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M. Bonitz

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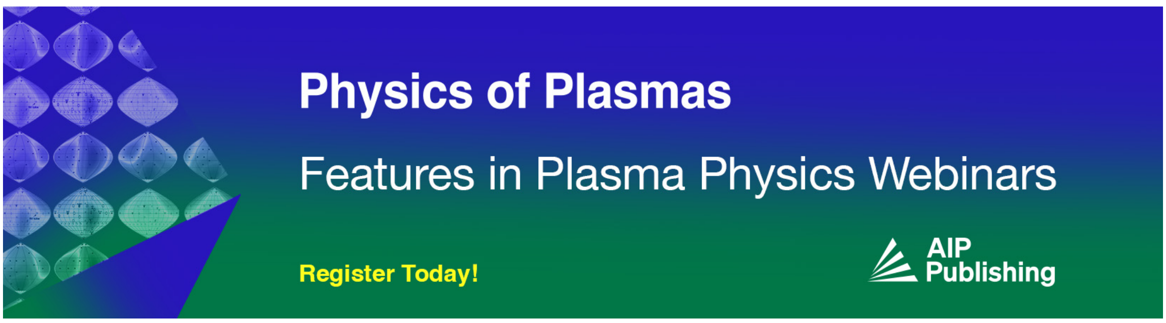

Impossibility of plasma instabilities in isotropic quantum plasmas

M. Bonitz[a)]
Optical Sciences Center, University of Arizona, Tucson, Arizona 85721

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It is well known that Vlasov plasmas with spherical momentum symmetry cannot exhibit longitudinal plasma instabilities, regardless of the monotonic behavior of the distribution function. It is shown that this property holds for quantum plasmas also, at least within the random phase approximation.

I. INTRODUCTION

Since the pioneering work of Vlasov and Landau,[1–3] the questions of plasmons and instabilities have been extensively studied, within the framework of the Vlasov dielectric function (DF), e.g., Refs. 4 and 5. The Vlasov DF is the appropriate starting point for an analysis of high-temperature gaseous plasmas, as well as for low-density plasmas (like in the atmosphere of the earth). It can also be used for the study of long wavelength excitations in solid state plasmas. However, analyzing plasma instabilities using the Vlasov DF restricts one to almost ideal and nondegenerate plasmas. On the other hand, nonideal plasmas have become an area of substantial current interest, e.g., in astrophysics or in connection with ion-beam and laser compression experiments. Nonideality effects are of importance also for electron-hole plasmas in metals or semiconductors. In these systems, many-particle effects, such as degeneracy, dynamical screening and Pauli blocking have to be taken into account. The consequences of this include changes in both the thermodynamic and, more importantly, the nonequilibrium (transport) properties.[6] Furthermore, collective excitations are significantly influenced by many-body effects, in particular by quantum effects, e.g., Ref. 7. So, the study of plasmons or instabilities in degenerate plasmas requires a quantum generalization of the Vlasov DF which is, in lowest order, given by the random phase approximation (RPA, Lindhard DF).[8]

The properties of the RPA DF in equilibrium have been investigated in detail, mainly in solid state physics. However, the possibility of plasma instabilities in Lindhard plasmas has been studied only recently.[7,9–13] These investigations are relevant for dense gaseous plasmas[14] and, in particular, for novel semiconductor materials, such as quantum well or quantum wire structures. Here, the generation of plasmons is expected to lead to possible device applications, e.g., Refs. 10 and 13. To this end it is essential to know which types of plasma symmetries may exhibit plasma instabilities in principle.

The aim of this paper is to demonstrate that isotropic quantum plasmas cannot exhibit longitudinal plasmon instabilities. This property is well known for classical plasmas which can be described by the Vlasov DF, e.g., Ref. 4. But from this, one can conclude only that quantum plasmas are stable against fluctuations with vanishing wave number. However, to the knowledge of the author, the stability of isotropic quantum plasmas with respect to longitudinal oscillations of *arbitrary* wave number has not yet been established. It is shown in this paper that this is the case for *arbitrary* distribution functions, including those for nonequilibrium states (e.g., functions having minima) and for an arbitrary number of components.

II. RPA DISPERSION RELATION FOR QUANTUM PLASMAS

We consider degenerate plasmas which are described by the RPA DF

$$\epsilon(\omega,\gamma,q)=1-\sum_a V_a(q)\Pi_a(\omega,\gamma,q). \quad (1)$$

Here, a denotes the charged-particles species, $V_a(q)$ is the Fourier transform of the Coulomb potential, and $\Pi_a(\omega,\gamma,q)$ is the Lindhard polarization function (cf. Sec. III). The retarded dielectric function is analytic in the upper half-plane of the complex frequency $\hat{\omega}=\omega-i\gamma$. Therefore, Eq. (1) is valid for arbitrary negative values of γ (positive values of Im ω). Longitudinal collective excitations are defined by the well-known dispersion relation

$$\epsilon(\omega,\gamma,q)=0. \quad (2)$$

This complex equation is fulfilled if Re ϵ and Im ϵ vanish simultaneously. The solutions of Eq. (2) are pairs of functions $\Omega_s(q)$, $\Gamma_s(q)$ (again the notation $\hat{\Omega}=\Omega-i\Gamma$ was used), $s=1,2,\ldots$. That means, $\Omega_s(q)$ defines the wave-vector dispersion of the plasmon mode s and $\Gamma_s(q)$ its damping. For $\Gamma_s>0$ the plasmon will be damped, for $\Gamma_s<0$ the mode would be unstable. It is obviously necessary for an instability to occur that both Re ϵ and Im ϵ change their sign in the upper frequency half-plane. So, for our purpose it is sufficient to show that one of them has always the same sign in this half-plane. As will be shown in the following, in fact, for $\gamma<0$ always Im $\epsilon>0$, for arbitrary isotropic distribution functions. To show this it is sufficient to demonstrate that Im $\Pi_a(\omega,\gamma,q)<0$ for any plasma component.

[a)]Permanent address: Universität Rostock, Fachbereich Physik, Universitätsplatz 3, 18051 Rostock, Germany.

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III. IMAGINARY PART OF THE LINDHARD DF FOR ISOTROPIC PLASMAS

The RPA polarization function was obtained by Lindhard.[8] We consider only isotropic plasmas, i.e., distribution functions depending only on the absolute value of the particle momentum $f(\mathbf{q})=f(|\mathbf{q}|)$

$$\Pi_a(\omega,\gamma,\mathbf{q})=(2s_a+1)\int\frac{d^3k}{(2\pi)^3}\times\frac{f_a(|\mathbf{k}|)-f_a(|\mathbf{k}+\mathbf{q}|)}{E_a(\mathbf{k})-E_a(\mathbf{k}+\mathbf{q})+(\omega-i\gamma)+i\varepsilon},\quad(3)$$

where ε is an infinitesimal small positive number, s_a is the spin and $E_a(q)=\hbar^2q^2/2m_a$. This formula is valid for $\gamma<0$, so one may drop ε. Introducing spherical coordinates and the new variables $z=\cos\theta$, $y=kz$, $u=(m_a/q)\omega(\omega\geqslant0)$ and $\delta=(m_a/q)\gamma$, one angle integration can be carried out

$$\Pi_a(\omega,\gamma,q)=(2s_a+1)\frac{m_a}{q}\int_0^\infty\frac{dk}{(2\pi)^2}kf_a(k)\times\int_{-k}^{k}dy\left(\frac{1}{y-q/2-(u-i\delta)}-\frac{1}{y+q/2-(u-i\delta)}\right).\quad(4)$$

The imaginary part of Eq. (4) is easily separated. After integration over y it can be written in the form

$$\mathrm{Im}\,\Pi_a(\omega,\gamma,q)=-(2s_a+1)\frac{m_a}{q}\int_0^\infty\frac{dk}{(2\pi)^2}kf_a(k)\times\{A_+-A_-\}\quad(5)$$

with

$$A_\pm=\arctan\frac{k-(\pm u-q/2)}{|\delta|}-\arctan\frac{k-(\pm u+q/2)}{|\delta|}.\quad(6)$$

Now, it remains to be shown that A_+-A_- does not change its sign for non-negative k. To see this, consider the difference $S=\arctan(x-a)-\arctan(x-b)$, with $b\geqslant a$. Obviously, $0\leqslant S\leqslant\pi$, with S having a single maximum at $x=(a+b)/2$. Notice that both A_+ and A_- are of the form S, with maxima at $k=u$ and $k=-u$, $(u\geqslant0)$, respectively. Hence,

$$A_+-A_-\begin{cases}<0, & k<0,\\ =0, & k=0,\\ >0, & k>0.\end{cases}$$

We therefore conclude that $\mathrm{Im}\,\Pi_a\leqslant0$.

This result does not depend on the carrier species a. Thus also for a many-component plasma $\mathrm{Im}\,\epsilon>0$, cf. Eq. (1), like in equilibrium. This result is true for arbitrary distribution functions, including those for nonequilibrium situations with one or several minima. This means, (at least) within the RPA, an isotropic plasma cannot have unstable longitudinal collective excitations.

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[1] A. A. Vlasov, Zh. Eksp. Teor. Fiz. **8**, 291 (1937); J. Phys. (U.S.S.R.) **9**, 25 (1945).
[2] A. A. Vlasov, J. Phys. (U.S.S.R.) **9**, 25 (1945).
[3] L. D. Landau, J. Phys. (U.S.S.R.) **10**, 25 (1946).
[4] G. Ecker, *The Theory of Fully Ionized Plasmas* (Academic, New York, 1972).
[5] R. C. Davidson, in *Basic Plasma Physics: Selected Chapters*, edited by M. N. Rosenbluth and R. Z. Sagdeev (Elsevier, New York, 1983).
[6] D. Kremp, M. Schlanges, M. Bonitz, and T. Bornath, Phys. Fluids B **5**, 216 (1993).
[7] M. Bonitz, R. Binder, S. W. Koch, and D. Kremp, to appear in Phys. Rev. E (1994).
[8] J. Lindhard, Kgl. Danske Videnskab. Selskab. Mat. Fys. Medd. **28**, 8 (1954).
[9] P. Bakshi, J. Cen, and K. Kempa, J. Appl. Phys. **64**, 2243 (1988).
[10] P. Bakshi, J. Cen, and K. Kempa, Solid State Commun. **76**, 835 (1990).
[11] K. ElSayed, R. Binder, D. C. Scott, and S. W. Koch, Phys. Rev. B **47**, 16 (1993).
[12] M. Bonitz, R. Binder, and S. W. Koch, Phys. Rev. Lett. **70**, 3788 (1993).
[13] K. Kempa, P. Bakshi, and H. Xie, Phys. Rev. B **48**, 9158 (1993).
[14] D. Kremp, K. Morawetz, M. Schlanges, and V. Rietz, Phys. Rev. E **47**, 635 (1993).