Response to "Comments on 'Impossibility of plasma instabilities in isotropic quantum plasmas' " [Phys. Plasmas 1, 832 (1994)]

M. Bonitz
Universität Rostock, Fachbereich Physik, Universitätsplatz 3, 18051 Rostock, Germany

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In his Comment, Ref. 1, Nieves gives a useful summary of analytic properties of the dielectric function (DF), cf. Eqs. (2)–(5). These properties are valid also for generalized susceptibilities. In the case of anisotropic media, the quantities ϵ_d and ϵ_a, cf. Eq. (1) of Ref. 1, have to be replaced by the Hermitian ϵ^H and anti-Hermitian ϵ^A parts of the dielectric tensor, respectively. In Ref. 2 it was proved, that a quantum plasma with isotropic momentum distribution cannot have unstable longitudinal plasmon modes, just as its classical counterpart. This was shown by proving that the imaginary part of the analytic continuation of the dielectric function [i.e., $v(x,y)$ in Ref. 1] is always positive in the upper frequency half-plane. In his Comment, Nieves demonstrates that for this to be true, it is sufficient, that the imaginary part of the DF ϵ_a is always positive on the real frequency axis, for $\omega>0$. This brings us back to the well-known general statement: if $\epsilon_a(\omega,k)>0$, for Im $\omega=0$, Re $\omega>0$, then there are no longitudinal plasma instabilities possible. Indeed, the energy W exchanged between the oscillation and the plasma is proportional to ϵ_a (or ϵ^A). In particular, in equilibrium $\epsilon_a>0$ always (we consider in the following only positive frequencies), corresponding to dissipation of wave energy in the plasma.

However, for a plasma in nonequilibrium, e.g., in the presence of particle beams, the imaginary part of the DF, of course, can be negative. This essentially depends on the distribution function of the carriers. For instance, distribution functions with a second maximum permit plasma instabilities (e.g., bump-on-tail instability). The simplified explanation in terms of resonant particle–wave interaction is the following: A plasma wave with the phase velocity $v_{\rm ph}$ on the increasing part of the distribution $[df(v_{\rm ph})/dv>0]$ will meet more particles with $v>v_{\rm ph}$ than slower ones. Hence, the wave will gain energy and be amplified.[3] In view of this standard explanation, the general stability of any spherically symmetric distribution of a quantum plasma (within the random phase approximation, RPA),[2] must sound rather surprising. An extension of these stability investigations is indeed desirable. Generalizations are necessary in three directions:

(1) Better approximations in the interaction or density, that go beyond the RPA;
(2) lower symmetries (no isotropy) of the momentum distribution;
(3) reduced dimensionality of the plasma, e.g., two-dimensional (2-D) or one-dimensional (1-D) quantum plasmas.

In particular the last point is currently of high interest for quantum plasmas in solids. Therefore, it deserves a remark.

Influence of the dimensionality on instabilities in quantum plasmas. Consider a wave with frequency Ω and wave number $\mathbf{k}_0$. Electrons interacting with the wave will change their momentum $\mathbf{k}_0$. The net energy balance of the wave is then determined by the difference of the number ΔN of electrons emitting a plasmon minus those that absorb a plasmon,

$$\Delta N(\Omega,\mathbf{k}_0)\sim\int d\mathbf{q}\;\delta\left(\frac{m\Omega}{\mathbf{k}_0}-q\cos\theta\right)\left\{f\left(\mathbf{q}+\frac{\mathbf{k}_0}{2}\right)\times\left[1-f\left(\mathbf{q}-\frac{\mathbf{k}_0}{2}\right)\right]-f\left(\mathbf{q}-\frac{\mathbf{k}_0}{2}\right)\left[1-f\left(\mathbf{q}+\frac{\mathbf{k}_0}{2}\right)\right]\right\}. \tag{1}$$

The terms in brackets give the probability of scattering an electron out of state $\mathbf{q}+\mathbf{k}_0/2$ into state $\mathbf{q}-\mathbf{k}_0/2$ and vice versa. Obviously, the Pauli blocking terms cancel. This formula corresponds to the level of the random phase approximation. The delta function takes into account only resonant interactions, i.e., those involving electrons with $v\cos\theta=v_{\rm ph}$, with θ being the angle between $\mathbf{k}_0$ and $\mathbf{q}$, ($\hbar=1$). Notice that ΔN is related to the imaginary part of the DF by $\Delta N\sim-\text{Im}\,\epsilon$. (One immediately sees that the resonance of the velocities is, up to a factor q, equivalent to energy conservation $E_{\mathbf{q}+\mathbf{k}_0/2}-E_{\mathbf{q}-\mathbf{k}_0/2}=\Omega$, with $E_k=k^2/2m$.)

In the following we will assume $f(\mathbf{q})=f(q)$, but otherwise consider arbitrary nonequilibrium distribution functions.

(i) In the case of quasi-1-D electron gas (organic chains, quantum wires, etc.) or homogeneous three-dimensional (3-D) plasmas (homogeneity in the plane perpendicular to $\mathbf{k}_0$), Eq. (1) yields

$$\Delta N^{\rm 1D}(\Omega,\mathbf{k}_0)\sim f\left(\frac{m\Omega}{k_0}+\frac{k_0}{2}\right)-f\left(\frac{m\Omega}{k_0}-\frac{k_0}{2}\right). \tag{2}$$

In the long wavelength limit ($k_0\to0$) we recover the well-known result for classical plasmas.

(ii) In a quasi-2-D plasma (electrons on liquid helium, thin films, quantum wells, etc.), the vectors $\mathbf{k}_0$ and $\mathbf{q}$ lie in one plane. Carrying out in Eq. (1) the integration over θ from $-\pi/2$ to $\pi/2$, one finds

$$\Delta N^{\rm 2D}(\Omega,\mathbf{k}_0)\sim\int_{q_0}^{\infty}dq\;q\,\frac{f_+-f_-}{\sqrt{q^2-q_0^2}}, \tag{3}$$

with $f_\pm=f(\sqrt{q^2+k_0^2\pm m\Omega})$ and $q_0=m\Omega/k_0$.

(iii) For 3-D plasmas, i.e., the situation considered in Ref. 2, the angle integrations in Eq. (1) can be carried out again:

$$\Delta N^{3D}(\Omega,\mathbf{k}_0)\sim\int_{q_0}^{\infty}dq\ q[f_+-f_-]. \quad (4)$$

One sees that phase space effects change the situation drastically. With increasing dimensionality d the number of nonequilibrium carriers increases with the dth power. However, the fraction of fast carriers that is able to transfer its energy resonantly to the wave, drops even stronger. In 3D, the phase space factor multiplying the difference $f_+ - f_-$ is a growing function of q, whereas in 2D, it decreases monotonically. Thus, one readily sees that ΔN^{3D} is negative for arbitrary distribution functions. Waves are always damped. The situation in 2D and 1D is qualitatively different. Here, nonequilibrium distributions can lead to positive ΔN and, therefore to negative $\mathrm{Im}\,\epsilon$. Waves with $v_{ph}\sim v_0$ and $k_0 < m\Delta v$, with v_0 and Δv being, respectively, the center of the increasing part of the distribution and the widths of its minimum, may become unstable.

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[1] J. F. Nieves, Phys. Plasmas **2**, 1015 (1995).
[2] M. Bonitz, Phys. Plasmas **1**, 832 (1994).
[3] In fact, for an instability to occur, the minimum has to be of sufficient depth, given, e.g., by the Penrose criterion.