V_{13}+V_{23},F_{123}]). \tag{8}$$

It is easy to show that the r.h.s. of Eq. (8) vanishes if the three–particle density operator has the property of permutation invariance with respect to the three particles involved [44]

$$P_{123} F_{123} = F_{123}, \tag{9}$$

where P_{123} denotes the three-particle permutation operator. In that case,

$$\frac{d}{dt} (\langle T \rangle + \langle V \rangle) = 0.$$

Constancy of the total energy in the course of the evolution does, of course, not mean constancy of kinetic energy only. In particular, during the buildup of correlations (formation of bound states etc.), potential energy changes, and so does the kinetic energy. Energy conservation and kinetic energy relaxation were investigated numerically by direct integration of the non-Markovian Landau equation in [24] and by molecular dynamics techniques [45].

The permutation invariance is, of course, fulfilled for the exact F_{123} (full hierarchy). We want to underline, however, that the conservation law holds also for any permutation invariant truncation approximation (energy conserving approximation) to the hierarchy. Notice that Eq. (9) is a sufficient condition only, and there are situations not covered by Eq. (4). An important question is, why, contrary to the exact hierarchy, the usual Boltzmann equation does not conserve total energy. We want to consider the question "where" energy conservation is lost. We will show that this property is not determined by the symmetry of the closure relation alone. It is also connected with the time structure of the density operator; i.e., it is essentially connected with the memory effects in the kinetic theory [46].

B. *Generalized Binary Collision Approximation*; *Selfenergy*

The central physical problem in deriving a closed equation for F_1, i.e. a kinetic equation, is the choice of the appropriate decoupling approximation of the Bogolyubov hierarchy. For this purpose it is advantageous to rewrite Eqs. (4) in terms of correlation operators $g_{1 \dots s}$ which are introduced by

$$F_{12} = F_1 F_2 + g_{12}, \tag{10}$$

$$F_{123} = F_1 F_2 F_3 + F_1 g_{23} + F_2 g_{13} + F_3 g_{12} + g_{123}, \tag{11}$$

and so on. The first two equations of the hierarchy now read

$$ih \frac{d}{dt} F_1 - [H_1 + \Sigma_1^{\mathrm{HF}}, F_1] = n \operatorname*{Tr}_2 [V_{12}, g_{12}]$$

$$ih \frac{d}{t} g_{12} - [H_{12}^0 + \Sigma_1^{\mathrm{HF}} + \Sigma_2^{\mathrm{HF}} + V_{12}, g_{12}] - [V_{12}, F_1 F_2] \tag{12}$$

$$= n \operatorname*{Tr}_3 \{[V_{13}, F_1 g_{23}] + [V_{23}, F_2 g_{13}] + [V_{13} + V_{23}, g_{123}]\}, \tag{13}$$

where we introduced the Hartree–Fock self-energy operator Σ_1^{HF} and the free two-particle Hamiltonian H_{12}^0 by

$$\Sigma_1^{\mathrm{HF}} = n \operatorname*{Tr}_2 (V_{12} F_2), \qquad H_{12}^0 = H_1 + H_2 . \tag{14}$$

Equations (12) and (13) are exact. They couple to the rest of the hierarchy via the three-particle correlation operator. The hierarchy truncation problem is now reduced to find approximations for g_{123}. A commonly used approximation for the derivation of kinetic equations is $g_{123} = 0$. This means that the many-particle system is fully determined by the one-particle and two-particle density operators. Any pair of particles does not feel the rest of the system. As a result, two-particle states and also initial correlations have an infinite lifetime. That is, of course, an unphysical long-time behavior, as we will discuss below in detail.

The solution is, to take a different approximation for g_{123}, which accounts for the interaction of the particles 1 and 2 with the surrounding medium, i.e. self-energy (damping) effects. In order to include self-energy in binary collision approximation, the equation of motion for g_{123} has to be taken in the following approximation:

$$\begin{aligned} ih \frac{d}{dt} g_{123} - \{ H_{123}^{0\mathrm{eff}} g_{123} - g_{123} H_{123}^{0\mathrm{eff}\dagger} \} - [V_{12} + V_{13} + V_{23}, g_{123}] \\ = [V_{12} + V_{23}, F_3 g_{12}] \end{aligned} \tag{15}$$

Thus, we neglected 4–particle contributions related to higher orders in the density (n Tr_4-terms) and accounted only for interactions between particle 1 with particle 3, and 2 with 3, respectively, retaining the full correlation g_{12} between particles 1 and 2. Notice that the three-particle ladder terms (third term on the l.h.s.) are included to achieve a consistent treatment of the collision integral and self-energy [38, 48]. The simpler case of self-energy in the second Born approximation has been developed in [27, 26, 49] and is studied in detail in [50].

Damping effects have also been derived for the electron–phonon problem in [22].

The effective Hamiltonian $H_{123}^{0\mathrm{eff}}$ is defined as

$$H_{123}^{0\mathrm{eff}} = \bar{H}_1 + \bar{H}_2 + \bar{H}_3, \tag{16}$$

where $\bar{H}_1$ is the effective one-particle Hamiltonian, which is yet to be determined. This means, we follow a self-consistent scheme, as a result of which we will obtain the effective one-particle, two-particle, and three-particle Hamiltonians and the corresponding propagators. We obtain the formal solution of Eq. (15) for g_{123},

$$g_{123}(t) = \frac{1}{ih} \int_{t_0}^{\infty} d\bar{t}\, U_{123}^{+}(t\bar{t}) [V_{13} + V_{23}, F_3 g_{12}]_{\bar{t}} U_{\bar{1}\bar{2}\bar{3}}^{-}(\bar{t}t), \tag{17}$$

where the initial value term has been dropped. The retarded propagator U^{+}_{123} is the solution of the equation

$$i\hbar \frac{d}{dt} U^{+}_{123}(tt') - (H^{0\text{eff}}_{123} + V_{12} + V_{13} + V_{23})\, U^{+}_{123}(tt') = i\hbar\delta(t-t'), \tag{18}$$

and the advanced propagator U^{-}_{123} obeys the adjoint equation and is related to the retarded propagator by $U^{-}_{123}(t, t') = [\, U^{+}_{123}(t', t)\,]^{\dagger}$.

Expression (17) for g_{123} has to be inserted into the equation for g_{12}, Eq. (13),

$$i\hbar \frac{d}{d} g_{12} - [\, H_1 + \Sigma^{\text{HF}}_1 + H_2 + \Sigma^{\text{HF}}_2 + V_{12}, g_{12}\,] - [\, V_{12}, F_1 F_2\,]$$

$$= \frac{n}{i\hbar} \operatorname*{Tr}_3 \left[V_{13} + V_{23}, \int_{t_0}^{\infty} d\bar{t} U^{+}_{123}(t\bar{t})[\, V_{13} + V_{23}, F_3(\bar{t})\, g_{12}(\bar{t})\,]\, U^{-}_{123}(\bar{t}t) \right]. \tag{19}$$

Since we are interested in the binary collision approximation, we neglected the polarization terms (first and second terms on the r.h.s. of Eq. (13)). Neglecting under the trace off-diagonal contributions (with products $V_{13} V_{23}$ which correspond to higher order correlations), there remain four (due to the commutator) terms that contain V^2_{13} or V^2_{23}. We consider one of them in detail:

$$S_1(t) = \frac{n}{i\hbar} \operatorname*{Tr}_3 \int_{t_0}^{\infty} d\bar{t}\, V_{13} U^{+}_{123}(t\bar{t})\, V_{13} F_3(\bar{t})\, g_{12}(\bar{t})\, U^{-}_{123}(\bar{t}t). \tag{20}$$

In consistency with approximation (15), the propagators in Eq. (20) factorize according to

$$V_{13}\, U^{+}_{123} = V_{13}\, U^{+}_{2}\, U^{+}_{13}, \qquad g_{12}\, U^{-}_{123} = g_{12}\, U^{-}_{12} U_3{}^{-}. \tag{21}$$

Again, the two-particle propagators U^{+}_{12} etc. are still unknown and will be determined self-consistently with the corresponding one- and two-particle equations, that will be discussed in the next section. With the factorizations (21), S_1 can be rewritten in a more familiar way,

$$S_1(t) = \int_{t_0}^{\infty} d\bar{t}\, \Sigma^{+}_1(t\bar{t})\, U^{+}_2(t\bar{t})\, g_{12}(\bar{t})\, U^{-}_{12}(\bar{t}t). \tag{22}$$

Here we introduced the retarded self-energy function

$$\Sigma^{+}_1(t\bar{t}) = \frac{n}{i\hbar} \operatorname*{Tr}_3 \{\, V_{13} U^{+}_{13}(t\bar{t})\, V_{13} F_3(\bar{t})\, U^{-}_3(\bar{t}t)\}. \tag{23}$$

We will see immediately, that Σ^{+}_1, in fact, renormalizes the one-particle energies, so that it is appropriate to call it self-energy. As we will show below, this quantity is indeed the well-known retarded self-energy function, familiar from Green's functions theory, here, given in binary collision approximation.

Let us now insert the four expressions of type S_1 into the second hierarchy equation. The equation for g_{12} has again the form of the binary collision approximation, but now with renormalized one-particle Hamiltonians,

$$ih\frac{d}{dt}g_{12}-\{(\bar{H}_1+\bar{H}_2)\,g_{12}-g_{12}\,(\bar{H}_1^{\dagger}+\bar{H}_2^{\dagger})\}-[V_{12},g_{12}]=[V_{12},F_1F_2], \tag{24}$$

where the effective single-particle Hamiltonian $\bar{H}_1$ (quasi–particle Hamiltonian) is given by

$$\bar{H}_1\,g_{12}=H_1\,g_{12}+\Sigma_1^{\mathrm{HF}}\,g_{12}+\int_{t_0}^{\infty}d\bar{t}\,\Sigma_1^{+}(t\bar{t})\;U_2^{+}(t\bar{t})\,g_{12}(\bar{t})\;U_{12}^{-}(\bar{t}t). \tag{25}$$

Thus, we have generalized the binary collision approximation in an important direction. Having included self-energy effects, we took into account an essential feature of many-particle systems, the coupling of the particle pair 1–2 to the surrounding medium. As a result, the energy of the pair will be renormalized and become complex, showing a finite lifetime of the two-particle state.

C. (*Anti*)-*Symmetrization for Fermions/Bosons*

Another many-particle effect of quantum systems, which has to be incorporated, is related to the spin statistics theorem. For Bose or Fermi systems, we have to consider in Eq. (24) only two-particle states belonging to the subspaces $\mathscr{H}^{\pm}$, i.e., symmetric or antisymmetric subspaces of the Hilbert space $\mathscr{H}$, respectively. This fact leads to a restricted availability of the phase space referred to as Pauli-blocking and to exchange effects [51]. An alternate approach is to work in the full Hilbert space, but to modify the operators instead. According to Dufty and Boercker [43], Pauli-blocking and exchange may be accounted for by a modification of the Hamiltonian and the interaction potential leading to a modification of the equations of motion for F_1, g_{12}, and g_{123}, Eqs. (12), (13), (15). We will use the latter concept. We have then, instead of Eq. (24),

$$ih\frac{d}{dt}\,g_{12}(t)-(H_{12}^{\mathrm{eff}}\,g_{12}-g_{12}\,H_{12}^{\mathrm{eff}\dagger})=(N_{12}\,V_{12}F_{12}^{<}-F_{12}^{<}V_{12}\,N_{12})\,\Lambda_{12}^{\pm}. \tag{26}$$

Here and in the following, we shall use the notations

$$F_1^{<}=F_1,\qquad F_1^{>}=1\pm nF_1 \tag{27}$$

$$F_{12}^{\gtrless}=F_1^{\gtrless}F_2^{\gtrless}. \tag{28}$$

The effective two-particle Hamiltonian in Eq. (26) accounts for both, self-energy and Pauli-blocking,

$$H_{12}^{\mathrm{eff}}=\bar{H}_1+\bar{H}_2+N_{12}\,V_{12}\,, \tag{29}$$

and the Pauli-blocking operator reads for bosons (+) and fermions (−)

$$N_{12} = 1 \pm nF_1 \pm nF_2 \,. \tag{30}$$

$\Lambda_{12}^{\pm}$ is the two-particle exchange operator, $\Lambda_{12}^{\pm} = 1 \pm P_{12}$, with the permutation operator P_{12} defined by $P_{12}\,|12\rangle = |21\rangle$. Furthermore, the Hartree–Fock term in $\bar{H}_1$ explicitly contains an exchange part,

$$\Sigma_1^{\mathrm{HF}} = n \operatorname*{Tr}_2 (V_{12} F_2)\, \Lambda_{12}^{\pm}. \tag{31}$$

We mention that we have to perform an (anti-)symmetrization also in the third hierarchy equation (15). Without going into details, we give the final result. The three-particle Hamiltonian has to be decomposed as

$$H_{123}^{\mathrm{eff}} = \bar{H}_1 + \bar{H}_2 + \bar{H}_3 + N_{12} V_{12} + N_{13} V_{13} + N_{23} V_{23} \,, \tag{32}$$

and in the inhomogeneity on the r.h.s. of Eq. (15) we have to replace (for details see [50])

$$V_{13} F_3 g_{12} \to N_{13} V_{13} F_3 g_{12} \mp F_1 F_3 V_{13} g_{12} \tag{33}$$

and similarly for the terms with $1 \leftrightarrow 2$. This gives rise to Pauli-blocking and exchange contributions in the retarded self-energy Σ^{+} and to an additional contribution (due to the second term on the r.h.s. of Eq. (33)),

$$\Sigma_1^{+}(t\bar{t}) = \frac{n}{i\hbar} \operatorname*{Tr}_3 \left\{ V_{13} U_{13}^{+}(t\bar{t}) (N_{13} V_{13} F_3 \mp F_1 F_3 V_{13}) |_{\bar{t}}\; U_3^{-}(\bar{t}t) \right\} \Lambda_{13}^{\pm} \,. \tag{34}$$

In Section IV, this expression will be transformed to the familiar self-energy in binary collision approximation, introducing the concepts of scattering theory and the T-operator.

Let us summarize the main results of this section. We introduced a new closure to the BBGKY-hierarchy, given by Eq. (15) for g_{123}. This allowed us to incorporate self-energy into the density operator formalism. Moreover, we derived equations of motion for the one- and two-particle density operators, given by Eqs. (12) and (26). This system is closed and includes the complete binary collision scenario (ladder approximation) and, moreover, essential many-particle effects, such as self-energy and Pauli-blocking. The system (12), (26) describes the coupled dynamics of the distribution function and the binary correlations. It is applicable to the whole time range, including short times, where $t < \tau^{\mathrm{corr}}$. Notice that Eqs. (12), (26) are local in time. If supplemented with initial conditions for F_1 and g_{12}, they define an initial value problem for these quantities which, in principal, is well suited for numerical analysis. However, this is currently not feasable. We proceed differently and consider first a formal solution for g_{12} (Section III) which can be inserted into the equation for F_1 to yield a closed kintic equation (Section IV).

III. TWO-PARTICLE DENSITY OPERATOR IN GENERALIZED BINARY COLLISION APPROXIMATION

A. *Formal Solution for the Binary Correlation Operator*

The most important quantity of a many-particle system with binary interactions is the reduced two-particle density operator F_{12}. This operator determines the mean value of the potential energy and the collision integral of kinetic equations for the one-particle density operator F_1. If we include degeneracy effects associated with Bose or Fermi statistics and the self-energy contribution in binary collision approximation, as discussed in the previous section, the equation for the binary correlation operator g_{12} can be written as

$$ih\frac{d}{dt}g_{12}-\{H_{12}^{\mathrm{eff}}(t)\,g_{12}-g_{12}H_{12}^{\mathrm{eff}\dagger}(t)\}$$
$$=\{N_{12}(t)\;V_{12}F_{12}^{<}(t)-F_{12}^{<}(t)\;V_{12}N_{12}(t)\} \tag{35}$$

with the initial condition

$$g_{12}(t)\,|_{t=t_0}=g_{12}^0.$$

Here and in the following, we drop the exchange terms ($\Lambda_{12}^{\pm}\to 1$), since they do not alter our derivation. Equations (35) and (36) define an initial value problem for $g_{12}(t)$, with t_0 being an arbitrary starting point for the evolution. Moreover, in principle, arbitrary values for g_{12}^0 are possible, with the only restriction that g_{12}^0 has to be consistent with the initial value of F_1. This allows one to describe a great variety of nonequilibrium situations which are of interest as a starting state for the relaxation [52].

The formal solution of the linear problem (35), (36) reads

$$g_{12}(t)=U_{12}^{+}(tt_0)\;g_{12}^0\;U_{12}^{-}(t_0t)$$
$$+\frac{1}{ih}\int_{t_0}^{\infty}d\bar{t}\;U_{12}^{+}(t\bar{t})\;[N_{12}(\bar{t})V_{12}F_{12}^{<}(\bar{t})-F_{12}^{<}(\bar{t})V_{12}N_{12}(\bar{t})]\;U_{12}^{-}(\bar{t}t). \tag{37}$$

The first term on the r.h.s. follows from the homogeneous part of Eq. (35) and describes the dynamics of initial correlations according to the effective two-particle Hamiltonian H_{12}^{eff}. The second term in Eq. (37) gives the contribution of two-particle correlations being built up after the initial time t_0, during the relaxation process.

It is interesting to note that the solution (37) describes a reversible dynamics of the statistical quantity $g_{12}(t)$, if self-energy effects beyond Hartree–Fock are neglected. Then Eq. (35) itself is invariant with respect to reversal of time. It is another attractive feature of the density operator approach that the transition from reversible to irreversible dynamics can be traced in detail. Irreversibility follows if

the initial value problem is replaced by an asymptotic condition for $t_0 \to -\infty$, as it was introduced by Bogolyubov who considered the special case of total weakening of initial correlations $g_{12}^0 = 0$ [53]. Less restrictive conditions are possible too (partial weakening of initial correlations) [14, 15]. This allows one to consider the effect of long-living correlations, such as bound states or large-scale fluctuations etc. If self-energy effects beyond Hartree–Fock are included, the effective Hamiltonian is no longer hermitian. Then the question of irreversibility is more subtle and depends on the evolution of the eigenvalues of H_{12}^{eff}.

Despite the simple form of the solution (37), its internal structure (basically the propagators $U_{12}^{\pm}$) is quite complex. This is due to the complicated time dependence of the quantities in Eq. (35) and the fact that the Hamilton operator contains the potential (binary collision approximation). Therefore, a detailed analysis of Eq. (37) is necessary; that is what we are going to do now. U_{12}^{+} and U_{12}^{-} are retarded and advanced effective two-particle propagators, respectively. The retarded one satisfies the differential equation

$$\left(i\hbar \frac{d}{dt} - H_1 - H_2 - N_{12}(t)\, V_{12} \right) U_{12}^{+}(tt') - \int_{t_0}^{\infty} d\bar{t} [\Sigma_1^{+}(t\bar{t})\, U_2^{+}(t\bar{t}) + \Sigma_2^{+}(t\bar{t})\, U_1^{+}(t\bar{t})]\, U_{12}^{+}(\bar{t}t') = i\hbar\delta(t-t') \tag{38}$$

with the self-energies Σ_1^{+}, Σ_2^{+} given by Eq. (34). The propagators $U_{12}^{+}(tt')$ and $U_{12}^{-}(tt')$ are related by

$$[U_{12}^{+}(tt')]^{\dagger} = U_{12}^{-}(t't). \tag{39}$$

Therefore, all relations for an advanced quantity which will be needed below, may be obtained from the equation for the corresponding adjoint quantity. Obviously, Eq. (38) represents a Schrödinger-like equation of motion for interacting quasiparticles with a free two–particle Hamiltonian modified by the one-particle self-energy contributions Σ_1^{+} and Σ_2^{+}. In the case of an attractive binary interaction, it includes the possibility of bound states which are modified by medium effects.

B. *One-Particle Propagators and the Renormalized Energy Spectrum*

Before continuing the analysis of Eq. (38), let us consider the free propagators $U_{12}^{0\pm}$ which obey Eq. (38) without the interaction potential,

$$\left(i\hbar \frac{d}{dt} - H_1 - H_2 \right) U_{12}^{0+}(tt') - \int_{t_0}^{\infty} d\bar{t}\, [\Sigma_1^{+}(t\bar{t})\, U_2^{+}(t\bar{t}) + \Sigma_2^{+}(t\bar{t})\, U_1^{+}(t\bar{t})]\, U_{12}^{0+}(\bar{t}t') = i\hbar\delta(t-t'). \tag{40}$$

The structure of this equation suggests making the ansatz for U_{12}^{0+}

$$U_{12}^{0+}(tt') = U_1^+(tt')\, U_2^+(tt').$$

The equation for the single-particle propagators is

$$\left(ih\frac{d}{dt} - H_1\right) U_1^+(tt') - \int_{t_0}^{\infty} d\bar{t}\, \Sigma_1^+(t\bar{t})\, U_1^+(\bar{t}t') = ih\delta(t-t'). \tag{41}$$

This equation corresponds to the well-known Dyson equation of Green's function theory with a self-energy function here in binary collision approximation, according to Eq. (34) ($U^{\pm}$ correspond to the retarded/advanced Green's functions $ihG^{R/A}$).

We give a brief discussion of some important properties of $U_1^{\pm}$, starting from the coordinate representation of Eq. (41). For this purpose it is useful to introduce the microscopic and macroscopic variables by $r = r_1 - r_1'$, $\tau = t - t'$ and $R = \frac{1}{2}(r_1 + r_1')$, $T = \frac{1}{2}(t + t')$, respectively. The Fourier transform with respect to the microscopic variables is then defined by

$$U_1^{\pm}(p\omega, RT) = \int dr\, d\tau\, e^{-(i/h)\,pr + i\omega\tau} U_1^{\pm}(r\tau, RT).$$

The analytic properties of the propagators are well known. First, the propagators may be continued analytically into the complex ω-plane. The analytic continuation of $U_1^{\pm}$ may be written as a Cauchy-type integral

$$U_1^{\pm}(pz, RT) = i \int \frac{d\bar{\omega}}{2\pi} \frac{A_1(p\bar{\omega}, RT)}{z - \bar{\omega}} \tag{42}$$

with the spectral function

$$A_1(p\omega, RT) = U_1^+(p\omega, RT) - U_1^-(p\omega, RT). \tag{43}$$

$U_1^{\pm}$ are analytic in the upper/lower half plane and may be continued into the lower (upper) half plane by

$$U_1^{\pm}(pz, RT) = U_1^{\mp}(pz, RT) \pm A_1(pz, RT).$$

The propagators can be determined explicitly, if the local approximation is applied to Eq. (41), i.e., if all quantities depend only on the difference variables. Then, the solution of Eq. (41) is given by

$$U_1^{\pm}(p\omega, RT) = 1 \Big/ \left(\hbar\omega - \frac{p^2}{2m} - \Sigma_1^{\pm}(p\omega, RT) \pm i\varepsilon\right). \tag{44}$$

Using the result of Eq. (44) and Eq. (43), we obtain for the spectral function

$$A_1(p\omega, RT) = \frac{\gamma_1(p\omega, RT)}{[\hbar\omega - p^2/2m - \mathrm{Re}\,\Sigma_1^+(p\omega, RT)]^2 + [\frac{1}{2}\gamma_1(p\omega, RT)]^2}. \quad (45)$$

with $\gamma_1 = -2\,\mathrm{Im}\,\Sigma_1^+$. This is a rather general result. In order to demonstrate the meaning of the spectral function, Eq. (45), we consider a further simplification; we calculate $\mathrm{Re}\,\Sigma_1^+$ and γ_1, substituting the argument $\hbar\omega \to p^2/2m$. This leads to a Lorentz shape of the spectral function,

$$A_1(p\omega RT) = \frac{\gamma_1(pRT)}{[\hbar\omega - E_1(pRT)]^2 + [\frac{1}{2}\gamma_1(pRT)]^2}. \quad (46)$$

This result for the spectral function yields the following expression for the single-particle propagators:

$$U_1^{\pm}(\tau) = \Theta(\pm\tau) e^{-(i/\hbar)(E_1 \mp i\gamma_1)\tau}. \quad (47)$$

Although Eq. (47) has the familiar exponential form, there are some important differences in comparison to the propagator of a free particle. We have effective one-particle energies given by

$$E_1(pRT) = \frac{p^2}{2m} + \mathrm{Re}\;\Sigma_1^+(p\omega RT)|_{\omega = E_1(pRT)} \quad (48)$$

and damping of the one-particle states given by the imaginary part of the self-energy, γ_1. Therefore Eq. (47) describes the propagator of damped quasiparticles.

C. *Connection between the Propagators and the Scattering Matrix*

After having determined the quasiparticle propagators, let us now return to the exact equations, Eqs. (38) and (40). The appropriate theoretical formalism to describe binary collisions in a many-particle system is given by a generalization of the quantum scattering theory. This requires us to define the quantities of scattering theory, such as the Møller operator and the scattering operator (T-operator) and to generalize them to scattering in a medium. To do this, it is convenient to transform the differential equations for the two-particle propagators $U_{12}^{\pm}$ into integral equations.

It is easy to prove that the effective two–particle propagator obeys the following integral equations:

$$\begin{aligned} U_{12}^{+}(tt') &= U_{12}^{0+}(tt') - \frac{i}{\hbar}\int_{-\infty}^{\infty} d\bar{t}\; U_{12}^{0+}(t\bar{t})\, N_{12}(\bar{t})\, V_{12}\, U_{12}^{+}(\bar{t}t') \\ &= U_{12}^{0+}(tt') - \frac{i}{\hbar}\int_{-\infty}^{\infty} d\bar{t}\; U_{12}^{+}(t\bar{t})\, N_{12}(\bar{t})\, V_{12}\, U_{12}^{0+}(\bar{t}t'). \end{aligned} \quad (49)$$

The corresponding equations for U_{12}^{-} follow from the adjoint equations, using the property (39). Equation (49) has a clear physical meaning. While the first term on the r.h.s. is related to free quasi-particles (which is exact in the case of weak coupling or Born approximation), the integral terms account for the coupling between the two particles. Equation (49) is a many-particle generalization of the propagator equation of scattering theory [54, 55]. In particular, it contains in addition the Pauli-blocking factors $N_{12}(t)$. To establish a closer relation to standard scattering theory, it is useful to consider, instead of $U_{12}^{\pm}$, new propagators defined as

$$G_{12}^{\pm}(tt') = \pm\Theta[\pm(t-t')]\{\mathscr{G}_{12}^{>}(tt') - \mathscr{G}_{12}^{<}(tt')\} \tag{50}$$

with

$$\mathscr{G}_{12}^{\gtrless}(tt') = U_{12}^{+}(tt')\, F_{12}^{\gtrless}(t') + F_{12}^{\gtrless}(t) U_{12}^{-}(tt'). \tag{51}$$

Then we can rewrite Eq. (50) as

$$G_{12}^{\pm}(tt') = \pm\Theta[\pm(t-t')]\{U_{12}^{+}(tt')N_{12}(t') + N_{12}(t)U_{12}^{-}(tt')\}. \tag{52}$$

We emphasize that the definition of the new quantities $\mathscr{G}_{12}^{\gtrless}$ does not contain any approximations. Thus the equations for $\mathscr{G}_{12}^{\gtrless}$ are fully equivalent to the equations for $U_{12}^{\pm}$. Similarly, we have for the free propagators $G_{12}^{0\pm}$,

$$G_{12}^{0\pm}(tt') = \pm\Theta[\pm(t-t')]\{\mathscr{G}_{12}^{0>}(tt') - \mathscr{G}_{12}^{0<}(tt')\} \tag{53}$$

with

$$\mathscr{G}_{12}^{0\gtrless}(tt') = U_{12}^{0+}(tt')\, F_{12}^{\gtrless}(t') + F_{12}^{\gtrless}(t)\, U_{12}^{0-}(tt') \tag{54}$$

and also

$$G_{12}^{0\pm}(tt') = \pm\Theta[\pm(t-t')]\{U_{12}^{0+}(tt')\, N_{12}(t') + N_{12}(t)\, U_{12}^{0-}(tt')\} \tag{55}.$$

The propagators $G_{12}^{\pm}$ and $G_{12}^{0\pm}$ have the advantage to "absorb the spin statistics," contained in the Pauli-blocking factors N_{12}, so that the structure of the resulting equations is now similar to that of quantum scattering theory for spinless particles. But in comparison to conventional scattering theory, $G_{12}^{\pm}$ describe *in-medium scattering*, accounting for Bose or Fermi statistics and self-energy effects. Using Eq. (49) and the definition (52), we obtain an integral equation for G_{12}^{+}:

$$G_{12}^{+}(tt') = G_{12}^{0+}(tt') - \frac{i}{\hbar}\int_{-\infty}^{+\infty} d\bar{t}\; G_{12}^{0+}(t\bar{t})\; V_{12} G_{12}^{+}(\bar{t}t'). \tag{56}$$

As before, the adjoint equation yields the integral equation for the advanced propagator G_{12}^{-}.

It is useful to rewrite (56) as

$$G_{12}^{+}(tt') = \int_{-\infty}^{+\infty} d\bar{t}\; G_{12}^{0+}(t\bar{t}) \left\{ \delta(t'-\bar{t}) - \frac{i}{\hbar}\; V_{12} G_{12}^{+}(\bar{t}t') \right\}. \tag{57}$$

To make use of the methods of quantum scattering theory, we now introduce a generalized Møller operator Ω_{12}^{+} by

$$\Omega_{12}^{+}(tt') = \delta(t-t') - \frac{i}{\hbar}\; G_{12}^{+}(tt')\; V_{12} \tag{58}$$

$$= \delta(t-t') - \frac{i}{\hbar}\; U_{12}^{+}(tt')\; N_{12}\; V_{12}. \tag{59}$$

The integral equation for the propagator G_{12}^{+} can be written in a compact form

$$G_{12}^{+}(tt') = \int_{-\infty}^{+\infty} d\bar{t}\; \Omega_{12}^{+}(t\bar{t})\; G_{12}^{0+}(\bar{t}t'). \tag{60}$$

In the same manner, we get for the advanced quantity

$$G_{12}^{-}(tt') = \int_{-\infty}^{+\infty} d\bar{t}\; G_{12}^{0-}(t\bar{t})\; \Omega_{12}^{-}(\bar{t}t'). \tag{61}$$

Analogously, we obtain for the propagators $U_{12}^{\pm}$

$$U_{12}^{+}(tt') = \int_{-\infty}^{+\infty} d\bar{t}\; \Omega_{12}^{+}(t\bar{t})\; U_{12}^{0+}(\bar{t}t') \tag{62}$$

and

$$U_{12}^{-}(tt') = \int_{-\infty}^{+\infty} d\bar{t}\; U_{12}^{0-}(t\bar{t})\; \Omega_{12}^{-}(\bar{t}t'). \tag{63}$$

Finally, let us introduce the central quantity of the binary collision approximation, the T-operator, by defining

$$T_{12}^{+}(tt') = V_{12}\Omega_{12}^{+}(tt'); \qquad T_{12}^{-}(tt') = \Omega_{12}^{-}(tt')V_{12}\,. \tag{64}$$

With Eqs. (60) and (64), we can express the propagator G^{+} by the T-operator

$$V_{12} G_{12}^{+}(tt') = \int_{-\infty}^{\infty} d\bar{t}\; T_{12}^{+}(t\bar{t})\; G_{12}^{0+}(\bar{t}t'), \tag{65}$$

and with Eqs. (56) and (58), also the Møller operator can be written in terms of the T-operator,

$$\Omega_{12}^{+}(tt') = \delta(t-t') - \frac{i}{\hbar}\int_{-\infty}^{\infty} d\bar{t}\; G_{12}^{0+}(t\bar{t})\; T_{12}^{+}(\bar{t}t'). \tag{66}$$

What is left now, is to derive the two fundamental equations for the generalized T-operator, the Lippmann–Schwinger equation and the optical theorem. First, combining Eqs. (60), (61), (64), and (65), we obtain the well-known Lippmann–Schwinger equation:

$$\begin{aligned} T_{12}^{+}(tt') &= V_{12}\delta(t-t') - \frac{i}{\hbar}\; V_{12} G_{12}^{+}(tt')\; V_{12} \\ &= V_{12}\delta(t-t') - \frac{i}{\hbar}\int_{-\infty}^{+\infty} d\bar{t}\; V_{12}\; G_{12}^{0+}(t\bar{t})\; T_{12}^{+}(\bar{t}t'). \end{aligned} \tag{67}$$

We will further need the corresponding equation for the advanced operator,

$$\begin{aligned} T_{12}^{-}(tt') &= V_{12}\delta(t-t') - \frac{i}{\hbar}\; V_{12} G_{12}^{-}(tt')\; V_{12} \\ &= V_{12}\delta(t-t') - \frac{i}{\hbar}\int_{-\infty}^{+\infty} d\bar{t}\; T_{12}^{-}(t\bar{t})\; G_{12}^{0-}(\bar{t}t)\; V_{12}\,. \end{aligned} \tag{68}$$

Second, we derive from Eqs. (67) and (68) the optical theorem in time representation

$$T_{12}^{+}(tt') - T_{12}^{-}(tt') = -\frac{i}{\hbar}\int d\bar{t}\, d\bar{\bar{t}}\; T_{12}^{+}(t\bar{t})\{G_{12}^{0+}(\bar{t}\bar{\bar{t}}) - G_{12}^{0-}(\bar{t}\bar{\bar{t}})\}\; T_{12}^{-}(\bar{\bar{t}}t'). \tag{69}$$

The difference of the quasiparticle propagators entering Eq. (69) can be rewritten in terms of the propagators $U^{0\pm}$, according to Eq. (55),

$$G_{12}^{0+}(tt') - G_{12}^{0-}(tt') = U_{12}^{0+}(tt')\; N_{12}(t') - N_{12}(t)\; U_{12}^{0-}(tt').$$

With the above equations, we have obtained a closed system for the scattering quantities and quasiparticle propagators. It should be mentioned again that, in comparison to scattering theory for an isolated pair of particles, here, in-medium effects are incorporated (cf. Eqs. (67) and (69)). With the generalized two-particle propagators G_{12}^{+} and G_{12}^{0+}, many-body effects such as self-energy and degeneracy due to Bose or Fermi statistics are taken into account, providing for an important extension of conventional scattering theory.

D. *Correlation Operator in Binary Collision Approximation*

Let us now return to the determination of the two-particle correlation operator $g_{12}(t)$ which is given by the formal solution (37). Using the relations (62), (59), and (63), it is easy to show that $g_{12}(t)$ can be written in the form

$$g_{12}(t)+F_{12}^{<}(t)=\int_{t_0}^{\infty}\int_{t_0}^{\infty} d\bar{t}\, d\bar{\bar{t}}\, \Omega_{12}^{+}(t\bar{t})\, U_{12}^{0+}(\bar{t}t_0)\, g_{12}(t_0)\, U_{12}^{0-}(t_0\bar{\bar{t}})\, \Omega_{12}^{-}(\bar{\bar{t}}t)$$

$$+\int_{t_0}^{\infty}\int_{t_0}^{\infty} d\bar{t}\, d\bar{\bar{t}}\, \Omega_{12}^{+}(t\bar{t})\, \mathscr{G}_{12}^{0<}(\bar{t}\bar{\bar{t}})\, \Omega_{12}^{+}(\bar{\bar{t}}t), \tag{70}$$

where the generalized Møller operators $\Omega_{12}^{\pm}$ were defined by Eq. (58) and the operator $\mathscr{G}_{12}^{0<}$ is given by

$$\mathscr{G}_{12}^{0<}(tt')=U_{12}^{0+}(tt')\, F_{12}^{<}(t')+F_{12}^{<}(t)\, U_{12}^{0-}(tt'). \tag{71}$$

Notice that the sum on the l.h.s of Eq. (70) is just the reduced two-particle density operator F_{12}. With Eq. (70) we obtained an exact solution of the Bogolyubov hierarchy on the level of the binary collision approximation. Especially, there is no restriction with respect to the time. All nonequilibrium properties of the many-particle system can be derived from this expression in well-known manner. So, we can determine the collision integral in the equations of motion for the single-particle density operator, Eq. (12), which allows us to calculate the time evolution of the distribution function and all one-particle observables. Furthermore, it is possible to evaluate the dynamics of all two-particle properties from g_{12}, e.g., the mean potential energy, Eq. (7).

At this point it is instructive to discuss some properties of the solution given by the expression (70). First, we have to notice here that the quantities $U_{12}^{0\pm}$ are two-particle propagators of free damped quasiparticles. In general, these propagators are to be determined from Eq. (40). In the simplest approximation, we get, according to Eq. (47),

$$U_{12}^{0\pm}(\tau)=\Theta(\pm\tau)\, e^{-(i/\hbar)[E_{12}\mp i(\gamma_{12})]\tau}, \tag{72}$$

where we introduced the short notation $E_{12}=E_1+E_2$ and $\gamma_{12}=\gamma_1+\gamma_2$.

Second, the binary correlation operator is influenced by its value at $t=t_0$, which means $g_{12}(t)$ depends on the dynamics of a correlated initial state. This contribution follows from the first term on the r.h.s. of Eq. (70). But, the effect of the initial correlations is weakened because the quasiparticle propagators are damped. This can be seen most easily from approximation (72). Then, the initial correlation term reads

$$\int_{t_0}^{\infty}\int_{t_0}^{\infty} d\bar{t}\, d\bar{\bar{t}}\, \Omega_{12}^{+}(t\bar{t})\, e^{-(i/\hbar)[E_{12}-i\gamma_{12}](\bar{t}-t_0)}\, g_{12}(t_o)\, e^{-(i/\hbar)[E_{12}+i\gamma_{12}](t_0-\bar{\bar{t}})}\, \Omega_{12}^{-}(\bar{\bar{t}}t)$$

which gives us an estimate for the time scale on which the initial correlations decay,

$$\tau_{\text{corr}} \sim 1/\gamma_{12} \,. \tag{73}$$

Therefore, for $t \gg \tau_{\text{corr}}$, the Bogolyubov assumption of weakening of initial correlations holds, and the nonequilibrium properties of the many-particle system can be described by simpler "conventional" Markovian equations. Our result is of interest for the understanding of the short-time behavior of the system. It shows that the Bogolyubov (kinetic) regime is established dynamically after relaxation of the correlations (see also [24]). We thus made important progress: We do not need to postulate the Bogolyubov condition; it follows from our approach directly as a result of the dynamics of the system confirming the assumption made by Bogolyubov and others.

We want to mention, however, that the damping of one- and two-particle states is, in general, very complex. Only in Born approximation is it reduced to one-particle damping. Otherwise, the propagators $U_{12}^{0\pm}$, as derived in our approach, yield only qualitatively correct results for the damping. Furthermore, it is clear that various types of correlations have different decay times. In particular, bound state correlations or large scale fluctuations may have a rather long lifetime. A correct treatment of the latter type of correlations again requires the inclusion of damping effects resulting from two-particle dynamics. We will not dicsuss this problem here but mention the main results of such analysis [14, 38]: It turns out that bound states are affected by the surrounding medium much less than continuum (scattering) states; i.e., the former are damped less than the latter. The reason is a rather complex compensation mechanism between different many-particle (damping) effects for bound states. This compensation does not occur for continuum states.

A third property of the solution (70) is that the binary correlation operator is given by an expression which is nonlocal in time. At the actual time t, the operator is determined not only by its actual value, but also by its values in the past; that means, there are memory effects which can essentially influence the relaxation behavior of the system. This nonlocality can be seen from the last expression on the r.h.s. of Eq. (70) which gives the contribution of correlations built up from the initial time t_0 up to the actual time t. But, if we look more closely at the special correlation function $\mathscr{G}_{12}^{0<}$, Eq. (71), and apply for simplicity approximation (72), we see that the memory has a "finite depth" which is again determined by the damping.

E. *Gradient Expansion of g_{12} and Conservation Laws*

The physical consequences of the memory effects in g_{12} can be conveniently studied if $g_{12}(t)$ is expanded with respect to the retardation in time. In particular, this allows us to evaluate the first corrections to the local (Markovian) behavior in explicit form. Let us discuss this expansion with two simplifying assumptions:

(i) We consider the special case

$$\lim_{t_0 \to -\infty} g_{12}(t_0) = 0$$

(complete weakening of initial correlations).

(ii) The time dependence of the Møller operator and the related retarded and advanced quantities is given by

$$\Omega^{\pm}_{12}(tt') = \Omega^{\pm}_{12}(t - t'),$$

what means that in the scattering quantities Pauli blocking is neglected.

In order to perform the retardation (gradient) expansion we introduce "center of mass" and relative variables, t and τ, respectively,

$$F_{12}(t) = \int_{-\infty}^{+\infty} \int_{-\infty}^{+\infty} d\tau\, d\bar{\tau}\, \Omega^{+}_{12}(\tau)\, \mathcal{G}^{0<}_{12}\left(-(\tau + \bar{\tau}),\, t + \frac{\bar{\tau} - \tau}{2}\right) \Omega^{-}_{12}(\bar{\tau}), \tag{74}$$

where $\tau = t - t'$ and $\bar{\tau} = \bar{\bar{t}} - t'$. Taylor expansion up to first order in the relative times around t and Fourier transformation with respect to τ yields

$$\begin{aligned} F_{12}(t) = & \int \frac{d\omega}{2\pi}\, \Omega^{+}_{12}(\omega)\, \mathcal{G}^{0<}_{12}(\omega, t)\, \Omega^{-}_{12}(\omega) \\ & + \frac{i}{2} \int \frac{d\omega}{2\pi} \left[\frac{d}{d\omega}\, \Omega^{+}_{12}(\omega)\, \frac{\partial}{\partial t}\, \mathcal{G}^{0<}_{12}(\omega, t)\, \Omega^{-}_{12}(\omega) \right. \\ & \left. - \Omega^{+}_{12}(\omega)\, \frac{\partial}{\partial t}\, \mathcal{G}^{0<}_{12}(\omega, t)\, \frac{d}{d\omega}\, \Omega^{-}_{12}(\omega) \right]. \end{aligned} \tag{75}$$

Here, we have to account for the expansion of $\mathcal{G}^{0<}_{12}$ which follows from Eq. (71),

$$\begin{aligned} \mathcal{G}^{0<}_{12}(\tau, t) = & \left[U^{0+}_{12}(\tau) + U^{0-}_{12}(\tau) \right] F^{<}_{12}(t) \\ & - \frac{\tau}{2} \left[U^{0+}_{12}(\tau) - U^{0-}_{12}(\tau) \right] \frac{d}{dt}\, F^{<}_{12}(t). \end{aligned} \tag{76}$$

If, furthermore, the damping in the propagators is being neglected, we obtain after Fourier transformation and using the Dirac identity

$$U^{0+}_{12}(\omega) + U^{0-}_{12}(\omega) = -2\pi i\hbar\, \delta(\hbar\omega - E_{12}),$$

$$U^{0+}_{12}(\omega) - U^{0-}_{12}(\omega) = -2\hbar\, \frac{P}{\hbar\omega - E_{12}},$$

which yields the gradient expansion of $\mathscr{G}^{0<}_{12}$

$$\mathscr{G}^{0<}_{12}(\omega, t) = -2\pi i\hbar\, \delta(\hbar\omega - E_{12})\, F^{<}_{12}(t) + \hbar\, \frac{d}{d\omega}\, \frac{P}{\hbar\omega - E_{12}}\, \frac{d}{dt}\, F^{<}_{12}(t). \tag{77}$$

Here, $E_{12} = E_1 + E_2$ is the two-particle energy. Finally, we can write down the full gradient expansion up to the first order for the binary density operator,

$$\begin{aligned} F_{12}(t) = & -i\, \Omega^{+}_{12}(E_{12})\, F^{<}_{12}(t)\, \Omega^{-}_{12}(E_{12}) \\ & + \hbar \int \frac{d\omega}{2\pi}\, \Omega^{+}_{12}(\omega)\, \frac{d}{d\omega}\, \frac{P}{\hbar\omega - E_{12}}\, \frac{d}{dt}\, F^{<}_{12}(t)\, \Omega^{-}_{12}(\omega) \\ & + \frac{\hbar}{2} \left\{ \frac{d\Omega^{+}_{12}}{dE_{12}}\, \frac{dF^{<}_{12}}{dt}\, \Omega^{-}_{12} - \Omega^{+}_{12}\, \frac{dF^{<}_{12}}{dt}\, \frac{d\Omega^{-}_{12}}{dE_{12}} \right\}. \end{aligned} \tag{78}$$

The first term on the r.h.s. of Eq. (78) represents the so-called local approximation. This contribution leads to the usual quantum Boltzmann collision integral in the equation of the one-particle density operator including self-energy corrections and degeneracy due to Bose or Fermi statistics. The further contributions are the first-order gradient expansion terms.

Having determined the binary density operator, we can calculate all macroscopic observables with correlation contributions in binary collision approximation fully included. For example, let us determine the mean value of the potential energy,

$$\langle V \rangle = \frac{n^2}{2} \operatorname*{Tr}_{12}\, V_{12} F_{12}. \tag{79}$$

With the local approximation for F_{12}, it follows that

$$\langle V \rangle = \frac{n^2}{2} \operatorname*{Tr}_{12} \{ V_{12}\, \Omega^{+}_{12} F^{<}_{12}\, \Omega^{-}_{12} \}. \tag{80}$$

Using the relation between Ω^{+}_{12} and T^{+}_{12} according to Eq. (66) and the invariance of the trace, Eq. (80) may be transformed to

$$\langle V \rangle = -i\, \frac{n^2}{4} \operatorname*{Tr}_{12} \{ T^{+}_{12}(E)\, F^{<}_{12}(t)\, \Omega^{-}_{12}(E) + \Omega^{+}_{12}\, F^{<}_{12}(t)\, T^{-}_{12}(E) \}. \tag{81}$$

Using again Eq. (66), it follows easily that

$$\langle V \rangle = i\, \frac{n^2}{2} \operatorname*{Tr}_{12} \left\{ \operatorname{Re}\, T^{+}_{12}(E)\, F^{<}_{12}(t) + T^{+}_{12}(E)\, \frac{\mathrm{P} N_{12}}{E - \bar{E}}\, F^{<}_{12}(t)\, T^{-}_{12}(E) \right\}. \tag{82}$$

This expression explains once more the character of the approximations used in our theory. From relation (82) we are able to determine the mean value of the potential

energy for an arbitrary nonequilibrium situation once the single-particle density operator $F_1(t)$ is known. Equation (82) is a rather general result. Further simplifications are possible in limiting cases only. In particular, in thermodynamic equilibrium, the density operator is known explicitly, and we can replace

$$n^2 F_{12}^{<} = n^2 F_1 F_2 = n_{12}(1 \pm f_1 \pm f_2), \tag{83}$$

where we introduced the Bose function $n_{12}(\omega) = 1/(\exp[\beta(\hbar\omega - \mu_1 - \mu_2)] - 1)$ and $f_{1,2}$ being Fermi functions. Inserting expression (83) into Eq. (82) we obtain, after partial integration,

$$\langle V \rangle = -\frac{kT}{2} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \ln |1 - z_1 z_2 e^{-\beta\omega}| \operatorname*{Tr}_{12} \left\{ \frac{\partial}{\partial\omega} \operatorname{Im}(G_0^+(\omega + i\varepsilon)\, T^+(\omega + i\varepsilon)) \right\}. \tag{84}$$

Here, $z_{1,2} = \exp[\beta\mu_{1,2}]$ are the fugacities. The corresponding expression for the equation of state may be derived from Eq. (82) using the charging procedure

$$\mathscr{V}(p - p_0) = -\int_0^1 \frac{d\lambda}{\lambda} \langle V \rangle_\lambda. \tag{85}$$

The λ-integration can be carried out with the help of the identity $N_{12}(\partial/\partial\omega)(G_0^+ T^+) = -(\partial/\partial\lambda)(G_0^+ T^+ G_0^+)$. The result is

$$\mathscr{V}(p - p_0) = \frac{kT}{2} \int_{-\infty}^{\infty} \frac{d\omega}{\pi} \ln |1 - z_1 z_2 e^{-\beta\omega}| \operatorname{Im} \operatorname*{Tr}_{12} \left\{ \frac{\partial G_0^+}{\partial\omega} T^+(\omega + i\varepsilon) \right\}. \tag{86}$$

Following the calculations given in Ref. [57], we arrive at the well-known result for the pressure

$$\begin{aligned} \mathscr{V}(p - p_0) = & -\frac{kT}{2} \sum_n \ln |1 - z_1 z_2 e^{-\beta E_n}| \\ & + \frac{kT}{2} \operatorname*{Tr}_{12} \left\{ N_{12} n_{12}(E) \operatorname{Re} T^+(E) + \ln |1 - z_1 z_2 e^{-\beta E}| \, \pi\delta(E - H) \right. \\ & \left. \times i \left(\frac{dT^+(E)}{dE} N_{12} T^-(E) - T^+(E) N_{12} \frac{dT^+(E)}{dE} \right) \right\}. \end{aligned} \tag{87}$$

The first term on the r.h.s. gives the contribution of the bound states. In the special case of a nondegenerate (classical) system, this is just the second cluster coefficient of the fugacity expansion of the pressure. Formulas of this type can be found, e.g., in the text book of Landau and Lifshits [56].

Finally, by using a partial wave expansion of the T-matrix, it follows the Beth–Uhlenbeck representation as shown in Refs. [57, 58].

F. *Recovery of the Generalized Kadanoff–Baym Ansatz*

It is interesting to compare the expression obtained for the binary density operator in binary collision approximation, Eq. (70), with the corresponding result which follows from the theory of nonequilibrium Green's functions [39]. We give a brief summary of this result. Using the Green's function formalism, one can derive an expression of the same form as Eq. (70). The main difference is that the quantity $\mathscr{G}_{12}^{0<}$ in Eq. (70) is now given by a product of two one-particle two-time correlation functions, i.e. (we suppress the momentum arguments),

$$\mathscr{G}_{12}^{0<}(tt') = ig_1^<(tt')\, g_2^<(tt'), \tag{88}$$

while, in the density operator approach, $\mathscr{G}_{12}^{0<}$ was given by Eq. (71). An agreement between our density operator result, Eq. (71), and the Green's function result, Eq. (88), can be achieved only if the one-particle correlation functions in Eq. (88) are "reconstructed" from their value on the time diagonal, $\mp ihg^<(tt) = F(t)$, according to the generalized Kadanoff–Baym ansatz proposed by Lipavský *et al.* [37],

$$\mp ihg^<(tt') = g^{\mathrm{R}}(tt')\, F(t') - F(t)\, g^{\mathrm{A}}(tt'), \tag{89}$$

where "$-(+)$" refers to fermions (bosons). Using this ansatz for $g_1^<$ and $g_2^<$ in Eq. (88), we obtain

$$\mathscr{G}_{12}^{0<}(tt') = \frac{-i}{h^2}\left\{g_1^{\mathrm{R}}(tt')\, g_2^{\mathrm{R}}(tt')\, F_1(t')\, F_2(t') + F_1(t)\, F_2(t)\, g_1^{\mathrm{A}}(tt')\, g_2^{\mathrm{A}}(tt')\right\}, \tag{90}$$

where we used the fact that products of retarded and advanced functions of the same arguments vanish. If we now recall that $U^{0\pm}(tt') = ihG^{\mathrm{R/A}}(tt')$, Eq. (90) coincides with Eq. (71) which was an exact result of the density operator approach. We thus showed that the Green's function formalism agrees with the density operator result only if the reconstruction ansatz of Lipavský *et al.* is used. We want to underline, however, that with the density operator approach we are not required to postulate this ansatz. It is the structure of the binary correlation operator, here in binary collision approximation, Eqs. (70), (71), which is exactly of the form of the product of two generalized Kadanoff–Baym ansatzes. Notice also that this agreement does not depend on the particular choice of the free propagators.

IV. COLLISION INTEGRAL WITH MEMORY EFFECTS

Let us now consider the derivation of the non-Markovian kinetic equation in binary collision approximation. For this, we start from the first equation of the

hierarchy, in which we can now insert the solution for the binary correlation operator, Eq. (70),

$$\frac{d}{dt} F_1(t) = n \operatorname*{Tr}_2 [V_{12}, g_{12}] = I_1^{IC}(t) + I_1(t), \tag{91}$$

where the second term is the conventional collision term and the first one is an additional contribution coming from the initial correlations (second and first integrals on the r.h.s. of Eq. (70), respectively). These collision integrals are given by

$$I_1(t) = n \operatorname*{Tr}_2 \left[V_{12}, \int_{t_0}^{\infty} \int_{t_0}^{\infty} d\bar{t}\, d\bar{\bar{t}}\, \Omega_{12}^{+}(t\bar{t})\, \mathscr{G}_{12}^{0<}(\bar{t}\bar{\bar{t}})\, \Omega_{12}^{-}(\bar{\bar{t}}t) \right] \tag{92}$$

$$I_1^{IC}(t) = n \operatorname*{Tr}_2 \left[V_{12}, \int_{t_0}^{\infty} \int_{t_0}^{\infty} d\bar{t}\bar{\bar{t}}\, \Omega_{12}^{+}(t\bar{t})\, U_{12}^{0+}(\bar{t}t_0)\, F_{12}(t_0)\, U_{12}^{0-}(t_0\bar{\bar{t}})\, \Omega_{12}^{-}(\bar{\bar{t}}t) - F_{12}^{<}(t) \right]. \tag{93}$$

We first consider the integral I_1. The transformation of I_1^{IC} can be performed analogously. The integral I_1 can be cast into a more convenient form by introducing the T-operator which is related to Ω according to Eqs. (64), (66). Then we get

$$\begin{aligned} I_1(t) = n \operatorname*{Tr}_2 \int_{t_0}^{t} d\bar{t} \Big\{ & [T_{12}^{+}(t\bar{t}) - V_{12}\delta(t-\bar{t})]\, \mathscr{G}_{12}^{0<}(\bar{t}t) \\ & + \frac{i}{\hbar} \int_{t_0}^{t} d\bar{\bar{t}}\, dt'\, T_{12}^{+}(t\bar{t})\, \mathscr{G}_{12}^{0<}(\bar{t}\bar{\bar{t}})\, T_{12}^{-}(\bar{\bar{t}}t')\, G_{12}^{0-}(t't) \Big\} \\ & - \text{adjoint}. \end{aligned} \tag{94}$$

Now we transform the first term of (94) with the help of the optical theorem (69) and get

$$\begin{aligned} I_1(t) = \frac{i}{\hbar} n \operatorname*{Tr}_2 \int_{t_0}^{t} d\bar{t}\, d\bar{\bar{t}}\, dt' \{ & T_{12}^{+}(t\bar{t})[G_{12}^{0+}(\bar{t}\bar{\bar{t}}) - G_{12}^{0-}(\bar{t}\bar{\bar{t}})]\, T_{12}^{-}(\bar{\bar{t}}t')\, \mathscr{G}_{12}^{0<}(t't) \\ & - \mathscr{G}_{12}^{0<}(t\bar{t})\, T_{12}^{+}(\bar{t}\bar{\bar{t}})[G_{12}^{0+}(\bar{\bar{t}}t') - G_{12}^{0-}(\bar{t}\bar{\bar{t}}]\, T_{12}^{-}(t't) \\ & + T_{12}^{+}(t\bar{t})\, \mathscr{G}_{12}^{0<}(\bar{t}\bar{\bar{t}})\, T_{12}^{-}(\bar{\bar{t}}t')\, G_{12}^{0-}(t't) - G_{12}^{0+}(t\bar{t})\, T_{12}^{+}(\bar{t}\bar{\bar{t}})\, \mathscr{G}_{12}^{0<}(\bar{\bar{t}}t')\, T_{12}^{-}(t't) \}. \end{aligned} \tag{95}$$

Here we used $G_{12}^{0+} - G_{12}^{0-} = \mathscr{G}_{12}^{0>} - \mathscr{G}_{12}^{0<}$.

Equation (95) is very general and goes far beyond the usual Boltzmann equation. It still includes the full retardation and, thus, memory effects, and it includes self-energy and damping. We want to remark that the result derived in the framework of the density operator technique is in agreement with results derived with the Greens's functions method (see [39, 31]).

We now transform the collision integral into the momentum representation. With (69) and using the T-matrices we obtain for $I_1 = I(p_1)$

$$
\begin{aligned}
I(p_1, t) = \frac{i}{\hbar^2} \int dp_2\, d\bar{p}_1\, d\bar{p}_2 \int d\bar{t}\, d\bar{\bar{t}}\, dt' \\
&\times \{ \langle p_1 p_2 | T^+_{12}(t\bar{t}) | \bar{p}_2 \bar{p}_1 \rangle \; \bar{U}^{0+}_{12}(\bar{t}\bar{\bar{t}}) \langle \bar{p}_1 \bar{p}_2 | T^-_{12}(\bar{\bar{t}}t') | p_2 p'_1 \rangle \\
&\times U^{0-}_{12}(t't) [\bar{N}_{12}(\bar{\bar{t}})\, F^<_{12}(t') - \bar{F}^<_{12}(\bar{\bar{t}})\, N_{12}(t')] \\
&+ \langle p_1 p_2 | T^+_{12}(t\bar{t}) | \bar{p}_2 \bar{p}_1 \rangle \; \bar{U}^{0-}_{12}(\bar{t}\bar{\bar{t}}) \langle \bar{p}_1 \bar{p}_2 | T^-_{12}(\bar{\bar{t}}t') | p_2 p'_1 \rangle \\
&\times U^{0-}_{12}(t't) [\bar{N}_{12}(\bar{t})\, F^<_{12}(t') - - \bar{F}^<_{12}(\bar{t})\, N_{12}(t')] \\
&- U^{0+}_{12}(tt') \langle p_1 p_2 | T^+_{12}(t'\bar{\bar{t}}) | \bar{p}_2 \bar{p}_1 \rangle \; U^{0-}_{12}(\bar{\bar{t}}\bar{t}) \langle \bar{p}_1 \bar{p}_2 | T^-_{12}(\bar{t}t) | p_2 p'_1 \rangle \\
&\times [\bar{N}_{12}(\bar{\bar{t}})\, F^<_{12}(t') - \bar{F}^<_{12}(\bar{\bar{t}})\, N_{12}(t')] \\
&- U^{0+}_{12}(tt') \langle p_1 p_2 | T^+_{12}(t'\bar{\bar{t}}) | \bar{p}_2 \bar{p}_1 \rangle \; \bar{U}^{0+}_{12}(\bar{\bar{t}}\bar{t}) \langle \bar{p}_1 \bar{p}_2 | T^-_{12}(\bar{t}t) | p_2 p'_1 \rangle \\
&\times [\bar{N}_{12}(\bar{t})\, F^<_{12}(t') - \bar{F}^<_{12}(\bar{t})\, N_{12}(t')] \}. \qquad (96)
\end{aligned}
$$

Here we used the following short notations for the momentum dependence of the different quantities: $N_{12} = 1 - F_1 - F_2$, $\bar{N}_{12} = 1 - \bar{F}_1 - \bar{F}_2$, $U^{0\pm}_{12} = U^{0\pm}_1\, U^{0\pm}_2 = U^{0\pm}(p_1)\, U^{0\pm}(p_2)$, $\bar{U}^{0\pm}_{12} = \bar{U}^{0\pm}_1\, \bar{U}^{0\pm}_2$, $F^<_{12} = F_1 F_2$, $\bar{F}^<_{12} = \bar{F}_1 \bar{F}_2$, $F_1 = F(p_1)$, and $\bar{F}_1 = F(\bar{p}_1)$. The distribution functions are normalized according to $2\mathscr{V} \int dp/[(2\pi h)^3]\, F(p) = 1$, where $\mathscr{V}$ is the volume. Notice that the renormalized free propagators $U^{0\pm}$ are to be determined self-consistently from Eq. (41) and the adjoint equation, respectively.

The collision integral arising from initial correlations is obtained in complete analogy. We apply Eqs. (64) and (66), and the final form is

$$
\begin{aligned}
I^{IC} = n \operatorname*{Tr}_2 \int dt' [\, T^+_{12}(tt')\, \mathscr{K}_{12}(t't) - \mathscr{K}_{12}(tt')\, T^-_{12}(t't)\,] - n \operatorname*{Tr}_2 [\, V_{12}, F^<_{12}(t)\,] \\
&- n \frac{i}{\hbar} \operatorname*{Tr}_2 \int d\bar{t}\, d\bar{\bar{t}}\, dt' [\, G^{0+}_{12}(tt')\, T^+_{12}(t'\bar{t})\, \mathscr{K}_{12}(\bar{t}\bar{\bar{t}})\, T^-_{12}(\bar{\bar{t}}t) \\
&+ T^+_{12}(t\bar{t})\, \mathscr{K}_{12}(\bar{t}\bar{\bar{t}})\, T^-_{12}(\bar{\bar{t}}t')\, G^{0-}_{12}(t't)], \qquad (97)
\end{aligned}
$$

where we introduced the abbreviation

$$\mathscr{K}_{12}(tt') = U^{0+}_{12}(tt_0)\, g_{12}(t_0)\, U^{0-}_{12}(t_0 t').$$

Further simplifications may only be done if $g_{12}(t_0)$ is given explicitly.

The last step left to perform is to express the retarded self-energy in binary collision approximation in terms of the T-matrix, too. The derivation follows the same

lines as for the collision integral above. We start from Eq. (23) and use Eqs. (62) and (64), leading to the result

$$\Sigma_1^+(t\bar{t}) = \frac{n}{ih} \operatorname*{Tr}_3 \int_{-\infty}^{+\infty} dt'\, T_{13}^+(tt')\, U_{13}^{0+}(t'\bar{t}) \times \{N_{13}(\bar{t})\, V_{13}\, F_3(\bar{t}) \mp F_1(\bar{t})\, F_3(\bar{t}) V_{13}\}\, U_3^-(\bar{t}t). \tag{98}$$

Equation (98) is the ladder approximation to the self-energy which includes all ladder-type diagrams, except the one with one rung only and those which are "closed" by a single-particle propagator. In terms of Green's functions, this self-energy expression is discussed in [6, 38, 48].

Equation (96) is the collision integral of a very general kinetic equation for the evolution of the single-particle density matrix. We want to stress the fact that this kinetic equation follows from the solution of the initial value problem for g_{12}, Eq. (70), without any additional approximations. In particular, the time dependence was treated exactly. Therefore, this equation is valid without restrictions with respect to the time. This equation has the following remarkable properties:

(i) The equation is nonlocal in time; i.e., the distribution function at time t is determined by its values for the preceding times too. We have a memory effect with the "memory depth" of the order of $1/\gamma_{12}$.

(ii) For times $t < 1/\gamma_{12}$, initial correlations influence the behavior of the system significantly, while for $t \gg 1/\gamma_{12}$, they are being completely weakened.

(iii) For $\gamma_{12} \to 0$ we have a symmetric hierarchy closure, i.e., $P_{\{123\}} F_{123} = F_{123}$; i.e., total energy is conserved,

$$\frac{d}{dt} \langle T + V \rangle = 0. \tag{99}$$

(iv) Conventional Boltzmann-type two-particle scattering integrals, which conserve kinetic energy only do not include bound states because, in that case, energy and momentum cannot be conserved simultaneously [13, 14]. In our case of the non-Markovian collision integral (96) this restriction does not exist because kinetic energy is not a conserved quantity. This means, the nonlocality in time allows for the existence of bound states in the framework of a two-particle collision approximation already. In particular, bound states may exist in the system already at the initial moment t_0, which can be accounted for by the choice of the initial correlations.

It is known from scattering theory that, for weak interaction, the scattering cross section may be approximated rather well by the Born approximation of the latter. Let us thus consider a simplified equation by taking the first Born approximation for the matrix element of the T-operator,

$$\langle p_1 p_2 | T_{12}^{\pm}(tt') | \bar{p}_2 \bar{p}_1 \rangle = V(p_1 - \bar{p}_1)\, \delta(p_1 + p_2 - \bar{p}_1 - \bar{p}_2)\, \delta(t - t'),$$

leading to essential simplifications of the numerical analysis. We mention that this type of approximation retains all non-Markovian features of the time dependencies discussed above. However, it is worthwhile to discuss such an equation as it leads to considerable simplifications; e.g., by the occurrence of the delta distribution,

$$\frac{d}{dt} F(p_1 t) = I^{IC} + \frac{2\mathscr{V}^2}{h^2} \int \frac{dp_2}{(2\pi h)^3} \frac{d\bar{p}_1}{(2\pi h)^3} \frac{d\bar{p}_2}{(2\pi h)^3}$$

$$\times \int_0^{t-t_o} d\tau\ V^2(p_2 - \bar{p}_2)(2\pi h)^3\, \delta(p_1 + p_2 - \bar{p}_1 - \bar{p}_2)$$

$$\times \bar{U}_{12}^{0+}(t, t+\tau)\ U_{12}^{0-}(t+\tau, t)$$

$$\times \{\bar{F}_{12}^{<}(t-\tau)\, F_{12}^{>}(t-\tau) - F_{12}^{<}(t-\tau)\, \bar{F}_{12}^{>}(t-\tau)\}. \tag{100}$$

Here, the initial correlation term is given by

$$I^{IC} = \frac{2\mathscr{V}}{h} \int \frac{dp_2}{(2\pi h)^3} \frac{d\bar{p}_1}{(2\pi h)^3} \frac{d\bar{p}_2}{(2\pi h)^3}\ V(p_1' - p_1)$$

$$\times (2\pi h)^3\, \delta(\Delta\bar{p}_{12})\, \mathrm{Im}\{\bar{U}_{12}^{0+}(t, t_0)\ U_{12}^{0-}(t_0, t)\, g_0(p_1, p_2, \bar{p}_1, \bar{p}_2), \tag{101}$$

where $\delta(\Delta\bar{p}_{12}) = \delta(p_1 + p_2 - \bar{p}_1 - \bar{p}_2)$. In Eqs. (100), (101), we still retained the full renormalized propagators. This means that self-energy effects are still included in a fully self-consistent way.

A further simplification is possible if propagators for damped quasiparticles, Eq. (47), are used. Taking, furthermore, into account that the free propagators $U_{12}^{0\pm}$ factorize, we obtain

$$\frac{d}{dt} F(p_1 t) = I^{IC} + \frac{2\mathscr{V}^2}{h^2} \int \frac{dp_2}{(2\pi h)^3} \frac{d\bar{p}_1}{(2\pi h)^3} \frac{d\bar{p}_2}{(2\pi h)^3}$$

$$\times \int_0^{t-t_o} d\tau\ V^2(p_2 - \bar{p}_2)(2\pi h)^3\, \delta(p_1 + p_2 - \bar{p}_1 - \bar{p}_2)$$

$$\times \exp\left\{-\frac{\bar{\gamma}_{12} + \gamma_{12}}{h}\, \tau\right\} \cos\frac{(\bar{E}_{12} - E_{12})\tau}{h}\, \{\bar{F}_{12}^{<} F_{12}^{>} - F_{12}^{<} \bar{F}_{12}^{>}\}|_{t-\tau}. \tag{102}$$

Here, $E_{12} = E_1 + E_2$ and E_1 is the quasiparticle energy, $E_1 = p_1^2/2m + \mathrm{Re}\, \Sigma_1^+$, and $\gamma = -2\, \mathrm{Im}\, \Sigma_1^+$. In the quasiparticle approximation, the retarded self-energy in the second Born approximation is given by

$$\Sigma_1^+(p_1 t) = \frac{2\mathscr{V}^2}{h} \int \frac{dp_2}{(2\pi h)^3} \frac{d\bar{p}_1}{(2\pi h)^3} \frac{d\bar{p}_2}{(2\pi h)^3}$$

$$\times \int_0^{t-t_o} d\tau\ V^2(p_2 - \bar{p}_2)(2\pi h)^3\, \delta(p_1 + p_2 - \bar{p}_1 - \bar{p}_2)$$

$$\times \exp\left\{-\frac{\bar{\gamma}_{12} + \gamma_{12}}{h}\tau\right\} \cos\frac{(\bar{E}_{12} - E_{12})\tau}{h} \left\{\bar{F}_{12}^{<} F_2^{>} \mp F_2^{<} \bar{F}_{12}^{>}\right\} |_{t-\tau}. \quad (103)$$

Equation (102) is a generalization of the Landau equation which is often used in plasma and solid state physics. This equation, in the limit, where self-energy and initial correlations are neglected, has been derived, among others, by Klimontovich [8, 11]. It contains initial correlations, energy broadening $\cos[(E_{12} - \bar{E}_{12})(t-\tau)/h]$, self-energy, and retardation effects in the distribution function. In cases where the memory depth is small, it was shown that a qualitatively correct approximation is to neglect, at the same time initital correlations, the retardation in the distributions and self-energy (damping) effects [27, 26]. Then, one can take the distributions out of the time integral and the τ-integration can be performed. The result is given by [26, 32]

$$\frac{d}{dt} F(p_1, t) = \frac{2\mathscr{V}^2}{h} \int \frac{dp_2}{(2\pi h)^3} \frac{d\bar{p}_1}{(2\pi h)^3} \frac{d\bar{p}_2}{(2\pi h)^3}\ V^2(p_2 - \bar{p}_2)(2\pi h)^3\, \delta(p_1 + p_2 - \bar{p}_1 - \bar{p}_2)$$

$$\times \frac{\sin[(\bar{E}_{12} - E_{12})(t - t_o)/h]}{\bar{E}_{12} - E_{12}} \left\{\bar{F}_{12}^{<}(t) F_{12}^{>}(t) - F_{12}^{<}(t) \bar{F}_{12}^{>}(t)\right\}. \quad (104)$$

V. KINETIC EQUATIONS IN FIRST-ORDER GRADIENT EXPANSION

A. *Gradient Expansion of the Colision Integral*

In deriving Eq. (96) we have obtained a very general equation which is nonlocal in time (non-Markovian) and which is valid on arbitrary time scales. An essential question is now to investigate how the usual Boltzmann equation, which is local in time turns out to be an approximation of our nonlocal equation Eq. (96). To this end, it is useful to expand the collision integral (96) with respect to the retardation. To begin we consider the expansion under the preassumption that the quantities T and Ω depend only on time differences; e.g., $T(t, t') = T(t - t')$; the initial time is $t_0 \to -\infty$; and $\lim_{t_0 \to -\infty} g(t_0) = 0$ (Boltzmann limit). In order to make the retardation "visible" explicitly, we introduce the variables

$$\bar{t} = t - \tau, \qquad \bar{\bar{t}} = t - \bar{\tau}, \qquad t' = t - \bar{\bar{\tau}}.$$

Then we get

$$
\begin{aligned}
I(p_1, t) = \frac{1}{h^2} \int d\tau\, d\bar{\tau}\, d\bar{\bar{\tau}} \int \frac{dp_2\, d\bar{p}_1\, d\bar{p}_2}{(2\pi h)^9} \\
\times \{ & T_{12}^{+}(\tau)\, \bar{U}_{12}^{0+}(\bar{\tau}-\tau)\, T_{12}^{-}(\bar{\tau}-\bar{\bar{\tau}})\, U_{12}^{0-}(-\bar{\bar{\tau}}) \\
& \times [\bar{F}_{12}^{>}(t-\bar{\tau})\, F_{12}^{<}(t-\bar{\bar{\tau}}) - \bar{F}_{12}^{<}(t-\bar{\tau})\, F_{12}^{>}(t-\bar{\bar{\tau}})] \\
& + T_{12}^{+}(\tau)\, \bar{U}_{12}^{0-}(\bar{\tau}-\tau)\, T_{12}^{-}(\bar{\tau}-\bar{\bar{\tau}})\, U_{12}^{0-}(-\bar{\bar{\tau}}) \\
& \times [\bar{F}_{12}^{>}(t-\tau)\, F_{12}^{<}(t-\bar{\bar{\tau}}) - \bar{F}_{12}^{<}(t-\tau)\, F_{12}^{>}(t-\bar{\bar{\tau}})] \\
& - U_{12}^{0+}(\bar{\bar{\tau}})\, T_{12}^{+}(\bar{\bar{\tau}}-\bar{\tau})\, \bar{U}_{12}^{0-}(\tau-\bar{\tau})\, T_{12}^{-}(-\tau) \\
& \times [\bar{F}_{12}^{>}(t-\bar{\tau})\, F_{12}^{<}(t-\bar{\bar{\tau}}) - \bar{F}_{12}^{<}(t-\bar{\tau})\, F_{12}^{>}(t-\bar{\bar{\tau}})] \\
& - U_{12}^{0+}(\bar{\bar{\tau}})\, T_{12}^{+}(\bar{\bar{\tau}}-\bar{\tau})\, \bar{U}_{12}^{0+}(\tau-\bar{\tau})\, T_{12}^{-}(-\tau) \\
& \times [\bar{F}_{12}^{>}(t-\tau)\, F_{12}^{<}(t-\bar{\bar{\tau}}) - \bar{F}_{12}^{<}(t-\tau)\, F_{12}^{>}(t-\bar{\bar{\tau}})]\}. \qquad (105)
\end{aligned}
$$

The local approximation of Eq. (105) follows by expansion with respect to the retardations τ, $\bar{\tau}$, and $\bar{\bar{\tau}}$. The result up to first-order in the retardation is

$$
\frac{d}{dt} F_1(p_1) = I^0(p_1) + I^{(1)}(p_1). \qquad (106)
$$

For the zeroth order we get, then, with the application of the convolution theorem for the Fourier transforms and using formulae (69), (70), the usual Boltzmann collision integral ($E \equiv E_{12}$),

$$
\begin{aligned}
I^0(p_1, t) = \frac{2}{h} \int \frac{dp_2\, d\bar{p}_1\, d\bar{p}_2}{(2\pi h)^9} & |\langle p_1\, p_2|\, T_{12}^{+}(E+i\varepsilon)\, |\bar{p}_2\, \bar{p}_1\rangle|^2\, \delta(E_{12}-\bar{E}_{12}) \\
& \times \{\bar{F}_{12}^{>} F_{12}^{<} - \bar{F}_{12}^{<} F_{12}^{>}\}. \qquad (107)
\end{aligned}
$$

The integral (107), i.e., the first contribution of (106) describes the relaxation in an asymptotic manner, i.e., for times t sufficiently far away from the initial time t_0 such that $t \gg t^{\mathrm{corr}}$. The following contributions $I^{(1)}(p_1)$ in Eq. (106) are corrections which are of importance for higher orders of the density expansion of the collision integral and for the correct asymptotic value of the mean value of the total energy.

For the determination of the first-order retardation terms, I is first expanded with respect to $\bar{\bar{\tau}}$, yielding $I_1^{(1)}$. Using Eq. (44), we get, after a simple calculation,

$$
I_1^{(2)} = \frac{2}{h} \operatorname*{Tr}_2 \left\{ |T_{12}^{+}(\bar{E}+i\varepsilon)|^2\, \frac{d}{dE}\, \frac{P}{E-\bar{E}} \left[\bar{F}_{12}^{>}\, \frac{dF_{12}^{<}}{dt} - \bar{F}_{12}^{<}\, \frac{d}{dt}\, F_{12}^{>} \right] \right\}. \qquad (108)
$$

Another contribution to $I^{(1)}$ follows by expansion of $\bar{F}^{>}(t-\bar{\tau})$ and $\bar{F}^{>}(t-\tau)$ and reads

$$
\begin{aligned}
I_2^{(1)} = \frac{1}{\hbar} \operatorname*{Tr}_2 \int d\tau\, d\bar{\tau}\, d\bar{\bar{\tau}} \,[\, \{ ((\bar{\tau}-\tau)+\tau)\, T_{12}^{+}(\tau)\, \bar{U}_{12}^{0+}(\bar{\tau}-\tau)\, T_{12}^{-}(\bar{\tau}-\bar{\bar{\tau}})\, U_{12}^{0-}(-\bar{\bar{\tau}}) \\
+ \tau T_{12}^{+}(\tau)\, \bar{U}_{12}^{0-}(\bar{\tau}-\tau)\, T_{12}^{-}(\bar{\tau}-\bar{\bar{\tau}})\, U_{12}^{0-}(-\bar{\bar{\tau}}) \} \\
- \{ \tau U_{12}^{0+}(\bar{\bar{\tau}})\, T_{12}^{+}(\bar{\bar{\tau}}-\bar{\tau})\, \bar{U}_{12}^{0+}(\tau-\bar{\tau}) T_{12}^{-}(-\tau) \\
+ ((\bar{\tau}-\tau)+\tau)\, U_{12}^{0+}(\bar{\bar{\tau}})\, T_{12}^{+}(\bar{\bar{\tau}}-\tau)\, \bar{U}_{12}^{0-}(\tau-\bar{\tau})\, T_{12}^{-}(-\tau) \} \,] \\
\times \left[F^{<} \frac{d\bar{F}^{>}}{dt} - F^{>} \frac{d\bar{F}^{<}}{dt} \right].
\end{aligned} \tag{109}
$$

Again, we use the convolution theorem and the relations (44), (70). We get, after some algebra,

$$
\begin{aligned}
I_2^{(1)} = \frac{1}{\hbar} \int \frac{dp_2\, d\bar{p}_1\, d\bar{p}_2}{(2\pi\hbar)^9} \left\{ -|T_{12}^{+}(\bar{E})|^2 \frac{P'}{\bar{E}-E} - |T_{12}^{+}(E)|^2 \frac{P'}{\bar{E}-E} \right. \\
\left. + \pi i\, \delta(E-\bar{E}) \left[\frac{dT_{12}^{+}}{dE}\, T_{12}^{-} - T_{12}^{+} \frac{dT_{12}^{-}}{dE} \right] \right\} \left\{ F^{<} \frac{d\bar{F}^{>}}{dt} - F^{>} \frac{d\bar{F}^{<}}{dt} \right\}.
\end{aligned} \tag{110}
$$

With (109) and (110) we have

$$
I^{(1)} = I_1^{(1)} + I_2^{(1)} \tag{111}
$$

and, thus,

$$
\begin{aligned}
I^{(1)} = -\frac{1}{\hbar} \int \frac{dp_2\, d\bar{p}_1\, d\bar{p}_2}{(2\pi\hbar)^9} \left\{ 2\, |T_{12}^{+}(E)|^2 \frac{P'}{\bar{E}-E} \left[\frac{dF^{<}}{dt} \bar{F}^{>} - \bar{F}^{<} \frac{dF^{>}}{dt} \right] \right. \\
+ \left(|T_{12}^{+}(E)|^2 \frac{P'}{\bar{E}-E} + |T_{12}^{+}(\bar{E})|^2 \frac{P'}{\bar{E}-E} - \pi i\, \delta(E-\bar{E}) \left\{ \frac{dT_{12}^{+}}{dE}\, T_{12}^{-} - T_{12}^{+} \frac{dT_{12}^{-}}{dE} \right\} \right) \\
\left. \times \left[F^{<} \frac{d\bar{F}^{>}}{dt} - F^{>} \frac{d\bar{F}^{<}}{dt} \right] \right\}.
\end{aligned} \tag{112}
$$

In connection with Eq. (112), we may apply a useful relation which may be derived from the optical theorem (71) by differentiation. Using the dispersion relation for T_{12}^{+}, it follows that

$$
\begin{aligned}
T_{12}^{+}(\bar{E}) \frac{P'\, N_{12}}{\bar{E}-E}\, T_{12}^{-}(E) - |T_{12}^{+}(E)|^2 \frac{P'\, N_{12}}{\bar{E}-E} \\
= \pi i\, \delta(E-\bar{E}) \left\{ \frac{dT_{12}^{+}}{dE}\, N_{12} T_{12}^{-} - T_{12}^{+} N_{12} \frac{dT_{12}^{-}}{dE} \right\}.
\end{aligned} \tag{113}
$$

Eqution (113) determines the difference between the off-shell T-matrices for the energies $\bar{E}$ and E. We mention that the application of the optical theorem leads to a neglect of bound states. This can be avoided if the contribution of the discrete energy spectrum to the T-matrix is separated off first as discussed in Ref. [31].

Unfortunately, a drastic simplification of Eq. (112) is possible only in the case of nondegenerate quantum systems, i.e., if $(d/dt)(1-F_1-F_2)=0$ and $F_{12}^{>}=1$. In this case, we may write

$$I^{(1)}(p_1,t) = -\frac{1}{h}\int \frac{dp_2\, d\bar{p}_1\, d\bar{p}_2}{(2\pi h)^9} \times\left\{ |T_{12}^{+}(E)|^2 \frac{P'}{\bar{E}-E} + |T_{12}^{+}(\bar{E})|^2 \frac{P'}{\bar{E}-E} - i\pi\,\delta(E-\bar{E}) \times\left[\frac{dT_{12}^{+}}{dE}\, T_{12}^{-} - T_{12}^{+}\frac{dT_{12}^{-}}{dE}\right]\right\} \frac{d}{dt}\,[\bar{F}_{12}^{>}(t) - F_{12}^{<}(t)]. \tag{114}$$

With the help of Eq. (113), this expression may be reduced to the simpler form

$$I^{(1)}(p_1,t) = -\frac{2}{h}\frac{d}{dt}\int \frac{dp_2\, d\bar{p}_1\, d\bar{p}_2}{(2\pi h)^9}\, |T_{12}^{+}(E)|^2 \frac{\mathrm{P}'}{E-\bar{E}}\,[\bar{F}_{12}^{<}(t) - F_{12}^{<}(t)]. \tag{115}$$

There exists a close connection of expression (114) and $I^0(p_1)$. To find this connection, let us define

$$I^0(\varepsilon) = -\frac{1}{h}\int \frac{dp_2\, d\bar{p}_1\, d\bar{p}_2}{(2\pi h)^9}\int d\omega\; 2\delta^{\varepsilon}(E-\omega)\, 2\delta^{\varepsilon}(\bar{E}-\omega) \times |\langle p_1\, p_2|\; T_{12}^{+}(E+i\varepsilon)\; |\bar{p}_2\,\bar{p}_1\rangle|^2\,\{\bar{F}_{12}^{<}(t) - F_{12}^{<}(t)\}. \tag{116}$$

Here, $\delta^{\varepsilon}(x)$ is a broadened delta function,

$$\delta^{\varepsilon}(x) = \frac{1}{\pi}\,\frac{\varepsilon}{x^2+\varepsilon^2}.$$

Using this expression for δ^{ε}, it is easy to show that

$$\lim_{\varepsilon\to 0} I^0(\varepsilon) = I^0. \tag{117}$$

Furthermore, one can derive an interesting relation between the first-order retardation term $I^{(1)}(p_1)$ and the Boltzmann collision integral I^0 (in agreement with Peletminski [59]), which reads

$$I^{(1)}(p_1) = \frac{1}{2}\frac{d}{dt}\frac{d}{d\varepsilon}\, I^0(\varepsilon)\Big|_{\varepsilon\to 0}. \tag{118}$$

Here we used the relation

$$\lim_{\varepsilon \to 0} \frac{d}{d\varepsilon}\, \delta^{\varepsilon}(x) = \frac{P'}{x}\,.$$

Consequently, the kinetic equation in first order gradient expansion may be given for nondegenerate many-particle systems in the following compact shape:

$$\frac{d}{dt}\, F_1(p_1) = \left(1 + \frac{1}{2}\, \frac{d}{dt}\, \frac{d}{d\varepsilon}\right) I^0(\varepsilon)\,\Big|_{\varepsilon \to 0}\,. \tag{119}$$

As compared to Eq. (96), the result of Eq. (119) is less general, and especially the non-Markovian character is lost. Nevertheless, Eq. (119) goes far beyond the usual Boltzmann equation. We will demonstrate this by considering the conservation laws.

B. *Conservation Laws*

As already mentioned, the usual Boltzmann equation leads only to conservation laws for an ideal many-particle system. This follows from the delta-function $\delta(E_1 + E_2 - \bar{E}_1 - \bar{E}_2)$ which conserves only the single-particle energies. The first-order gradient terms do not include such a delta function. One can show that these gradient contributions lead to the correct conservation laws of correlated many-particle systems. We will show this for the conservation of the (total) energy.

To this end, we multiply the kinetic equation (106), together with Eqs. (107), (114), by the kinetic energy $p_1^2/(2m_1)$ and calculate the trace with respect to the free index 1. Furthermore, we symmetrize the resulting expression with respect to the variables 1, 2 and $\bar{1}$, $\bar{2}$. Then we get the relation

$$\frac{\partial}{\partial t} \left\{ \langle T \rangle + \frac{1}{2}\, \operatorname*{Tr}_{12} \left[T_{12}^{+}(E)\, \frac{\mathrm{P}}{E - \bar{E}}\, \bar{F}_{12}^{<}(t)\, T_{12}^{-}(\bar{E}) \right.\right.$$
$$\left.\left. + T_{12}^{+}(E)\, \frac{\mathrm{P}}{E - \bar{E}}\, T_{12}^{-}(\bar{E})\, F_{12}^{<}(t) \right] \right\} = 0, \tag{120}$$

and with the dispersion relation

$$2\ \mathrm{Re}\ T_{12}^{+}(E) = T_{12}^{+}(E)\, \frac{\mathrm{P}}{E - \bar{E}}\, T_{12}^{-}(\bar{E}), \tag{121}$$

we get the energy conservation

$$\frac{d}{dt} \left\{ \langle T \rangle + \langle V \rangle \right\} = 0. \tag{122}$$

Here we used

$$\langle V\rangle = \frac{n^2}{2}\operatorname*{Tr}_{12}\left\{\operatorname{Re}\, T_{12}^{+}(E)\, F_{12}^{<}(t) - T_{12}^{+}(E)\,\frac{\mathrm{P}}{E-\bar{E}}\,\bar{F}_{12}^{<}(t)\, T_{12}^{-}(\bar{E})\right\}. \tag{123}$$

Equation (123) is, according to formula (82), just the mean value of the potential energy in binary collision approximation. Thus, the first order-gradient terms provide for the consistency of the character of approximation of the kinetic equation with that of the conservation laws.

After having considered a simplified time behavior, let us now go back to the more general case of a finite initial time t_0 and arbitrary initial correlations $g(t_0)$. But now we will consider an approximation with respect to the strength of the interaction; i.e., we consider a retardation expansion of the Born approximation (Landau equation). An expansion of the distribution functions with respect to the retardation leads to the equation

$$\frac{dF_1(p_1)}{dt} = I^{IC} + I^0 + I^{(1)}.$$

Here, the zeroth-order term is given by

$$I^0 = \frac{2\mathscr{V}}{h}\int \frac{dp_2\, d\bar{p}_1\, d\bar{p}_2}{(2\pi h)^9}\,|V(\bar{p}_1 - p_1)|^2\,\delta(p_1+p_2-\bar{p}_1-\bar{p}_2)\,\frac{\sin\left[(E-\bar{E})(t-t_0)/h\right]}{E-\bar{E}}$$
$$\times\left\{\bar{F}_{12}^{>}(t)\, F_{12}^{<}(t) - \bar{F}_{12}^{<}(t)\, F_{12}^{>}(t)\right\}. \tag{124}$$

For the first-order retardation term we may write

$$I^{(1)} = \frac{2\mathscr{V}}{h}\int \frac{dp_2\, d\bar{p}_1\, d\bar{p}_2}{(2\pi h)^9}\,|V(\bar{p}_1 - p_1)|^2\,\delta(p_1+p_2-\bar{p}_1-\bar{p}_2)$$
$$\times\frac{d}{dE}\,\frac{\cos\left[(E-\bar{E})(t-t_0)/h\right]-1}{E-\bar{E}}$$
$$\times\,\frac{d}{dt}\left\{\bar{F}_{12}^{>}(t)\, F_{12}^{<}(t) - \bar{F}_{12}^{<}(t)\, F_{12}^{>}(t)\right\}. \tag{125}$$

The initial correlation term I^{IC} is given by Eq. (101). This equation was discussed in [24]. In the Boltzmann limit, i.e. $t_0 \to -\infty$, initial correlations are being neglected, $\lim_{t_0\to-\infty} g_{12}(t_0) = 0$. Then, using $\lim_{\tau\to\infty}(\sin x\tau/x) = \delta(x)$ and $\lim_{t\to\infty}((\cos a\tau - 1)/a) = \mathrm{P}/x$, we get the Born approximations of the collision integral contributions I^0, Eq. (107), and $I^{(1)}$, Eq. (112).

VI. NUMERICAL RESULTS

To illustrate our theoretical results on non-Markovian effects in kinetic equations, we present some numerical examples. Unfortunately, the non-Markovian Boltzmann equation is still not feasable for numerical solution. Therefore, we have to restrict ourselves to approximations. In this section we present numerical results for the solution of the kinetic equation in Born approximation. We expect, however, that qualitative features, like the interplay between retardation (memory) and damping effects will remain the same also in the case of the T-matrix approximation.

We studied the relaxation of a dense electron gas starting from an initial non-equilibrium distribution using various approximations of the kinetic equation in second Born approximation, such as the Markovian Landau equation, the non-Markovian Landau equation in zeroth-order retardation approximation, Eq. (124), and the non-Markovian Landau equation with full memory but without self-energy. Some of these results were presented in Ref. [24], where also a comparison with solutions of the Kadanoff–Baym equations was given. We refer to this paper for numerical details. Here, we focus on the influence of self-energy effects on the relaxation. To this end, we solved the non-Markovian Landau equation with full memory, Eq. (102), where the retarded self-energy was calculated self-consistently, according to Eq. (103). The calculation of the self-energy was performed using the local approximation for the free propagators and different approximations to the time dependencies, including the Markov approximation and the full non-Markovian level, Eq. (103). These results are compared to the relaxation without damping (Fig. 1), whereas Fig. 2 shows the relaxation according to the non-Markovian calculation of the self-energy. The results for the real and imaginary parts of the retarded self-energy, used in Fig. 2, are shown in Fig. 3. One clearly sees the buildup of correlations. At the initial time, $t=0$, $\mathrm{Re}\,\Sigma^{+}=\mathrm{Im}\,\Sigma^{+}=0$. During the relaxation, the damping coefficient ($\mathrm{Im}\,\Sigma^{+}$) and the energy shift ($\mathrm{Re}\,\Sigma^{+}$) increase until they reach an almost stationary value after about 100 *fs*, which is of the order of the correlation time τ_{corr}. With the Markov limit for the self-energy, the relaxation starts already with nonzero values for Σ^{R} which results in an overestimation of the damping.

The effect of self-energy on the relaxation can be seen by comparing Figs. 1 and 2, which show the evolution of the distribution function and its time derivative. (The derivative is of interest since it is much more sensitive to retardation and damping effects than the distribution function.) As one can see, the effect of self-energy is twofold. First, the relaxation is slowed down, because the memory depth is reduced, which leads to a reduction of the scattering cross section. Second, the damping terms lead to a broadening of the spectral function. In the long-time limit, the system relaxes toward a broadened energy delta function (δ^{ε} with $\varepsilon\sim\mathrm{Im}\,\Sigma^{+}$). This Lorentzian shape of the spectral function leads to increased scattering into high momentum states, which can be seen by comparing the high momentum tails of the time derivatives df/dt in Fig. 2, as compared to Fig. 1. Furthermore, this broadening causes a continuous increase of kinetic energy (cf. Fig. 4). On the other

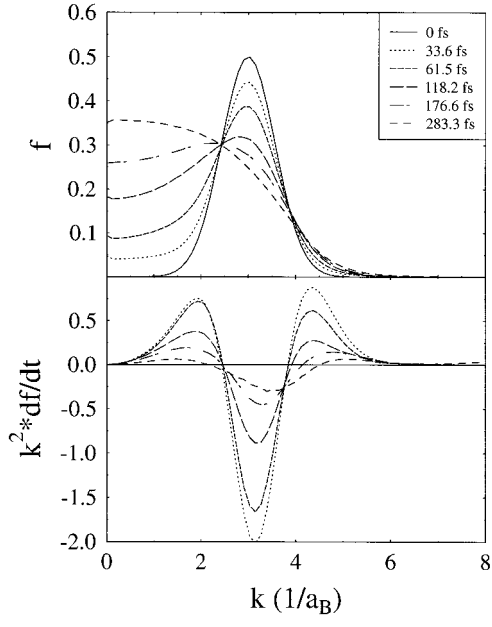

FIG. 1. Relaxation of the distribution function (upper figure) and its time derivative (lower figure), calculated from the non-Markovian Landau equation (102) without self-energy.

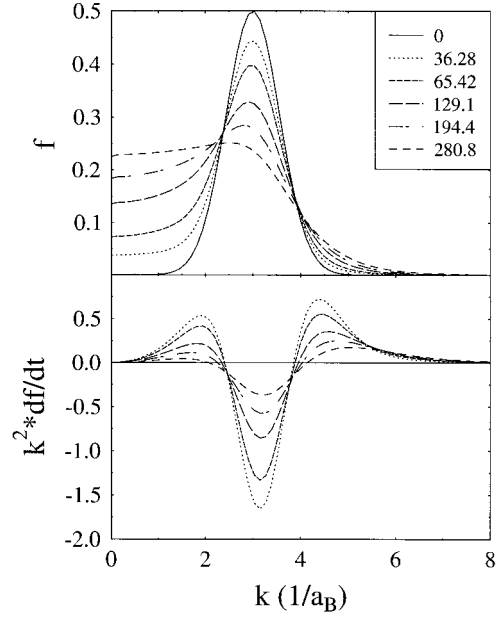

FIG. 2. Same as Fig. 1, but for the non-Markovian Landau equation (102) with full self-consistently calculated self-energy according to Eq. (103).

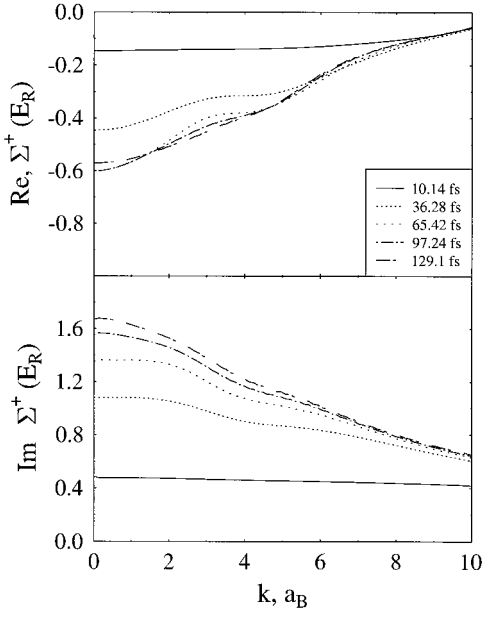

FIG. 3. Evolution of the real and imaginary parts (upper and lower figures, respectively) of the retarded self-energy, Eq. (103), corresponding to Fig. 2.

hand, after times of the order of the correlation time, we expect that the relaxation reaches the kinetic (Markovian) stage, where kinetic energy does not change further [24]. This indicates that the Lorentzian form of the spectral function is not a good approximation in the long-time limit. This is in agreement with the recent analysis of Haug and Banyai, who discussed improved spectral functions [60] which decay faster at high momentum values.

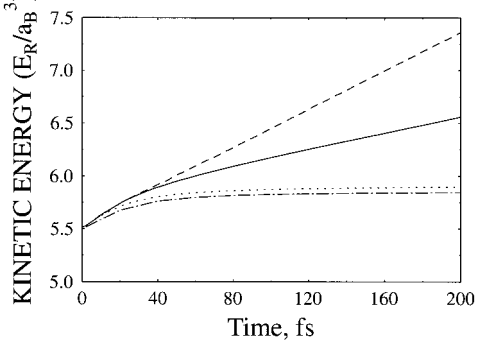

FIG. 4. Relaxation of the kinetic energy for different scattering models: non-Markovian Landau equation with self-energy (corresponding to Fig. 2, dashes), non-Markovian Landau equation without self-energy (corresponding to Fig. 1, full line), zeroth-order retardation approximation (Eq. (104), dots), and Kadanoff–Baym equations (from Ref. [26], dash-dots).

Furthermore, we mention that for times of the order of the correlation time, one can avoid the solution of non-Markovian equations and solve, instead, modified Markovian equations using the so-called correlation time approximation [61].

VII. CONCLUDING REMARKS

In this paper we presented a derivation of a quantum kinetic equation in binary collision approximation which generalizes the conventional quantum Boltzmann equation for the Wigner distribution function in several directions. First, we extended the kinetic equation to the regime of ultrashort times by including the full dynamics of the binary correlations. This lead us to a kinetic equation which contains memory (retardation) effects and initial correlations. On the other hand, we saw that the coupled dynamics of one-particle and two-particle quantities alone (i.e., the dynamics of an isolated pair of particles) leads to unphysical long-time behavior, which is characterized by time reversibility and by an inifinite lifetime of the initial correlations. Therefore, a second generalization was necessary, namely to account for the influence of many-body (medium) effects on the two-particle interaction, which was accomplished by including relevant contributions from three-particle correlations. This was the motivation for the generalized closure relation of the hierarchy, Eq. (4). Solving the BBGKY-hierarchy with this closure, opened up a straightforward way to incorporate self-energy effects, and in particular, damping, into the density operator formalism.

Our main result was the kinetic Eq. (91), which includes all the effects just mentioned. Moreover, by avoiding any assumption about weakness of the interaction, our results are valid for systems with strong coupling, too. Therefore, our analysis applies to strongly correlated systems, in particular to systems containing bound states, and thus generalizes previous investigations of memory effects which were confined to the Born approximation.

Of course, there remain numerous open questions. It remains a challenging yet unfeasible task to solve the kinetic equation (91) numerically. This would allow one to investigate the ultrashort time dynamics of strongly correlated systems. On the other hand, the very interesting problem of the dynamics of bound state correlations, their formation and break up requires further extensions of the theory. For this it is necessary to include three-particle scattering integrals, two-particle self-energy contributions, but also to account for the dynamical character of the interaction.

ACKNOWLEDGMENTS

We acknowledge discussions with Th. Bornath and S.V. Peletminski. This work is supported by the Deutsche Forschungsgemeinschaft (SFB 198 "Kinetik partiell ionisierter Plasmen" und Schwerpunkt "Quantenkohärenz in Halbleitern) and a grant for CPU time at the HLRZ Jülich.

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52. In cases, where the particles are created during some excitation process (e.g., laser pulse excitation in solids, ionization in plasmas), one may, alternatively, move the starting time t_0 back before the start of the excitation. Then the initial state is characterized by low density and $g^0_{12} = 0$, and the excitation dynamics has to be included in the calculation. However, this is not always possible or even desirable, in particular, if particles are present in the system permanently. Then, the initial state could be, e.g. a (correlated) equilibrium or stationary state, and the dynamics after an excitation is essentially influenced by these initial correlations [17, 29].
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