

Two-temperature relaxation in nonideal partially ionized plasmas

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Evolution equations for the coupled relaxation of densities and temperatures for the components in nonideal partially ionized plasmas are given. In these equations many-body effects, such as screening, self-energy, and lowering of the binding energy, are included. The coupled equations are solved numerically for a hydrogen plasma consisting of electrons, protons, and atoms. Impact ionization, three-body recombination, and elastic processes are taken into account. Thermal relaxation times are determined and the results are compared with those from the literature. The influence of many-body effects on the evolution process are discussed. In some cases, a significantly increased lifetime of the two-temperature regime is found. © 1996 American Institute of Physics. [S1070-664X(96)00704-7]

I. INTRODUCTION

In nonideal partially ionized plasmas, many-particle effects, such as screening, self-energy, bound states, and lowering of the ionization energy, play an important role.^{1,2} In Ref. 3, the influence of these effects on the reaction-diffusion process in nonideal plasmas was studied. Reaction-diffusion equations with generalized expressions for the coefficients of ionization, recombination, and diffusion were derived. Because of the nonideality, these coefficients are density dependent. Nonlinear behavior like nonlinear diffusion, bistability, and running ionization fronts were discussed. A limitation of Ref. 3 was that the density relaxation was considered in the case of homogeneous constant temperature.

In the next paper⁴ we took into account the temperature evolution. We derived hydrodynamic equations from generalized quantum kinetic equations for quasiparticles. With these equations one can describe the relaxation of the density and temperature of the plasma species in nonideal partially ionized plasmas. We only considered dense plasmas that are collision dominated. To highlight the effect of temperature on the relaxation process, we analyzed the simplest case—that of a single temperature common for all components. As a result, we found strong correlations between the evolution of chemical composition and temperature.

Now we are interested in the class of plasmas where light and heavy particles have not yet relaxed toward a common temperature. This is the case when the plasma is exposed to the action of electric fields⁵ or particle beams that generate highly energetic electrons. Such a plasma can also arise when a short pulse laser radiation interacts with solid matter.⁶ To focus on the evolution of the macroscopic quantities, we assume here that the quasihydrodynamic regime is reached.⁷ This means, each component has relaxed toward its own quasiequilibrium momentum distribution characterized by a specific temperature. At the same time, these temperatures are not yet in equilibrium with each other and chemical equilibrium has not yet been reached. This regime is justified for gaseous plasmas with large mass differences ($m_e \ll m_h$).

In this paper we study the relaxation process in the two-

temperature model.⁸ The validity of this model is discussed in Ref. 6. We analyze the coupled evolution of density, electron temperature, and temperature of heavy particles, assuming the same temperature for ions and atoms nonideal partially ionized plasmas. We study the evolution processes for large electron densities. That means we consider plasmas, which are collision dominated, and therefore the radiation processes can be neglected. We determine relaxation times and compare the results with those from the literature (see Refs. 7, 8, and 9). Of special interest is the influence of bound states on the temperature relaxation process, which is included in our approach.

Furthermore, we study many-particle effects in detail. We will show that the dominating effects are screening of the Coulomb interaction and lowering of the binding energy. As a result, the lifetime of the two-temperature regime is extended significantly.

Our paper is organized as follows. First we derive from the hydrodynamic equations the equations for densities and temperatures in the two-temperature model (Sec. II). Here, we take into account many-body effects. Then we apply these equations in the case of a nonideal hydrogen plasma. The transport coefficients, impact ionization, and three-body recombination coefficients for hydrogen are given in Sec. III. In Sec. IV we present numerical results for the solution of the coupled equations of density, electron temperature, and temperature of heavy particles in a spatially homogeneous hydrogen plasma. Finally, a discussion of the results is given in Sec. V.

II. HYDRODYNAMIC EQUATIONS FOR TWO-TEMPERATURE NONIDEAL PLASMA

In Ref. 4, hydrodynamic equations for nonideal partially ionized plasmas were derived. If we consider a system of electrons (e), singly charged ions (i), and atoms (ei), the resulting equations for the densities of free ($a=e,i$) and bound particles in the spatially homogeneous case are

$$\frac{\partial n_a}{\partial t} = W_a, \quad (1)$$

$$\frac{\partial n_{(ei)}}{\partial t} = \sum_j W_{(ei)}^j. \quad (2)$$

Here $n_{(ei)}$ is the total number density of atoms and is given by $n_{(ei)} = \sum_j n_{(ei)}^j$ (j -atomic level). Nonideality effects enter Eqs. (1) and (2) via the source functions of free particles W_a and of atoms $W_{(ei)}^j$. These functions are related by

$$W_e = W_i = - \sum_j W_{(ei)}^j = - \sum_j \int \frac{d^3 P}{(2\pi\hbar)^3} \sum_c I_{(ei)c}. \quad (3)$$

The source function of electrons, for instance, is given by

$$W_e = \sum_{c=e,i} \sum_j (\alpha_j^c n_c n_{(ei)}^j - \beta_j^c n_c n_e n_i). \quad (4)$$

The summation is over the impact particles and the quantum numbers of the atom. Here α_j^c and β_j^c are the coefficients of impact ionization and three-body recombination of the atomic level j . Explicit expressions for these rate coefficients can be found in Ref. 10.

The resulting equations for the energy densities in the nondegenerate case can be written as

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{3}{2} n_a k_B T_a + \frac{1}{2} \int \frac{d^3 p}{(2\pi\hbar)^3} \text{Re} \Sigma_a^R(p, \omega = \epsilon_a, t) f_a(p, t) \right) \\ = \int \frac{d^3 p}{(2\pi\hbar)^3} \epsilon_a(p, t) \left(\sum_b I_{ab} + \sum_{bc} I_{abc} \right), \end{aligned} \quad (5)$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{3}{2} n_{(ei)} k_B T_{(ei)} + \sum_j E_j n_{(ei)}^j \right) \\ = \sum_j \int \frac{d^3 P}{(2\pi\hbar)^3} \left(\frac{P^2}{2m_{(ei)}} + E_j \right) \sum_c I_{(ei)c}. \end{aligned} \quad (6)$$

The integrals I_{ab} , I_{abc} , and $I_{(ei)c}$ describe the two- and three-particle scattering processes and are given in Refs. 11 and 12. Because we are interested in the description of dense plasmas, radiation processes can be neglected.

The free particles (electrons and ions) were considered in a quasiparticle picture in which their energy is given by

$$\epsilon_a(p, t) = \frac{p^2}{2m_a} + \text{Re} \Sigma_a^R(p, \omega = \epsilon_a(p, t)). \quad (7)$$

It was shown in Refs. 1 and 13 that the energy shift of the bound states is small. Therefore this shift was neglected in Eq. (6).

As shown in Ref. 4, the system of Eqs. (1), (2), (5), and (6) has the important property that it conserves the total density of particles $n_a + n_{(ei)} = \text{const}$ with $a = e, i$ as well as the total energy density,

$$\begin{aligned} E^{\text{tot}} = \sum_{a=e,i} \left(\frac{3}{2} n_a k_B T_a + \frac{1}{2} \int \frac{d^3 p}{(2\pi\hbar)^3} \right. \\ \left. \times \text{Re} \Sigma_a^R(p, \omega = \epsilon_a, t) f_a(p, t) \right) \\ + \frac{3}{2} n_{(ei)} k_B T_{(ei)} + \sum_j E_j n_{(ei)}^j. \end{aligned} \quad (8)$$

In contrast to an approach starting from the ordinary Boltzmann equation, in Eq. (8) nonideality contributions in quasiparticle approximation are included.

The real part of the self-energy function $\text{Re} \Sigma_a^R$, which accounts for the interaction with the surrounding medium, can be calculated in the framework of the Green's functions techniques. An appropriate approximation is the so-called V^s approximation,¹ with V^s denoting the screened potential. In Eq. (5) there is needed the self-energy averaged with the distribution function. For local thermodynamic equilibrium one obtains in the lowest order the result^{1,14}

$$\int \frac{d^3 p}{(2\pi\hbar)^3} \text{Re} \Sigma_a^R(p, \omega = \epsilon_a, t) f_a(p, t) = - \frac{e^2 n_a}{2r_0}, \quad (9)$$

with the screening length

$$\frac{1}{r_0^2} = 4\pi \sum_b \frac{n_b e^2}{k_B T_b}. \quad (10)$$

Thus we have for the total energy density, conserved by Eqs. (5) and (6), the approximation

$$\begin{aligned} E^{\text{tot}} = \sum_{a=e,i} \left(\frac{3}{2} n_a k_B T_a - \frac{e^2 n_a}{4r_0} \right) + \frac{3}{2} n_{(ei)} k_B T_{(ei)} \\ + \sum_j E_j n_{(ei)}^j. \end{aligned} \quad (11)$$

The temporal change of the energy densities of the various species is determined by the collision integrals on the right-hand side (RHS) of Eqs. (5) and (6). The calculation of these terms involves the quasiparticle energies. In order to simplify the calculations, we will use the so-called rigid shift approximation for the quasiparticle energies,¹⁴

$$\epsilon_a(p, t) = \frac{p^2}{2m_a} + \Delta_a(t), \quad (12)$$

where the momentum-independent shift is given approximately by

$$\Delta_a(t) = \frac{1}{n_a} \int \frac{d^3 p}{(2\pi\hbar)^3} \text{Re} \Sigma_a^R(p, \omega = \epsilon_a, t) f_a(p, t). \quad (13)$$

Using Eq. (9), we have

$$\Delta_a = - \frac{e^2}{2r_0}. \quad (14)$$

Now we can derive the evolution equations of the temperatures for the light and heavy particles in the two-temperature model from Eqs. (5) and (6). The relaxation process is determined by the elastic and inelastic collisions between equal and different particle species. The energy transfer between electrons and heavy species (singly charged ions and atoms) is not very effective, because of the great mass difference. Therefore the thermal relaxation times between particles of great mass difference (electron-ion and electron-atom) are greater than the times between particles of equal mass (electron-electron, ion-ion, and atom-atom) or nearly equal mass (ion-atom). That means there exists a

stage where the electrons and heavy particles are in quasi-equilibrium with the respective temperatures T_e and T_h .

We obtain the temperature equation for the electrons from Eq. (5), together with Eq. (9). The time derivative on the left-hand side (LHS) can be easily calculated. The RHS of Eq. (5) describes the energy transfer due to the various collision processes. There are some useful approximations. The energy transfer terms of the elastic collisions of three free particles can be neglected in comparison to the elastic two-particle collisions. The impact ionization by ions is much less effective compared to the electron impact ionization. Therefore we restrict us to the latter process. Some details of the derivation are discussed in the Appendix. From the energy balance equation (5) for the electrons, we obtain

$$\begin{aligned} & \left(\frac{3}{2} k_B n_e + \frac{n_e e^2 T_{he}}{8 T_e^2 r_0} \right) \frac{\partial T_e}{\partial t} + \frac{n_e e^2 T_{he}}{8 T_h^2 r_0} \frac{\partial T_h}{\partial t} \\ & = \sum_j \left(\frac{3}{2} k_B T_e - \frac{7 e^2}{8 r_0} - E_j \right) W_{(ei)}^j + Z_{ei} + Z_{e(ei)}. \end{aligned} \quad (15)$$

The integrals Z_{eb} , $b=i, (ei)$ stand for the energy transfer following from the elastic collisions between electrons and ions,

$$\begin{aligned} Z_{ei} &= \frac{1}{\hbar V} \int \frac{d^3 p_e}{(2\pi\hbar)^3} \frac{d^3 p_i}{(2\pi\hbar)^3} \frac{d^3 \bar{p}_e}{(2\pi\hbar)^3} \frac{d^3 \bar{p}_i}{(2\pi\hbar)^3} \frac{p_e^2}{2m_e} \\ & \times |\langle p_e p_i | T_{ei} | \bar{p}_e \bar{p}_i \rangle|^2 2\pi \delta(E_{ei} - \bar{E}_{ei}) (\bar{f}_e \bar{f}_i - f_e f_i) \end{aligned} \quad (16)$$

and electron-atom collisions

$$\begin{aligned} Z_{e(ei)} &= \frac{1}{\hbar V} \sum_j \sum_{\bar{j}} \int \frac{d^3 p_e}{(2\pi\hbar)^3} \frac{d^3 P_{(ei)}}{(2\pi\hbar)^3} \frac{d^3 \bar{p}_e}{(2\pi\hbar)^3} \frac{d^3 \bar{P}_{(ei)}}{(2\pi\hbar)^3} \\ & \times \frac{p_e^2}{2m_e} |\langle p_e P_{(ei)} j | T_{e(ei)}^{11} | \bar{p}_e \bar{P}_{(ei)} \bar{j} \rangle|^2 2\pi \delta(E_{e(ei)j} \\ & - \bar{E}_{e(ei)\bar{j}}) (\bar{f}_e \bar{f}_{\bar{j}(ei)} - f_e f_{j(ei)}). \end{aligned} \quad (17)$$

The temperature equation of the heavy particles is obtained by summing up equations (5) for ions and (6) for atoms,

$$\begin{aligned} & \left(\frac{3}{2} k_B (n_e + n_{(ei)}) + \frac{n_e e^2 T_{he}}{8 T_h^2 r_0} \right) \frac{\partial T_h}{\partial t} + \frac{n_e e^2 T_{he}}{8 T_e^2 r_0} \frac{\partial T_e}{\partial t} \\ & = \frac{e^2}{8 r_0} \sum_j W_{(ei)}^j + Z_{ie} + Z_{(ei)e} + X_{(ei)e}. \end{aligned} \quad (18)$$

Here T_{he} is given by $T_{he} = T_h T_e / (T_h + T_e)$. In addition, Z_{ie} follows from Z_{ei} by substituting $p_i^2 / (2m_i)$ for $p_e^2 / (2m_e)$. If we replace $p_e^2 / (2m_e)$ by $p_{(ei)}^2 / (2m_{(ei)})$ in $Z_{e(ei)}$, we obtain $Z_{(ei)e}$. The integral $X_{(ei)e}$ is only important for excitation and deexcitation reactions and can be written as

$$\begin{aligned} X_{(ei)e} &= \sum_j E_j \sum_{\bar{j}} \frac{1}{\hbar V} \int \frac{d^3 p_e}{(2\pi\hbar)^3} \frac{d^3 P_{(ei)}}{(2\pi\hbar)^3} \frac{d^3 \bar{p}_e}{(2\pi\hbar)^3} \\ & \times \frac{d^3 \bar{P}_{(ei)}}{(2\pi\hbar)^3} |\langle p_e P_{(ei)} j | T_{e(ei)}^{11} | \bar{p}_e \bar{P}_{(ei)} \bar{j} \rangle|^2 2\pi \delta(E_{e(ei)j} \\ & - \bar{E}_{e(ei)\bar{j}}) \times (\bar{f}_e \bar{f}_{\bar{j}(ei)} - f_e f_{j(ei)}). \end{aligned} \quad (19)$$

In the following, we consider only the ground state ($\bar{j}=1$). This is a good approximation for the high densities we are interested in because the excited states have already vanished due to the Mott effect.

The LHS of the Eqs. (15) and (18) contain the derivatives of the temperature of both (electrons and heavy) species. The solution of these system of equations can be written as

$$\begin{aligned} \frac{\partial T_e}{\partial t} &= \frac{1}{k_1 k_4 - k_2 k_3} \left\{ k_1 \left[\left(\frac{3}{2} k_B T_e - \frac{7 e^2}{8 r_0} - E_1 \right) W_{(ei)}^1 + Z_{ei} \right. \right. \\ & \left. \left. + Z_{e(ei)} \right] - k_3 \left[\frac{1 e^2}{8 r_0} W_{(ei)}^1 + Z_{ie} + Z_{(ei)e} \right] \right\}, \end{aligned} \quad (20)$$

$$\begin{aligned} \frac{\partial T_h}{\partial t} &= \frac{1}{k_1 k_4 - k_2 k_3} \left\{ -k_2 \left[\left(\frac{3}{2} k_B T_e - \frac{7 e^2}{8 r_0} - E_1 \right) W_{(ei)}^1 \right. \right. \\ & \left. \left. + Z_{ei} + Z_{e(ei)} \right] + k_4 \left[\frac{1 e^2}{8 r_0} W_{(ei)}^1 + Z_{ie} + Z_{(ei)e} \right] \right\}. \end{aligned} \quad (21)$$

The terms k_1 , k_2 , k_3 , and k_4 are given by

$$k_1 = \frac{3}{2} k_B (n_e + n_{(ei)}) + k_3, \quad (22)$$

$$k_2 = \frac{n_e e^2 T_{he}}{8 T_e^2 r_0}, \quad (23)$$

$$k_3 = \frac{n_e e^2 T_{he}}{8 T_h^2 r_0}, \quad (24)$$

$$k_4 = \frac{3}{2} k_B n_e + k_2. \quad (25)$$

For comparison, we give the corresponding equations for an ideal plasma, i.e. nonideality effects are neglected,

$$\frac{\partial n_e}{\partial t} = -W_{(ei)}^1,$$

$$\frac{3}{2} k_B n_e \frac{\partial T_e}{\partial t} = \left(\frac{3}{2} k_B T_e - E_1 \right) W_{(ei)}^1 + Z_{ei} + Z_{e(ei)}, \quad (26)$$

$$\frac{3}{2} k_B (n_e + n_{(ei)}) \frac{\partial T_h}{\partial t} = Z_{ie} + Z_{(ei)e}.$$

Similar expressions are given in Ref. 5.

The calculation of the integrals Z_{ab} [see Eqs. (16) and (17)] is difficult. But we can use the quasihydrodynamic approximation. For a nondegenerate system with local equilibrium distribution functions for each species, the expressions Z_{ab} can be simplified,⁷

$$Z_{ab} = \frac{8k_B n_a n_b \mu_{ab}}{(m_a + m_b)} \sqrt{\frac{2}{\pi \varphi_{ab}}} Q_{ab}(T_b - T_a), \quad (27)$$

with $Z_{ba} = -Z_{ab}$, and

$$\mu_{ab} = \frac{m_a m_b}{(m_a + m_b)}, \quad \varphi_{ab} = \frac{\varphi_a \varphi_b}{(\varphi_a + \varphi_b)}. \quad (28)$$

The term φ_a can be written as $\varphi_a = m_a / (k_B T_a)$. Here Q_{ab} is an integral over the transport cross section and is given by

$$Q_{ab} = \int_0^\infty z^5 \exp(-z^5) Q_{ab}^T(z) dz, \quad (29)$$

with $z^2 = \varphi_{ab} \hbar^2 k^2 / (2\mu_{ab}^2)$. The transport cross section Q_{ab}^T can be calculated from the scattering phase shifts according to

$$Q_{ab}^T = \frac{4\pi}{k^2} \sum_l (l+1) \sin^2(\delta_{l+1} - \delta_l). \quad (30)$$

The numerical calculation of the phase shifts is described in the following section.

III. HYDROGEN PLASMA: RATE COEFFICIENTS AND TRANSPORT CROSS SECTIONS

In the following we consider a partially ionized hydrogen plasma, because we want to study the influence of non-ideality effects on the relaxation process in a simple example. In order to have an analytic expression for the reaction function $W_{(ep)}^1$, we use the results of Ref. 10. The ionization coefficient is then given by ($j=1$)

$$\alpha_1 = \alpha_1^{id} \exp[-(\Delta_e - \Delta_p) / k_B T_e], \quad (31)$$

$$\alpha_1^{id} = \alpha_0 g(|E_1| / k_B T_e). \quad (32)$$

Here α_0 and g are defined by

$$\alpha_0 = \frac{(10\pi a_B^2 |E_1|^{1/2})}{(2\pi m_e)^{1/2}}, \quad (33)$$

$$g(x) = x^{1/2} \int_x^\infty \left(\frac{\exp(-t)}{t} \right) dt. \quad (34)$$

There is a strong influence of many-body effects on the ionization coefficient. This coefficient is an increasing function with increasing electron density. The ionization and the recombination coefficients are related by

$$\beta_1 = \alpha_1 \lambda_e^3 \exp[I_1^{\text{eff}} / (k_B T_e)], \quad (35)$$

with λ_e being the thermal wavelength, and the effective ionization energy is $I_1^{\text{eff}} = |E_1| + \Delta_e + \Delta_p$. It was shown in Refs. 1 and 13 that the energy shift of the bound states is small. Therefore this shift was neglected in Eqs. (31) and (35). If we insert Eq. (31) into Eq. (35), it follows that β_1 is a function of T_e only. That means in this approximation the recombination coefficient is independent of the density. In Refs. 15–17, rate coefficients were calculated in a more rigorous way. The numerical results show that the many-body effects have only a small influence on the recombination in comparison with the ionization coefficients.

In the coupled set of balance equations [see Eqs. (1), (20), and (21)], we have, along with the reaction function $W_{(ep)}^1$, the contributions Z_{ep} and $Z_{e(ep)}$. They mainly determine the energy transfer in the collision process between electrons and heavy particles. In order to calculate these quantities, the corresponding transport cross sections Q_{ab}^T that can be expressed by the phase shifts [see Eq. (30)] must be known.

The phase shifts for the electron–proton collisions were determined by numerical solution of the radial Schrödinger equation using the Numerov method. The effective interaction potential between the charged particles was assumed to be a statically screened Coulomb potential,

$$V_{ep}^{\text{eff}}(q,0) = \frac{V_{ep}(q)}{\epsilon(q,0)}, \quad \epsilon(q,0) = 1 + \frac{\kappa^2}{q^2}, \quad (36)$$

where $V_{ep}(q) = -4\pi e^2 / q^2$ is the Fourier transform of the Coulomb potential and $\epsilon(q,0)$ is the RPA (random phase approximation) dielectric function in static approximation. Here κ is the inverse screening length [see Eq. (10)].

The elastic scattering of electrons on hydrogen atoms was treated on the basis of the close coupling equations of quantum scattering theory. Within perturbation theory, this system of equations can be reduced to an integrodifferential equation for the electron scattering wave function that describes the scattering of the electrons in an effective atomic potential.¹⁸ Neglecting exchange effects we obtain, instead of an integrodifferential equation,

$$\frac{d^2}{dr^2} f_l(r) + \left(k^2 - \frac{l(l+1)}{r^2} - V_{eH}^{\text{st}}(r) - V_{eH}^{\text{pol}}(r) \right) f_l(r) = 0. \quad (37)$$

This is a radial Schrödinger equation for the electron scattering wave function $f_l(r)$, where the electron–atom interaction is determined by a static and a polarization contribution. The static potential is given by

$$V_{eH}^{\text{st}}(r) = -e^2 \left(\frac{2}{r} + \frac{2}{a_B} \right) \exp\left(\frac{-2r}{a_B} \right). \quad (38)$$

In dipole approximation, assuming static Debye screening, the polarization potential is¹⁹

$$V_{eH}^{\text{pol}}(r) = -\frac{e^2}{2} \frac{\alpha}{(r^2 + r_1^2)^2} (1 + \kappa r)^2 \exp(-2\kappa r). \quad (39)$$

Here $\alpha = 4.5a_B^3$ is the atomic polarizability, and the parameter r_1 was chosen to be $r_1 = 1.456a_B$, which interpolates the behavior for small distances. As in the case of electron–proton scattering, we have calculated the transport cross section $Q_{e(ep)}^T$ from the scattering phase shifts, which were determined from Eq. (37) by numerical integration.

Numerical results are given in Refs. 20 and 21.

IV. NUMERICAL RESULTS FOR THE DENSITY AND TEMPERATURE EVOLUTION IN A HYDROGEN PLASMA

Let us study the kinetics of density and temperatures in a dense partially ionized hydrogen plasma. We consider a closed system, where the total number of electrons

$n = n_e + n_{(ep)}$ is constant. We use the density equation (1) and the temperature equations (20) and (21). We take into account the approximations (27) for the elastic collisions and Eqs. (31) and (35) for the rate coefficients. The energy shifts are given by Eq. (14). It is convenient to use dimensionless variables for density, time, and temperatures,

$$c = \frac{n_e}{n}, \quad \tau = \frac{t}{t_0}, \quad \theta_e = \frac{k_B T_e}{|E_1|}, \quad \theta_h = \frac{k_B T_h}{|E_1|}. \quad (40)$$

c is the degree of ionization, n is the total electron density, τ is the dimensionless time [$t_0 = (\alpha_0 n)^{-1}$], θ_e is the dimensionless electron temperature, and θ_h is the dimensionless temperature of heavy particles. Now we transform the evolution equations (1), (20), and (21) into dimensionless form,

$$\frac{\partial c}{\partial \tau} = c g f, \quad (41)$$

$$\begin{aligned} \frac{\partial \theta_e}{\partial \tau} = & \frac{1}{m_1 m_4 - m_2 m_3} \left\{ m_1 \left[\left(\frac{7}{12} \sqrt{c} \eta \theta_e - \theta_e - \frac{2}{3} \right) c g f \right. \right. \\ & \left. \left. + \frac{2}{3 |E_1| n^2 \alpha_0} (Z_{ep} + Z_{e(ep)}) \right] - m_3 \right. \\ & \left. \times \left(-\frac{1}{12} \sqrt{c} \eta \theta_e c g f + \frac{2}{3 |E_1| n^2 \alpha_0} (Z_{pe} + Z_{(ep)e}) \right) \right\}, \quad (42) \end{aligned}$$

$$\begin{aligned} \frac{\partial \theta_h}{\partial \tau} = & \frac{1}{m_1 m_4 - m_2 m_3} \left\{ -m_2 \left[\left(\frac{7}{12} \sqrt{c} \eta \theta_e - \theta_e - \frac{2}{3} \right) c g f \right. \right. \\ & \left. \left. + \frac{2}{3 |E_1| n^2 \alpha_0} (Z_{ep} + Z_{e(ep)}) \right] + m_4 \right. \\ & \left. \times \left(-\frac{1}{12} \sqrt{c} \eta \theta_e c g f + \frac{2}{3 |E_1| n^2 \alpha_0} (Z_{pe} + Z_{(ep)e}) \right) \right\}. \quad (43) \end{aligned}$$

The term $g = g(\theta_e)$ is given by Eq. (34), and the integrals $Z_{ab} = Z_{ab}(c, \theta_e, \theta_h, n)$ follow from Eq. (27). The function $f(c, \theta_e, \theta_h, n)$ is

$$f(c, \theta_e, \theta_h, n) = (1 - c) \exp[\eta(\theta_e, \theta_h, n) \sqrt{c}] - c^2 \chi(\theta_e, n), \quad (44)$$

where $\eta(\theta_e, \theta_h, n)$ and $\chi(\theta_e, n)$ are given by

$$\begin{aligned} \eta(\theta_e, \theta_h, n) &= \frac{2e^3}{|E_1| \theta_e} \left(\frac{\pi n}{|E_1| \theta_{he}} \right)^{1/2}, \\ \chi(\theta_e, n) &= \lambda_e^3(\theta_e) n \exp\left(\frac{1}{\theta_e}\right). \quad (45) \end{aligned}$$

The terms m_1 , m_2 , m_3 , and m_4 can be written as

$$m_1 = 1 + m_3, \quad m_2 = \frac{c \sqrt{c} \eta(\theta_e, \theta_h, n) \theta_{he}}{12 \theta_e}, \quad (46)$$

$$\begin{aligned} m_3 &= \frac{c \sqrt{c} \eta(\theta_e, \theta_h, n) \theta_e \theta_{he}}{12 \theta_h^2}, \quad m_4 = c + m_2, \\ \theta_{he} &= \frac{\theta_h \theta_e}{\theta_h + \theta_e}. \quad (47) \end{aligned}$$

The system of equations (41)–(43) describes the relaxation process in a dense two-temperature hydrogen plasma. Terms that contain the quantity $\eta = \eta(\theta_e, \theta_h, n)$ arise from the non-ideality. Furthermore, we take into account nonideality effects in Z_{ab} . All these terms show the influence of the many-body effects on the kinetics. The corresponding expressions for an ideal hydrogen plasma follow from Eqs. (41)–(43), if we set $\eta = 0$ and if we take transport cross sections for a small screening parameter λ_D .

In the following we want to study the influence of elastic and ionization (recombination) processes on the relaxation process. Also, we look at the modifications in partially ionized plasma, and we will compare our results with the low-density ideal case.

A. Effect of elastic collisions in a fully ionized plasma

First we study the evolution without inelastic collisions in a fully ionized plasma. Therefore, we set $c g f = 0$ in Eqs. (41)–(43). In this case the temperature relaxation is driven by the elastic $e-p$ collisions alone. From Eq. (41), it follows that the degree of ionization is constant. We solved Eqs. (42) and (43) numerically under the condition $c = \text{const} = 1$. From the numerical results, we have determined thermal relaxation times. First, we calculated the temperatures $\theta_e(\tau)$ and $\theta_h(\tau)$. Then we calculated the temperature difference ($\theta_e - \theta_h$) as a function of time. From the slope of this function, we obtained the thermal relaxation time similar to Ref. 22. Here $\tau_{\lambda_D}^1$ was determined, including nonideality effects. On the other hand, we neglected these effects in τ^1 . That means we set $\eta = 0$ in Eqs. (42) and (43) and we take transport cross sections for a small screening parameter λ_D .

These relaxation times can be compared with expressions from the literature. In fully ionized hydrogen plasmas ($c = 1$), the rate of energy transfer is given by the Landau-Spitzer (LSP) relaxation time,^{8,9}

$$\tau_{\text{LSP}}^1 = \frac{3 m_e m_p}{8 (2\pi)^{1/2} n_p e^4 \ln \Lambda} \left(\frac{k_B T_e}{m_e} \right)^{3/2}. \quad (48)$$

The Coulomb logarithm is $\ln \Lambda = \ln(l_\infty / l_0)$. Here l_∞ is equivalent to the screening length r_0 [see Eq. (10)], and $l_0 = \hbar / (2\pi \mu_{ep} k_B T_e)^{1/2}$.

The relaxation time can also be calculated according to Shdanov (SH),⁷

$$\tau_{\text{SH}}^1 = \frac{(m_e + m_p)}{2 \mu_{ep}} \tau_{ep}, \quad (49)$$

where τ_{ep} can be written as

$$\tau_{ep}^{-1} = \frac{4}{3} n_p \sqrt{\frac{8}{\pi \varphi_{ep}}} Q_{ep}, \quad (50)$$

and φ_{ep} is given by Eq. (28). The integral Q_{ep} is equivalent to Eq. (29). To calculate this integral, we need the transport cross sections Q_{ep}^T (see Sec. III).

TABLE I. Comparison of the thermal relaxation times in fully ionized hydrogen plasma ($c = 1$) without ionization and recombination processes. Here τ_{LSP}^1 is the Landau–Spitzer result.^{8,9} Also, τ_{SH}^1 is calculated from Ref. 7. Our results are $\tau_{\lambda_D}^1$, including nonideal effects and τ^1 the relaxation time in the low-density limit.

n (m^{-3})	T_e (K)	T_h (K)	τ_{LSP}^1 (s)	τ_{SH}^1 (s)	$\tau_{\lambda_D}^1$ (s)	τ^1 (s)
10^{26}	10 000	5000	2.08E-12	7.90E-12	3.48E-12	0.25E-12
10^{26}	30 000	5000	6.96E-12	16.55E-12	6.36E-12	0.87E-12
10^{26}	50 000	5000	13.01E-12	26.52E-12	9.03E-12	1.62E-12
10^{26}	50 000	5000	13.01E-12	26.52E-12	9.03E-12	1.62E-12
10^{26}	50 000	15 000	10.71E-12	21.11E-12	8.73E-12	1.62E-12
10^{26}	50 000	25 000	10.01E-12	19.26E-12	8.59E-12	1.63E-12
10^{27}	10 000	5000	4.22E-12	3.63E-12	1.64E-12	0.02E-12
10^{27}	30 000	5000	1.78E-12	4.09E-12	1.78E-12	0.09E-12
10^{27}	50 000	5000	2.77E-12	5.67E-12	2.03E-12	0.16E-12
10^{27}	50 000	5000	2.77E-12	5.67E-12	2.03E-12	0.16E-12
10^{27}	50 000	15 000	1.90E-12	3.97E-12	1.65E-12	0.13E-12
10^{27}	50 000	25 000	1.69E-12	3.54E-12	1.60E-12	0.14E-12

In Table I, the relaxation times τ_{LSP}^1 and τ_{SH}^1 according to Eqs. (48) and (49) and our results, $\tau_{\lambda_D}^1$ and τ^1 , are shown. The relaxation times show a qualitative agreement. With a constant temperature of heavy particles and increasing electron temperature, the relaxation is slower. There is one exception, because $\tau_{\text{LSP}}^1(T_e)$ can show a minimum (see Table I for the density $n = 10^{27} \text{ m}^{-3}$). In our results, we do not observe this behavior. On the other hand, with constant electron temperature and increasing temperature of heavy particles, the relaxation is faster. This is not valid for τ^1 , because non-ideality effects were neglected. Furthermore, we see that with increasing density the relaxation is quicker. Obviously, with decreasing interparticle distance, the collision frequency increases.

The Landau–Spitzer results τ_{LSP}^1 are in good agreement with our results $\tau_{\lambda_D}^1$. The relaxation times τ_{SH}^1 from formula (49) are too large.

The relaxation in nonideal plasma is slower in comparison with that in the low-density limit (compare $\tau_{\lambda_D}^1$ and τ^1). The explanation is that the two-particle interaction in nonideal plasmas is screened. The collisions are less effective compared to the low-density case.

B. Effect of elastic collisions in a partially ionized plasma

In the following we study the relaxation process in partially ionized plasmas ($c \neq 1$) again without inelastic collisions [$c_{gf} = 0$ in Eqs. (41)–(43), $c = \text{const}$], but taking into account additionally the $e-H$ scattering. The relaxation time can be calculated according to Ref. 7,

$$\tau_{\text{SH}}^2 = \frac{1}{2} \left(\frac{\mu_{ep} \tau_{ep}^{-1}}{m_e + m_p} + \frac{\mu_{eH} \tau_{eH}^{-1}}{m_e + m_H} \right)^{-1}, \quad (51)$$

where the τ_{ab}^{-1} are given by Eq. (50). For the calculation of τ_{SH}^2 , we need the $e-H$ transport cross sections (see Sec. III).

Table II shows the relaxation times τ_{SH}^2 in comparison to our results $\tau_{\lambda_D}^2$ and τ^2 . We solved Eqs. (42) and (43) under the conditions $c = \text{const} \neq 1$. But now, additionally to the $e-p$ scattering, we take into account the scattering between electrons and H atoms. We found the same qualitative behavior as in Table I. With a decreasing degree of ionization, the relaxation is slower because more bound states arise, and the $e-H$ scattering is less effective than the Coulomb scattering between electrons and protons.

TABLE II. Comparison of the thermal relaxation times in partially ionized hydrogen plasma ($c \neq 1$) without ionization and recombination processes. Here τ_{SH}^2 is calculated from Ref. 7. Our results are $\tau_{\lambda_D}^2$, including nonideal effects and τ^2 the relaxation time in the low-density limit.

n (m^{-3})	c	T_e (K)	T_h (K)	τ_{SH}^2 (s)	$\tau_{\lambda_D}^2$ (s)	τ^2 (s)
10^{27}	0.5	10 000	5000	4.39E-12	1.74E-12	0.05E-12
10^{27}	0.5	30 000	5000	5.29E-12	2.16E-12	0.18E-12
10^{27}	0.5	50 000	5000	7.19E-12	2.67E-12	0.38E-12
10^{27}	0.1	10 000	5000	5.27E-12	3.27E-12	0.33E-12
10^{27}	0.1	30 000	5000	6.91E-12	4.07E-12	0.97E-12
10^{27}	0.1	50 000	5000	9.17E-12	5.22E-12	1.74E-12
10^{27}	0.5	50 000	5000	7.19E-12	2.67E-12	0.38E-12
10^{27}	0.5	50 000	15 000	5.65E-12	2.61E-12	0.37E-12
10^{27}	0.5	50 000	25 000	5.20E-12	2.50E-12	0.37E-12
10^{27}	0.1	50 000	5000	9.17E-12	5.22E-12	1.74E-12
10^{27}	0.1	50 000	15 000	8.54E-12	5.11E-12	1.74E-12
10^{27}	0.1	50 000	25 000	8.31E-12	5.02E-12	1.75E-12

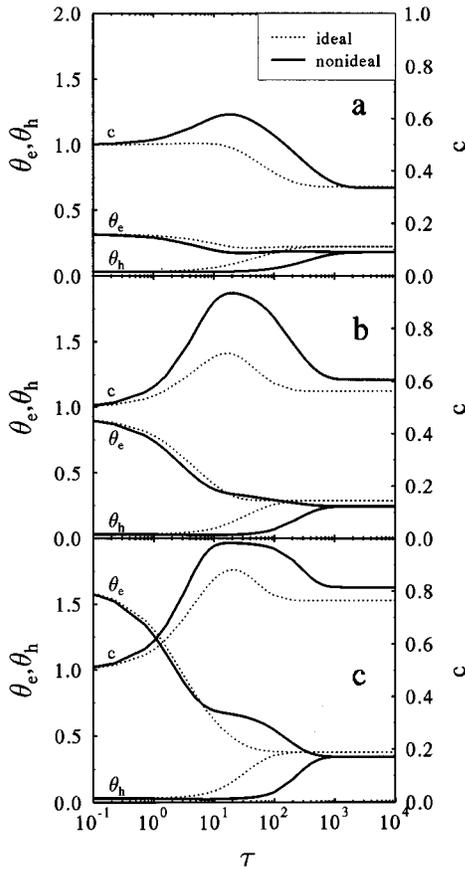


FIG. 1. Solution of the coupled density–temperature equations (41)–(43) for an ideal and nonideal hydrogen plasma. We start with the same initial values for the degree of ionization c and the temperature of heavy particles θ_h , but vary the electron temperature θ_e from (a) to (c). The total density is $n = 10^{27} \text{ m}^{-3}$ ($1\tau = 1.84 \times 10^{-14} \text{ s}$).

C. Relaxation including elastic and inelastic collisions

Let us now study the full relaxation process, including $e-p$ scattering, $e-H$ scattering, ionization by electrons, and three-body recombination. Starting with initial nonequilibrium values $c(\tau=0)$, $\theta_e(\tau=0)$, and $\theta_h(\tau=0)$, we can solve the system of Eqs. (41)–(43). Typical results are shown in Figs. 1 and 2. In these figures we demonstrate the relaxation of the degree of ionization c , the temperature of electrons θ_e , and heavy particles θ_h . In Fig. 1 we start with the same initial values for c and θ_h , but vary the electron temperature from Fig. 1(a) to 1(c). On the contrary, in Fig. 2 we vary the ionization degree c . For comparison, each run is also redone for the ideal (low-density) case.

Let us analyze the evolution process in detail. In the beginning of the relaxation process, we start with hot electrons. This is a typical situation. The energy of the electrons can be transformed in different channels: (i) elastic scattering between electrons and protons, (ii) elastic and inelastic scattering between electrons and H atoms. The elastic energy transfer is relatively slow for particles with a great mass difference. Such a situation is given in hydrogen plasmas. Therefore, it takes many collisions until the electrons have the same temperature as the heavy particles. On the other hand, we can observe a big energy transfer between an elec-

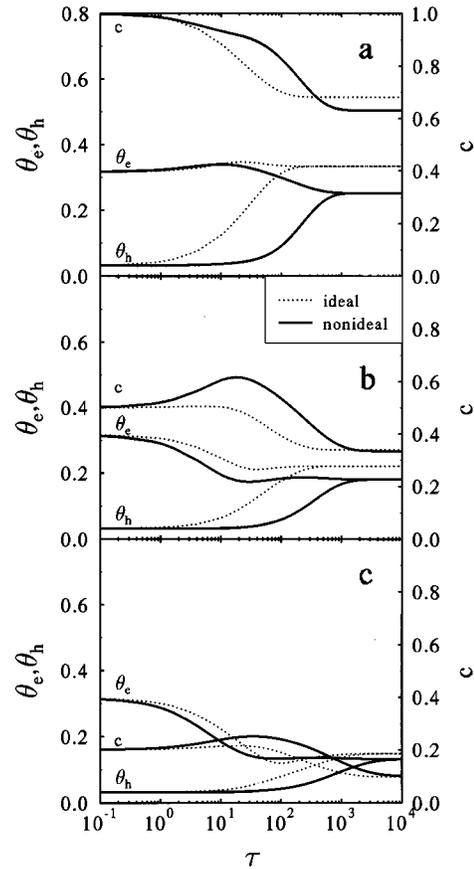


FIG. 2. The same as Fig. 1, but we vary the degree of ionization c .

tron and a H atom in a single inelastic collision, if $E_e^{\text{kin}} > I^{\text{eff}}$. Then the H atom is ionized. Therefore, the inelastic process cools the electrons more effectively than the elastic process.

Now we study the influence of nonideality effects on the relaxation process. The following effects have to be accounted for.

(1) Elastic scattering: (a) between electrons and protons: The Coulomb interaction between electrons and protons is screened. Therefore, the energy exchange is less effective compared to the ideal (low-density) case (see Refs. 20 and 21); (b) between electrons and H atoms: The influence of nonideality effects is very small (see Refs. 20 and 21).

(2) Ionization and recombination: In nonideal plasmas we observe a lowering of the ionization energy with increasing density. Therefore electrons with a kinetic energy lower compared to the ideal case can ionize the H atoms. That means, with increasing density, elastic collisions are becoming less effective compared to inelastic collisions.

Figures 1 and 2 show a two-time regime. The first fast regime is connected with the inelastic processes between electrons and H atoms. During this regime, the temperature of the heavy particles is nearly constant. We observe only a coupled relaxation of c and θ_e . The next time regime is determined by the elastic scattering (electron–proton and electron–H atom scattering). In this regime the temperature of electrons and heavy particles are adjusted.

In Fig. 1 the electron temperature is increased from Fig. 1(a) to 1(c). Because the electron distribution is a Maxwell function, there are more electrons with $E_e^{\text{kin}} > I^{\text{eff}}$ from Fig. 1(a) to 1(c) at the beginning of the relaxation process. The result is that the slope of the electron temperature curve decreases, and the slope of the curve of the ionization degree c increases from Fig. 1(a) to 1(c). The maximum of the ionization degree c is reached after $t = 3.4 \times 10^{-13}$ s. We can observe a plateau of c and θ_e . That means there is a saturation of c and θ_e , because nearly all H atoms are ionized by inelastic collisions. After the maximum, the elastic process is dominating. The temperature of electrons and heavy particles are adjusted. Thereby the degree of ionization is decreased to the equilibrium value.

From Fig. 2(a) to 2(c) the degree of ionization is decreased. The behavior in Figs. 2(b) and 2(c) is similar to Fig. 1. In Fig. 2(a) the ionization degree is only decreased to the equilibrium value, because all H atoms are ionized in the beginning. Due to the compensation of elastic and inelastic processes, we can observe a maximum in the curve $\theta_e(\tau)$.

The nonideality effects, which were discussed earlier, are clearly seen in Figs. 1 and 2. In nonideal plasmas, the equilibration process of θ_e and θ_h takes much longer than in ideal plasmas, because the Coulomb interaction is screened. This depends on the total density n . With increasing total density, the deviation from an ideal plasma increases. The influence of the effective ionization energy is clearly seen in Fig. 1. The ionization degree increases stronger than in ideal plasmas in the first time regime, because there are more electrons with $E_e^{\text{kin}} > I^{\text{eff}}$.

V. CONCLUSIONS

The aim of this paper was to study the influence of nonideality effects on the kinetics of macroscopic quantities like temperature and composition of a partially ionized plasma. Starting from hydrodynamic equations given in an earlier paper,⁴ evolution equations were derived for the temperatures of heavy and light particles in a nonideal three-component plasma. The nonideality corrections lead, in comparison to ideal plasmas, to an additional density dependence of the temperatures.

As a rather simple example, the coupled equations for densities and temperatures of the various species were numerically analyzed for a hydrogen plasma. In order to show the importance of the several processes, different levels of approximation were investigated. In a fully ionized hydrogen plasma, our results are in good agreement with the Landau–Spitzer relaxation times. Because of the screening, the relaxation in nonideal plasmas is slower compared with the low-density (ideal) case.

As a next step, a partially ionized plasma was studied, taking into account additionally only elastic electron–atom scattering. Because these collisions are less effective than the scattering between electrons and protons, the relaxation is slower with a decreasing degree of ionization.

Finally, the full relaxation process was analyzed, including elastic and inelastic collisions. In nonideal plasmas we found two competing influences of screening. First, the en-

ergy exchange between electrons and protons is less effective compared to the ideal case. On the other hand, we observe a lowering of the ionization energy with increasing density. Therefore, electrons with a lower kinetic energy, compared to the ideal case, are able to ionize the atoms. Hence, with increasing nonideality, hot electrons lose more and more energy in inelastic collisions (ionization), reducing the heating of the heavy particles. This leads to significantly increased lifetime of the two-temperature regime.

In our calculations, we restrict us to hydrogen atoms in the ground state. This is certainly a good approximation for very dense systems because excited states already disappeared due to the Mott effect. In an intermediate density region, one should, of course, take into account also all processes involving excited states. This will be done in a subsequent paper.

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APPENDIX: DERIVATION OF THE EVOLUTION EQUATIONS FOR THE TEMPERATURES OF ELECTRONS AND HEAVY PARTICLES

In this appendix, the derivation of Eqs. (15) and (18) is shown. At first we consider the electron temperature evolution. We start from Eq. (5) with the approximation (9). The time derivation of the LHS of Eq. (5) can be calculated,

$$\frac{\partial}{\partial t} \left(\frac{3}{2} n_e k_B T_e \right) = \frac{3}{2} n_e k_B \frac{\partial T_e}{\partial t} - \frac{3}{2} k_B T_e \sum_j W_{(ei)}^j, \quad (\text{A1})$$

$$\begin{aligned} \frac{\partial}{\partial t} \left(-\frac{e^2 n_e}{4r_0} \right) &= \frac{n_e e^2 T_{he}}{8T_e^2 r_0} \frac{\partial T_e}{\partial t} + \frac{n_e e^2 T_{he}}{8T_h^2 r_0} \frac{\partial T_h}{\partial t} \\ &+ \frac{3e^2}{8r_0} \sum_j W_{(ei)}^j. \end{aligned} \quad (\text{A2})$$

The RHS of Eq. (5) describes the energy transfer of the species due to elastic and inelastic collisions. The collision integrals on the RHS of this equation can be found, for instance, in the appendix of Ref. 4 [see Eqs. (A3)–(A6)]. The elastic electron–ion contribution to the energy transfer in the nondegenerate case can be written as

$$\begin{aligned} Z_{ei} &= \frac{1}{\hbar V} \int \frac{d^3 p_e}{(2\pi\hbar)^3} \frac{d^3 p_i}{(2\pi\hbar)^3} \frac{d^3 \bar{p}_e}{(2\pi\hbar)^3} \frac{d^3 \bar{p}_i}{(2\pi\hbar)^3} \epsilon_e \\ &\times |\langle p_e p_i | T_{ei} | \bar{p}_e \bar{p}_i \rangle|^2 2\pi \delta(E_{ei} - \bar{E}_{ei}) (\bar{f}_e \bar{f}_i - f_e f_i), \end{aligned} \quad (\text{A3})$$

with $E_{ei} = \epsilon_e(p_e) + \epsilon_i(p_i)$. The quasiparticle energies ϵ_e and ϵ_i are given by Eq. (12) with the momentum-independent shift from Eq. (14). In this approximation, the delta function in Eq. (A3) is given by

$$\delta(E_{ei} - \bar{E}_{ei}) = \delta \left(\frac{p_e^2}{2m_e} + \frac{p_i^2}{2m_i} - \frac{\bar{p}_e^2}{2m_e} - \frac{\bar{p}_i^2}{2m_i} \right). \quad (\text{A4})$$

Here Z_{ei} splits into two contributions. The first contribution is given by Eq. (16), whereas the second one vanishes.

The energy transfer terms due to elastic collisions of three free particles can be neglected in comparison to the two-particle collisions.

The electron–atom contribution to the energy transfer in the nondegenerate case can be written as

$$Z_{e(ei)} = \frac{1}{\hbar V} \sum_j \int \frac{d^3 p_e}{(2\pi\hbar)^3} \frac{d^3 P_{(ei)}}{(2\pi\hbar)^3} \frac{d^3 \bar{p}_e}{(2\pi\hbar)^3} \frac{d^3 \bar{p}_{(ei)}}{(2\pi\hbar)^3} \times \epsilon_e | \langle p_e P_{(ei)j} | T_{e(ei)}^{11} | \bar{p}_e \bar{P}_{(ei)j} \rangle |^2 2\pi \delta(E_{e(ei)j} - \bar{E}_{e(ei)j}) (\bar{f}_e \bar{f}_{j(ei)} - f_e f_{j(ei)}), \quad (\text{A5})$$

with $E_{e(ei)j} = \epsilon_e(p_e) + P_{(ei)j}/2m_{(ei)} + E_j$. The expression (17) follows similar to Eq. (16).

At last the reaction terms are considered. The impact ionization by ions is much less effective compared to the electron impact ionization. Therefore we restrict us to the latter process. Collecting the corresponding integrals to the ionization/recombination processes and using the properties of these integrals with respect to an interchange of the integration variables, we obtain

$$\int \frac{d^3 p_e}{(2\pi\hbar)^3} \epsilon_e(p_e) (2I_{e(ei)}^{01} + I_{e(ei)}^{02} + I_{e(ei)}^{03} + 2I_{(ei)e}^{10}) = \frac{1}{\hbar V} \sum_j \int \frac{d^3 p_e}{(2\pi\hbar)^3} \frac{d^3 P_{(ei)}}{(2\pi\hbar)^3} \frac{d^3 \bar{p}_e}{(2\pi\hbar)^3} \frac{d^3 \bar{p}_{(ei)}}{(2\pi\hbar)^3} \times \frac{d^3 \bar{p}_i}{(2\pi\hbar)^3} \left(-\frac{P_{(ei)}^2}{2m_{(ei)}} - E_j + \frac{\bar{p}_i^2}{2m_i} + \Delta_i(\bar{p}_i) \right) \times | \langle p_e P_{(ei)j} | T_{e(ei)}^{10} | \bar{p}_e \bar{p}_{(ei)} \bar{p}_i \rangle |^2 2\pi \delta(E_{e(ei)j} - \bar{E}_{e(ei)}) \times (\bar{f}_e \bar{f}_{(ei)} \bar{f}_i - f_e f_{(ei)}). \quad (\text{A6})$$

Using the adiabatic approximation and momentum-independent shifts, we can derive

$$\int \frac{d^3 p_e}{(2\pi\hbar)^3} \epsilon_e(p_e) (2I_{e(ei)}^{01} + I_{e(ei)}^{02} + I_{e(ei)}^{03} + 2I_{(ei)e}^{10}) = - \sum_j E_j W_{(ei)}^j - \frac{e^2}{2r_0} \sum_j W_{(ei)}^j. \quad (\text{A7})$$

From the results of this appendix, we can obtain the temperature equation of electrons [see Eq. (15)].

Similar considerations can be made for the temperature equation of heavy particles. We get Eq. (18) if we sum Eqs. (5) and (6) of ions and atoms, respectively.

¹W. D. Kraeft, D. Kremp, W. Ebeling, and G. Röpke, *Quantum Statistics of Charged Particle Systems* (Plenum, London, 1986).

²W. Ebeling, W. D. Kraeft, and D. Kremp, *Theory of Bound States and Ionization Equilibrium in Plasmas and Solids* (Akademie-Verlag, Berlin, 1976).

³D. Kremp, M. Schlanges, M. Bonitz, and Th. Bornath, *Phys. Fluids B* **5**, 216 (1993).

⁴Th. Ohde, M. Bonitz, Th. Bornath, D. Kremp, and M. Schlanges, *Phys. Plasmas* **2**, 3214 (1995).

⁵L. M. Bibermann, V. S. Vorob'ev, and I. T. Yakubov, *Kinetics of Nonequilibrium Low-Temperature Plasmas* (Nauka, Moscow, 1982; Consultants Bureau, New York, 1987), p. 110, p. 331.

⁶D. P. Kilcrease, *J. Quant. Spectrosc. Radiat. Transfer* **51**, 161 (1994).

⁷V. M. Shdanov, *Transport Processes in Multicomponent Plasmas* (Energoizdat, Moscow, 1982) (in Russian), pp. 15, 17, 19.

⁸E. M. Lifshitz and L. P. Pitaevskii, *Physical Kinetics* (Pergamon, Oxford, 1981), p. 184.

⁹L. Spitzer, Jr., *Physics of Fully Ionized Gases* (Interscience, New York, 1956).

¹⁰M. Schlanges, Th. Bornath, and D. Kremp, *Phys. Rev. A* **38**, 2174 (1988).

¹¹Yu. Klimontovich and D. Kremp, *Physica A* **109**, 517 (1981).

¹²Yu. Klimontovich, M. Schlanges, and Th. Bornath, *Contrib. Plasma Phys.* **30**, 349 (1990).

¹³R. Zimmermann, K. Kilimann, W. D. Kraeft, D. Kremp, and G. Röpke, *Phys. Stat. Sol. B* **90**, 175 (1978).

¹⁴R. Zimmermann, *Many-Particle Theory of Highly Excited Semiconductors* (Teubner, Leipzig, 1987).

¹⁵M. Schlanges and Th. Bornath, *Physica A* **192**, 262 (1993).

¹⁶Th. Bornath and M. Schlanges, *Physica A* **196**, 427 (1993).

¹⁷Th. Bornath, Th. Ohde, and M. Schlanges, *Physica A* **211**, 344 (1994).

¹⁸P. M. Stone and J. R. Reitz, *Phys. Rev.* **131**, 2101 (1963).

¹⁹D. Kremp, G. Röpke, and M. Schlanges, in *Transport Properties of Dense Plasmas*, edited by W. Ebeling, W.-D. Kraeft, and G. Röpke (Birkhäuser, Basel, 1984; Akademie-Verlag, Berlin, 1984), p. 156.

²⁰J. Starzynski, Diploma thesis, Universität Rostock, Rostock, 1992.

²¹J. Starzynski, R. Redmer, and M. Schlanges, "Electrical conductivity for nonideal partially ionized alkali plasmas," to appear in *Phys. Plasmas*.

²²J. P. Hansen and I. R. McDonald, *Phys. Lett. A* **97**, 42 (1983).