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# Kinetic energy relaxation and correlation time of nonequilibrium many-particle systems

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## Abstract

A simple method is proposed to calculate the correlation time of many-particle systems with binary collisions. It is based on the time evolution of kinetic energy and applies to arbitrary (quasi-)static interaction. The calculated nonequilibrium correlation time increases nearly linearly with the range of the interaction, similarly as in equilibrium.

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Short-time phenomena are currently of high interest in the context of laser–plasma interaction, ultrafast optical response of semiconductors [1] and nuclear matter as well [2]. These phenomena can be understood correctly only if the dynamics of one-particle and two-particle properties and their characteristic times, e.g. relaxation time  $\tau_R$  and correlation time  $\tau_{\text{cor}}$ , are known.

Traditionally, nonequilibrium properties of many-particle systems have been described quite successfully on the basis of Markovian kinetic equations for the one-particle distribution function, such as the Landau, Boltzmann or Lenard–Balescu equation. However, in neglecting the dynamics of binary and higher correlations, these equations are applicable only to time scales exceeding the correlation time of the system. At times  $t < \tau_{\text{cor}}$ , one has to consider the coupled relaxation of one-particle, two-particle and, possibly, higher order distribution functions.

The concept of a hierarchy of time scales is one of the fundamental principles in statistical physics. It is based on the experience that in most situations one observes a multi-stage process, where first the correlations relax (initial stage,  $t_0 \leq t < \tau_{\text{cor}}$ ), then the one-particle distribution (kinetic stage,  $\tau_{\text{cor}} < t < \tau_R$ ) and, finally, macroscopic quantities approach the stationary state (hydrodynamic stage,  $t > \tau_R$ ). Correspondingly, the theoretical description differs essentially regarding the relevant degrees of freedom of the system and the complexity of the models as well. Therefore, the choice of the appropriate description requires knowledge of the characteristic times. In particular, the correlation time sets the criterion whether a Markovian kinetic theory is applicable or whether more general kinetic equations have to be used [3,4]. On the other hand, processes with an overall duration shorter than the correlation time may be described even by significantly simpler collisionless theories, such as the Vlasov equation [5].

The correlation time  $\tau_{\text{cor}}$  is one of the shortest characteristic times in interacting many-body systems (exceptions may be systems consisting of species with large mass difference or partially ionized systems [6]). It is usually (within a classical picture) identified with the time one particle needs to fly through the interaction range  $r_{\text{int}}$  of a short range potential (range of a Yukawa potential, Debye radius of a plasma and so on) [7]. Thus, in equilibrium

$$\tau_{\text{cor}} = r_{\text{int}}/v_{\text{th}}, \quad (1)$$

where  $v_{\text{th}}$  denotes the thermal velocity. This is a very useful expression, which depends only on thermodynamic parameters, such as density and temperature. However, it is not clear how Eq. (1) is related to correlations in nonequilibrium. There, the situation is much more complex:

- (i) No quantity  $v_{\text{th}}$  is defined;
- (ii) The kinetic energy of the system may change in time due to correlation buildup (total energy conservation [4]);
- (iii) Density may change (e.g. in carrier generation processes in plasmas or semiconductors);
- (iv) The interaction range of the potential may change during the evolution (e.g. buildup or deformation of the screening cloud in a plasma subject to short laser pulses or external fields).

In this paper we will not consider the last two points, i.e. we deal with situations where density and also the interaction radius  $r_{\text{int}}$  are constant. In particular, we will not consider the specific situation of plasmas generated by ultra-short laser pulses, where the buildup of correlations is essentially modified by screening buildup (there, the time scale is the inverse plasma frequency, which follows immediately from Eq. (1) by inserting for  $r_{\text{int}}$  the Debye radius). Interesting first nonequilibrium results derived from a dynamical two-time approach to screening have been presented in Refs. [8,9].

For the calculation of the correlation time in nonequilibrium, we use a dynamic approach. We consider the evolution of the binary correlation function  $g_{12}$ . We define the correlation time as the time after which the correlations reach (quasi-)stationary form. This is a natural choice which is in analogy with the definition of the relaxation time as the equilibration time of the one-particle distribution function. To do this we notice that the binary correlation function has a two-fold time dependence. First, there is an explicit dependence arising from short-scale two-particle processes. On the other hand, the correlation function depends on the evolution of the one-particle distribution function  $F$ , which gives rise to an additional, in most cases slow time dependence. According to the picture of the hierarchy of time scales, we expect that first there is a relaxation of the fast processes (initial stage). After this stage is over (after the correlation time), the correlations depend on time only weakly, via the distribution functions (Bogolyubov's functional hypothesis [10]), and the evolution enters the kinetic regime. This means that

$$\begin{aligned} g_{12}(t) &= g_{12}(t, [F(t)]), & t_0 \leq t \leq \tau_{\text{cor}}, \\ &= g_{12}([F(t)]), & t > \tau_{\text{cor}}. \end{aligned} \quad (2)$$

Of course, the transition from one stage to the other is gradual and it is practically difficult to determine the correlation time from the continuous evolution of the binary correlation function. Moreover, this evolution is rather complex, since  $g_{12}$  depends on four variables (e.g. momenta) and time. Therefore, it is useful to consider the behavior of certain macroscopic quantities calculated as moments of the correlation function, such as the autocorrelation function or the potential energy. We will see below that, indeed, the evolution of the potential (or kinetic) energy is well suited for determining the correlation time in a nonideal many-particle system.

Though this concept for calculating the correlation time is rather generally applicable to non-Markovian relaxation, we will illustrate it on a simple example. We will consider the second Born approximation (Landau kinetic equation), for which the time evolution of the binary correlation function is known exactly. Moreover, the two-fold time dependence of the correlation function can be seen explicitly. Thus, the results will be valid for weakly nonideal systems.

The non-Markovian kinetic equation in the second Born approximation (Landau equation), Eq. (8), has been derived for classical systems by Klimontovich [7]. It was generalized to quantum systems by Haug and Eli who used the Green's functions technique [3]. An alternative derivation has been given in Ref. [4] in the framework of a density operator approach which in addition to the kinetic equation yields the dynamics of the binary correlation function, Eq. (7). Here we present only the final results, Eqs. (7), (8), along with the specific approximation to the BBGKY-hierarchy, which allows us to derive these results fully including self-energy effects (for details we refer to Ref. [4]).

The coupled equations for the one-particle density operator and the binary correlation operator in the second Born approximation are [4]

$$i\hbar \frac{d}{dt} F_1 - [H_1 + U^{\text{HF}}, F_1] = n \text{Tr}_2 [V_{12}, g_{12}], \quad (3)$$

$$i\hbar \frac{d}{dt} g_{12} - (H_{12}^{\text{eff}} g_{12} - g_{12} H_{12}^{\text{eff}\dagger}) = \hat{V}_{12} F_1 F_2 - F_1 F_2 \hat{V}_{12}^\dagger, \quad (4)$$

where  $V_{12}$  is the binary interaction potential,  $U^{\text{HF}}$  is the Hartree–Fock potential,  $\hat{V}_{12} = (1 \pm F_1 \pm F_2) V_{12}$ ,  $n$  is the density,  $H_1 = p^2/2m$  and  $H_{12}^{\text{eff}} = \bar{H}_1 + \bar{H}_2$ . Here we have neglected quantum mechanical exchange terms which are not important for the following considerations. The renormalized one-particle Hamiltonian is given by

$$\bar{H}_1 g_{12}(t) = \left( \frac{p_1^2}{2m_1} + \Sigma_1^{\text{HF}} \right) g_{12}(t) + \frac{1}{i\hbar} \int_{t_0}^t d\bar{t} \Sigma_1^+(t\bar{t}) U_2^+(t\bar{t}) g_{12}(\bar{t}) U_1^-(\bar{t}t) U_2^-(\bar{t}t), \quad (5)$$

where the self-energy in the second Born approximation,

$$\Sigma_1^+(t, t') = n \text{Tr}_3 V_{13} U_1^+(tt') U_3^+(tt') V_{13} F_3(t') U_3^-(t't), \quad (6)$$

is related self-consistently to the one-particle propagators  $U_1^\pm$ . The formal solution of the initial value problem for the binary correlation function in a spatially homogeneous system is given by

$$\begin{aligned} g(\mathbf{p}_1, \mathbf{p}_2, \bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2, t) &= \exp\left(-\frac{i}{\hbar}(E_{12} - \bar{E}_{12})(t - t_0)\right) \exp\left(-\frac{1}{\hbar}(\gamma_{12} + \bar{\gamma}_{12})(t - t_0)\right) g_0(\mathbf{p}_1, \mathbf{p}_2, \bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2) \\ &- \frac{1}{i\hbar} \int_0^{t-t_0} d\tau \exp\left(-\frac{i}{\hbar}(E_{12} - \bar{E}_{12})\tau\right) \exp\left(-\frac{1}{\hbar}(\gamma_{12} + \bar{\gamma}_{12})\tau\right) V(\mathbf{p}_1 - \bar{\mathbf{p}}_1) \\ &\times [\bar{F}_1 \bar{F}_2 (1 \pm F_1)(1 \pm F_2) - F_1 F_2 (1 \pm \bar{F}_1)(1 \pm \bar{F}_2)]|_{t-\tau}, \end{aligned} \quad (7)$$

where  $E_{12} = p_1^2/2m + p_2^2/2m$ ,  $F_1 = F(\mathbf{p}_1)$ ,  $\bar{F}_1 = F(\bar{\mathbf{p}}_1)$ . The term containing  $g_0$  is related to the initial correlations, whereas the integral reflects the correlation buildup due to binary collisions. The integral has a memory structure, i.e. it depends on the state of the system (distribution functions) at all previous times  $t_0 \leq \tau \leq t$ . However, the memory depth is limited by the damping exponents, where the damping coefficients are related to the imaginary part of the single-particle self-energy  $\gamma_{12} = \text{Im} \Sigma_1 + \text{Im} \Sigma_2$  [3,4].  $\gamma_{12}$  depends on the actual distribution function and has, in principle, to be calculated self-consistently with the solution of the kinetic equation. The damping terms assure the correct long-time behavior: weakening of initial correlations and transition to the Markovian regime. Here, only the formation of the correlations will be of interest to us, since the saturation of this process will take place just at  $t \sim \tau_{\text{cor}}$ . The contribution from the initial correlations only slightly alters the correlation buildup and we, therefore, do not need to consider it below ( $g_0 = 0$ ).

Eq. (7) clearly shows the two-fold time dependence of the binary correlations which was mentioned above. Along with the slow time dependence via the distribution functions, there is the fast time dependence, explicitly

given by the exponential factors. With increasing time, the oscillations of the exponents become more rapid (except for  $E_{12} = \bar{E}_{12}$ ) and decrease in amplitude. We expect that after a sufficiently long time these fast oscillations will be damped, and the evolution of all macroscopic quantities will be determined by the envelope which is given by  $F(t)$  alone.

To verify this expectation, one has to solve the corresponding non-Markovian kinetic equation which is obtained by inserting the solution (7) with  $g_0 = 0$  on the r.h.s. of Eq. (3),

$$\begin{aligned} \frac{d}{dt} F_1(t) &= \frac{2n}{\hbar^2} \int_0^{t-t_0} d\tau \int \frac{d\mathbf{p}_2}{(2\pi\hbar)^3} \int \frac{d\bar{\mathbf{p}}_1}{(2\pi\hbar)^3} \int \frac{d\bar{\mathbf{p}}_2}{(2\pi\hbar)^3} (2\pi\hbar)^3 \delta(\mathbf{p}_{12} - \bar{\mathbf{p}}_{12}) V^2(\bar{\mathbf{p}}_1 - \mathbf{p}_1) \\ &\times \cos\left(\frac{E_{12} - \bar{E}_{12}}{\hbar} \tau\right) e^{-(\gamma_{12} + \bar{\gamma}_{12})\tau/\hbar} [\bar{F}_1 \bar{F}_2 (1 \pm F_1)(1 \pm F_2) - F_1 F_2 (1 \pm \bar{F}_1)(1 \pm \bar{F}_2)]|_{t-\tau}, \end{aligned} \quad (8)$$

where  $\mathbf{p}_{12} = \mathbf{p}_1 + \mathbf{p}_2$ . Numerical solutions of this generalized kinetic equation have been performed under additional approximations for the self-energy, e.g. in Refs. [1,4]. Using the solution  $F(t)$ , one obtains from Eq. (7) the relaxation of the binary correlation function. It indeed shows some signature of the two-stage behavior mentioned above [4]. However, one finds that the correlation buildup is much more clearly seen in the relaxation of the potential energy  $\langle V_{12} \rangle$ , or, due to total energy conservation,  $d\langle T \rangle/dt = -d\langle V_{12} \rangle/dt$ , in the kinetic energy relaxation as well. The kinetic energy follows immediately from the solution  $F(\mathbf{p}, t)$  of the kinetic equation,

$$\langle T \rangle(t) = \int \frac{d^3 p}{(2\pi\hbar)^3} \frac{p^2}{2m} F(\mathbf{p}, t).$$

Therefore, the kinetic energy is well suited for the determination of the correlation time and will be the central quantity below.

This is a general concept which can be applied also to other non-Markovian kinetic equations, that are beyond the second Born approximation, as long as they conserve total energy. It applies also to two-time equations, such as the Kadanoff–Baym equations [12], which provide the most adequate description of the system.

The main practical problem is that the numerical effort to solve the non-Markovian kinetic equation or the Kadanoff–Baym equations, is quite large. Fortunately, various properties of the system are not very sensitive to many details of the non-Markovian model. This has been shown in Ref. [4] by comparing the time relaxation of the distribution function and the correlation function for different approximations of Eq. (7) to the solution of the Kadanoff–Baym equations. Important properties, such as the evolution of the kinetic and potential energies, are very well reproduced by the zeroth order retardation approximation without self-energy (see below). Therefore, this approximation is well suited for the calculation of the correlation time in non-Markovian relaxation (in second Born approximation) too, reducing, at the same time, computer time by 1 to 2 orders of magnitude.

The zeroth order retardation approximation follows from Eq. (7) by neglecting the retardation in the distribution functions ( $F(t - \tau) \approx F(t)$ ) in the integral term together with initial correlations and self-energy effects ( $g_0 = \gamma_{12} = 0$ ). In this approximation, the imaginary part of the binary correlation matrix element is

$$\begin{aligned} \text{Im } g_0(\mathbf{p}_1, \mathbf{p}_2, \bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2, t) &= \frac{1}{\hbar} \frac{\sin[(E_{12} - \bar{E}_{12})(t - t_0)/\hbar]}{E_{12} - \bar{E}_{12}} V(\mathbf{p}_1 - \bar{\mathbf{p}}_1) \\ &\times [\bar{F}_1 \bar{F}_2 (1 \pm F_1)(1 \pm F_2) - F_1 F_2 (1 \pm \bar{F}_1)(1 \pm \bar{F}_2)]|_t. \end{aligned} \quad (9)$$

The collision integral of the kinetic equation in zeroth retardation order is obtained by inserting Eq. (9) on the r.h.s. of Eq. (3),

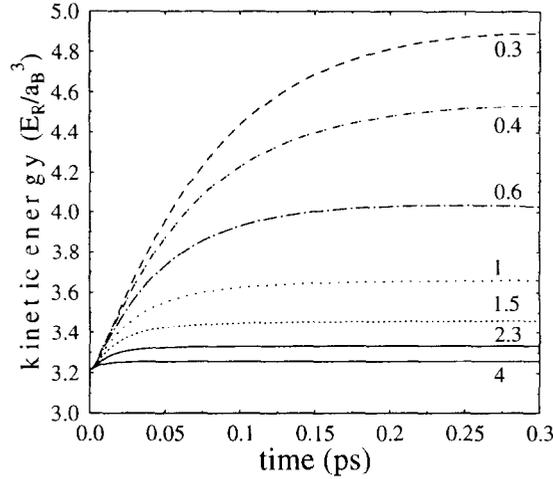


Fig. 1. Relaxation of the kinetic energy for different ranges of the static interaction potential. The numbers in the figure are the values of the potential parameter  $\kappa$  in units of  $a_B^{-1}$ .

$$\begin{aligned} \frac{d}{dt}F_1(t) &= \frac{2n}{\hbar} \int \frac{d\mathbf{p}_2}{(2\pi\hbar)^3} \int \frac{d\bar{\mathbf{p}}_1}{(2\pi\hbar)^3} \int \frac{d\bar{\mathbf{p}}_2}{(2\pi\hbar)^3} (2\pi\hbar)^3 \delta(\mathbf{p}_{12} - \bar{\mathbf{p}}_{12}) V^2(\bar{\mathbf{p}}_1 - \mathbf{p}_1) \\ &\times \frac{\sin\{[ (E_{12} - \bar{E}_{12})/\hbar](t - t_0)\}}{E_{12} - \bar{E}_{12}} [\bar{F}_1 \bar{F}_2 (1 \pm F_1)(1 \pm F_2) - F_1 F_2 (1 \pm \bar{F}_1)(1 \pm \bar{F}_2)]|_t, \end{aligned} \quad (10)$$

This approximation includes important properties of the full equation, such as the two-fold time dependence, the energy broadening and the correlation buildup. Thus, the short-time behavior is described correctly. In the long-time limit, in which we are not interested here, the neglect of retardation in Eqs. (9), (10) is not justified. However, due to the damping in Eqs. (7), (8), the actual memory depth is, in many cases, small [4] and there are only small deviations from the full equation.

We illustrate our approach by numerical results obtained from the solution of Eq. (10). We consider the spatially homogeneous situation and assume isotropy in momentum space,  $F(\mathbf{p}) = F(p)$ . To be specific, we choose, as an example, electrons in a GaAs bulk semiconductor. The parameters are the effective mass  $m = 0.067m_0$  ( $m_0$  is the electron mass in vacuum), and the Rydberg energy  $E_R = 4.2$  meV, corresponding to a background dielectric constant  $\epsilon_b = 13.998$  and an exciton Bohr radius  $a_B = 132$  Å. We use a Yukawa-type interaction potential with the Fourier transform  $V(q) = \gamma/(q^2 + \kappa^2)$ . This model covers a broad class of both neutral particles and plasmas with constant carrier density (neglecting, however, screening buildup [8,9], which is beyond the scope of this paper). The following results are obtained for  $\gamma = 4\pi e_0^2/\epsilon_b$  ( $e_0$  is the free electron charge). Modifying  $\kappa$ , we vary the range of the potential. We studied various types of initial nonequilibrium distributions. As a typical example, we present here a Gaussian centered around the momentum  $p_0 = 2\hbar/a_B$ , corresponding to a density of  $n = 1.49 \times 10^{18}$  cm $^{-3}$ , cf. the dotted line in Fig. 3.

Fig. 1 shows the time evolution of kinetic energy, which confirms the above prediction. At short times, retardation effects are negligible, and the approximation (9), (10) describes the relaxation correctly. We first observe an increase of (the absolute value of) the potential energy, which is due to the buildup of binary correlations in the initially uncorrelated system. Notice that despite the fact that we are dealing with a one-component plasma with repulsive interaction, we had to assume a neutralizing background. Otherwise, the system would be unstable. The total interaction within the whole system is attractive, thus  $\langle V_{12} \rangle < 0$ . Due to conservation of total energy, the increase of the absolute value of potential energy results in an increase of kinetic energy, cf. Fig. 1 [4]. When the system has become correlated, the kinetic energy increase saturates,

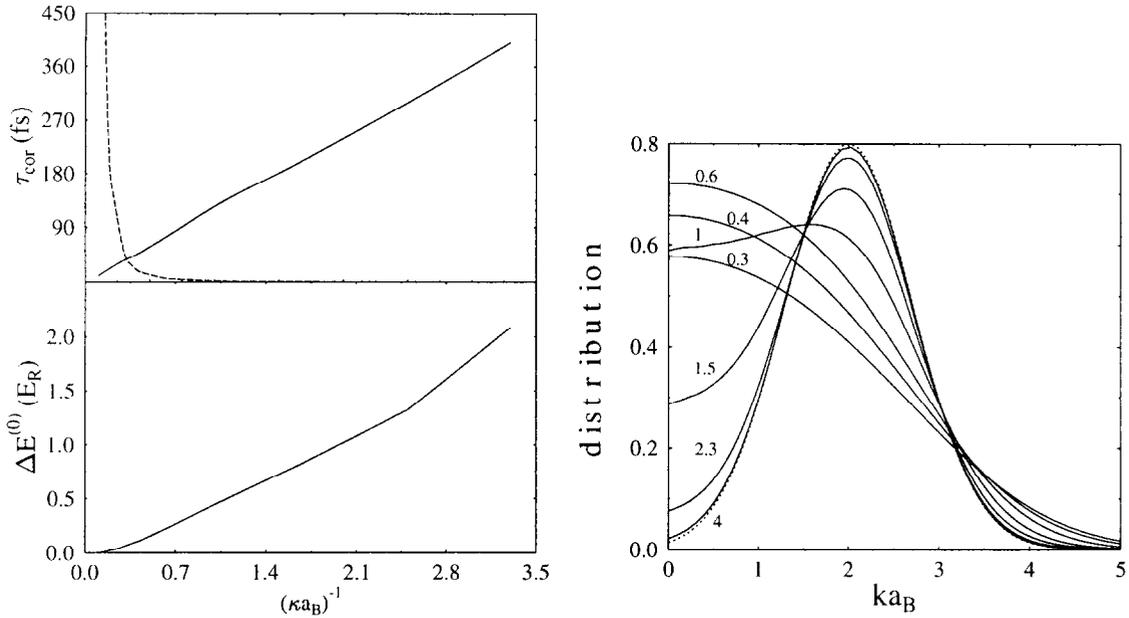


Fig. 2. (a) Nonequilibrium correlation time  $\tau_{\text{cor}}$  versus potential range, corresponding to the data of Fig. 1. Dashed line: the characteristic time  $t_{\text{dph}}$  calculated from the Markovian scattering rates, cf. Eq. (12). (b) Kinetic energy increase during the relaxation versus potential range.

Fig. 3. Relaxation of the distribution function for different values of  $\kappa$ , starting from a nonequilibrium distribution (dotted line). Snapshots of the distribution function after  $t = \tau_{\text{cor}}(\kappa)$ . The numbers in the figure are the values of  $\kappa$  in units of  $a_B^{-1}$ .

resulting in a crossover of the behavior of  $d\langle T \rangle/dt$ . This is just the transition from the initial to the kinetic stage of evolution, which occurs at  $t \sim \tau_{\text{cor}}$ .

In the long-time (Markov) limit, the kinetic energy does not change any longer since kinetic energy is conserved in each binary collision,

$$\frac{\sin[(E_{12} - \bar{E}_{12})t/\hbar]}{(E_{12} - \bar{E}_{12})/\hbar} \rightarrow \hbar\delta(E_{12} - \bar{E}_{12}).$$

In Fig. 2a, the corresponding crossover times extracted from Fig. 1 are shown versus the potential range  $1/\kappa$ . With increasing range of the static potential, the formation of correlations takes longer. This is in agreement with expectations, since the number of particles in the sphere of radius  $1/\kappa$ , which have to establish their pair correlations, increases ( $n = \text{const}$ ). Accordingly, the potential (and kinetic) energy increase grows with the interaction range, cf. Fig. 2b. The dependence of  $\tau_{\text{cor}}$  on  $\kappa^{-1}$  is almost linear, as in equilibrium, cf. Eq. (1), even though, in our case, the momentum distribution is far from an equilibrium one and no temperature (thermal velocity) has been established yet. Nevertheless, this agreement with equilibrium behavior is reasonable, if the characteristic velocity, e.g.  $\sqrt{\langle v^2 \rangle}$  changes only weakly within the correlation time. As one can see from Fig. 2b ( $\langle v^2 \rangle \sim \Delta E$ ), this holds for short range potentials. How fast  $\langle v^2 \rangle$  changes depends on the change of the distribution function during the interval  $t_0 \leq t \leq \tau_{\text{cor}}$ . Again, this change is more significant for long range potentials. In Fig. 3, we show along with the initial distribution  $F(t_0)$  (dotted line) snapshots of the distribution functions at  $t = \tau_{\text{cor}}$  for different values of  $\kappa$ , using the values  $\tau_{\text{cor}}(\kappa)$  from Fig. 2a. One clearly sees that during the equilibration of the binary correlations ( $t \leq \tau_{\text{cor}}$ ), there are only minor changes of the distribution for large  $\kappa$  ( $\geq 2/a_B$ ). On the other hand, for long range potentials ( $\kappa \leq 1/a_B$ ), the distribution

function is already close to its equilibrium shape at  $t \sim \tau_{\text{cor}}$ . In this case, there is no separation of one-particle and two-particle time scales at all, the correlation time approaches the relaxation time of the system.

Despite the specific choice of material parameters in our calculation, the qualitative behavior of the correlation time as a function of the potential range should rather generally hold. Moreover, a similar behavior should be expected also for strongly coupled systems, where the Born approximation does not apply, which is confirmed by molecular dynamics simulations of dense plasmas [11]. Furthermore, in each special case, the potential range will be defined by different system parameters. So in the case of a charged particle system in equilibrium, the potential range is the Debye radius (or Thomas–Fermi radius). In nonequilibrium, one can similarly define an inverse potential range  $\kappa$  using the (equilibrium) relation for the static limit of the RPA (random phase approximation) dielectric function,  $\epsilon(\mathbf{q}, \omega = 0) = 1 + \kappa^2/q^2 = 1 - V(q) \Pi(\mathbf{q}, \omega = 0)$ , where the polarization function  $\Pi$  is calculated with the nonequilibrium distribution function  $F$ . In an isotropic system and for  $q \rightarrow 0$  one obtains

$$\kappa^2(t) = \frac{4m}{\pi} \int_0^\infty dp F(p, t). \quad (11)$$

This is a useful quantity if the potential range itself does not (or only slowly) change during the correlation time, as it is the case in our example of a constant carrier density. For the parameters and initial distribution chosen in our case,  $\kappa = 2.3a_{\text{B}}^{-1}$  and  $\tau_{\text{cor}} \approx 45$  fs, cf. Fig. 2a. In this case, there is a clear separation of one-particle and two-particle time scales: during the correlation time, the distribution function undergoes almost no change, Fig. 3.

Finally, we mention a different approach to correlation time. Considering Eq. (7), it is clear that both weakening of initial correlations and the correlation buildup are essentially determined by the damping coefficients  $\gamma_{12}$  (or dephasing rates in optics terminology). The characteristic time

$$t_{\text{deph}}(\mathbf{p}_1, \mathbf{p}_2, \bar{\mathbf{p}}_1, \bar{\mathbf{p}}_2) = \frac{\hbar}{\gamma(\mathbf{p}_1) + \gamma(\mathbf{p}_2) + \gamma(\bar{\mathbf{p}}_1) + \gamma(\bar{\mathbf{p}}_2)}, \quad (12)$$

is related to the imaginary part of the self-energy in the second Born approximation  $\gamma(\mathbf{p}_1, t) = \text{Im} \Sigma(\mathbf{p}_1, t)$ . To avoid a complicated self-consistent non-Markovian calculation of self-energy and distribution function, one can try to approximate  $\Sigma$  by its Markov limit,

$$\begin{aligned} \text{Im} \Sigma^{\text{M}}(\mathbf{p}_1, t) &= \frac{2n}{\hbar} \int \frac{d\mathbf{p}_3}{(2\pi\hbar)^3} \frac{d\mathbf{q}}{(2\pi\hbar)^3} V^2(\mathbf{q}) \delta(E_{\mathbf{p}_3+\mathbf{q}} + E_{\mathbf{p}_1-\mathbf{q}} - E_{\mathbf{p}_1} - E_{\mathbf{p}_3}) \\ &\times \{F(\mathbf{p}_3)[1 \pm F(\mathbf{p}_1 + \mathbf{q})][1 \pm F(\mathbf{p}_3 - \mathbf{q})] + [F \leftrightarrow 1 \pm F]\}_t. \end{aligned} \quad (13)$$

Still the problem remains that  $t_{\text{deph}}$  depends on all momenta in a complicated way and that it changes with time. If the distribution function changes weakly on the scale of the correlation time, it is reasonable to use the initial distribution in Eq. (13). Also, the choice of the values for the momentum arguments of  $\text{Im} \Sigma$  has to agree with the initial distribution function. In our case, the relevant momentum is the center position  $p_0$  of the Gaussian, so we choose

$$t_{\text{deph}} \approx \frac{\hbar}{4 \text{Im} \Sigma^{\text{M}}(\mathbf{p}_0, t_0)}. \quad (14)$$

We calculated the dephasing time according to Eq. (14) for the static potentials used in the previous calculations. The result  $t_{\text{deph}}(\kappa^{-1})$  is shown in Fig. 2a (dashed line). The most striking result is the opposite dependence of  $\tau_{\text{cor}}$  and  $t_{\text{deph}}$  on the potential range, cf. also Ref. [12]. The reason is that the Markovian scattering rates ( $\text{Im} \Sigma^{\text{M}}$ , Eq. (13)) increase with increasing potential range  $\kappa^{-1}$ . On the other hand, the result for  $\tau_{\text{cor}}$  is in fair agreement with two-time calculations of the Kadanoff–Baym equations where the self-energy is treated

fully self-consistently [4]. This allows one to draw conclusions, when the Markov approximation for  $\text{Im } \Sigma$  and the damping coefficients  $\gamma_{12}$  in Eq. (7) is applicable. Interestingly, a crossing of the curves  $\tau_{\text{cor}}(\kappa^{-1})$  and  $t_{\text{deph}}(\kappa^{-1})$ , Fig. 2a, occurs around  $\kappa = 3a_{\text{B}}^{-1}$ , which is close to the value 2.3 of the screening parameter obtained from Eq. (11). In fact, the agreement is even better if one takes into account that  $t_{\text{deph}}$  is an exponential decay time, whereas  $\tau_{\text{cor}}$  has been defined as the saturation time of kinetic energy. If one takes as a basis the time during which kinetic energy increases up to  $(1 - 1/e)\Delta E$ , where  $\Delta E$  is the total increase, cf. Fig. 2b,  $\tau_{\text{cor}}$  comes rather close to  $t_{\text{deph}}(\kappa = 2.3) \approx 15$  fs.

In summary, we have discussed the crossover from two-particle to one-particle relaxation processes in non-Markovian kinetics and have proposed a simple approach to the time  $\tau_{\text{cor}}$  separating both regimes. The evolution of the kinetic energy well reflects the correlation buildup and transition from the initial to the kinetic stage. Therefore, the saturation time of kinetic energy is an appropriate measure for the nonequilibrium binary correlation time. We expect this approach to be applicable to more complicated systems beyond the second Born approximation too, including, for example systems with bound states or polarization dynamics. Of course, the choice of simplified models may differ from the present case and has to be justified each time by comparison with full two-time calculations.

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