

Quantum hydrodynamics for plasmas—quo vadis?

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Quantum plasmas are an important topic in astrophysics and high pressure laboratory physics for more than 50 years. In addition, many condensed matter systems, including the electron gas in metals, metallic nanoparticles, or electron-hole systems in semiconductors and heterostructures exhibit – to some extent – plasma-like behavior. Among the key theoretical approaches that have been applied to these systems are quantum kinetic theory, Green functions theory, quantum Monte Carlo, semiclassical and quantum molecular dynamics and, more recently, density functional theory simulations. These activities are in close contact with the experiments and have firmly established themselves in the fields of plasma physics, astrophysics and condensed matter physics.

About two decades ago, a second branch of quantum plasma theory emerged that is based on a quantum fluid description and has attracted a substantial number of researchers. The focus of these works has been on collective oscillations and linear and nonlinear waves in quantum plasmas. Even though these papers pretend to address the same physical systems as the more traditional papers mentioned above the former appear to form a rather closed community that is largely isolated from the rest of the field. The quantum hydrodynamics (QHD) results have – with a few exceptions – not found application in astrophysics or in experiments in condensed matter physics. Moreover, these results did practically not have any impact on the former quantum plasma theory community. One reason is the unknown accuracy of the QHD for dense plasmas. In this paper, we present a novel derivation, starting from reduced density operators that clearly points to the deficiencies of QHD, and we outline possible improvements.

It has also to be noted that some of the QHD results have attracted negative attention being criticized as unphysical. Examples include the prediction of “novel attractive forces” between protons in an equilibrium quantum plasma, the notion of “spinning quantum plasmas” or the new field of “quantum dusty plasmas”. In the present article we discuss the latter system in some detail because it is a particularly disturbing case of formal theoretical investigations that are detached from physical reality despite bold and unproven claims of importance for e.g. dense astrophysical plasmas or microelectronics. We stress that these deficiencies are not a problem of QHD itself which is a powerful and efficient method, but rather due to ignorance of its properties and limitations. We analyze the common flaws of these works and come up with suggestions how to improve the situation of QHD applications to quantum plasmas.

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I. INTRODUCTION

Quantum plasmas – charged particle systems in which at least the one component (typically the electrons) exhibit quantum degeneracy – are ubiquitous in nature, e.g. [1–4]. Examples are the matter in the interior of our Earth [5], of giant planets, e.g. [6–11] and brown and white dwarf stars [12–14], and the outer crust of neutron stars [15, 16]. In the laboratory, quantum plasmas are being routinely produced via compression of matter with the help of lasers, ion beams or X-rays. Examples include the Lawrence Livermore National Laboratory [17, 18], the Z-machine at Sandia National Laboratory [19, 20], the upcoming FAIR facility at GSI Darmstadt, Germany [21], the Omega laser at the University of Rochester [22], the Linac Coherent Light Source (LCLS) in Stanford [23, 24], the European free electron laser facilities FLASH and X-FEL in Hamburg, Germany [25, 26], and other laser and free electron laser laboratories. A particularly exciting application is inertial confinement

fusion [17–19] where electronic quantum effects are important during the initial phase. Another exotic example is the quark-gluon plasma that is now routinely produced at the relativistic heavy ion collider in Brookhaven and the Large Hadron Collider at CERN and exhibits interesting similarities with Coulomb plasmas, e.g. [27, 28].

Aside from dense plasmas, also many condensed matter systems exhibit – to some extent – quantum plasma properties. This concerns, for example, the electron gas in metals, e.g. [4, 29, 30] and electron-hole systems in semiconductors, e.g. [31, 32]. Among recent applications we mention nanoplasmonics which studies the interaction of quantum electrons in metallic nanostructures with electromagnetic radiation [33, 34].

The behavior of all these very diverse systems is characterized, among others, by electronic quantum effects, so their accurate description is of high importance and constitutes an actively developing field. Quantum effects of ions are relevant only at very high densities, in particular, for the phase diagram of light elements, such as hy-

drogen, e.g. [35], for ion crystals in white dwarfs, crystals of heavy holes in semiconductors [36, 37], or for the exotic matter in the interior of neutron stars [15]. Quantum effects of electrons are of relevance at low temperature and/or if matter is very highly compressed, such that the temperature is lower than the Fermi temperature (for the relevant parameter range, see Fig. 1 and, for the parameter definitions, see Sec. II). Quantum effects that have to be taken into account include the spatial delocalization (diffraction effects), exchange effects (antisymmetry of the wave function and Pauli blocking), electronic spin effects, bound states and their many-particle renormalization (continuum lowering) etc. [38, 39]. At the same time, quantum plasmas are characterized by correlations and finite temperature effects. Thus the theoretical description of quantum plasmas is challenging because standard approaches such as perturbation theory or ground state methods do not apply.

The development of a theory of quantum plasmas goes back to the 1950s and was based on quantum generalizations of kinetic equations derived by Bohm and Pines [40, 41] and Klimontovich and Silin [42–44], and others, see also Ref. [45]. In the mean time, improved generalized quantum kinetic equations have been derived starting from reduced density operators, e.g. [39, 46], or nonequilibrium Green functions (NEGF) [47–49]. Meanwhile many text books on quantum statistics and quantum kinetic equations are available that include applications to quantum plasmas, e.g. [38, 39, 50, 51] and references therein. Another direction in quantum plasma theory is first principle computer simulations such as quantum Monte Carlo [4, 52–57], semiclassical molecular dynamics with quantum potentials (SC-MD), e.g. [58] and various variants of quantum MD, e.g. [59–61]. More recently, also density functional theory (DFT) simulations were introduced to the field of quantum plasmas and have quickly gained importance due to their ability to treat realistic warm dense matter, e.g. [62–64]. Furthermore, also orbital-free DFT methods (OF-DFT) are being developed. Finally, also time-dependent methods such as TD-DFT, e.g. [65] and NEGF are being successfully applied, e.g. [66, 67].

Certainly, all these approaches are rather complicated reflecting the high complexity of realistic quantum plasmas, and they often require time consuming computer simulations. Therefore, simplified models that would allow for an approximate description in limiting cases are highly desirable. The situation is similar to classical plasma physics where an accurate description is based on kinetic theory (e.g. Boltzmann equation of particle in cell simulations) or particle simulations (e.g. molecular dynamics). Here a simplified description is achieved by transition to a fluid approach that is rigorously derived from nonequilibrium statistical mechanics and kinetic theory. The limitations of fluid models are well known (they are related e.g. to coarse graining effects, neglect of kinetic and collision effects), and the model results can be carefully verified against experiments, ki-

netic theory or first-principle simulations.

Thus it is natural to look for a fluid description also in the case of quantum plasmas. However, until now no rigorous derivation of fluid equations via moments of quantum kinetic equations exists that would parallel the quality of classical fluid equations, e.g. [68]. Instead, already long ago various semi-phenomenological approaches have been proposed, such as Landau's Fermi liquid theory [29, 30], Thomas-Fermi theory [69] or the Gross-Pitaevskii equation for bosons [70, 71]. In nanoplasmics recently fluid models are being used that are derived from Thomas-Fermi-von-Weizsäcker models, e.g. [33, 72, 73]. Finally, Madelung, Bohm and others showed that the single-particle Schrödinger equation can be rewritten identically as a fluid-type equation for the probability density, see Ref. [74], for an overview.

This latter equation was used more recently by Manfredi and Haas to motivate a fluid description of a *quantum many-fermion system* [75, 76]. The resulting quantum hydrodynamic (QHD) equations have the same form as the single-particle equation except for an additional mean field (Hartree) potential and a Fermi pressure term created by all particles. Thus, they neglect quantum exchange (in particular the Pauli principle), dissipation, correlations and finite temperature effects. (We note that exchange-correlation corrections were introduced in Ref. [77] but the accuracy of this phenomenological approach is unclear.) Moreover, the derivation of Ref. [75] employed several strong assumptions giving rise to a limited applicability range, e.g. [78–80]. For example, the QHD equations of Ref. [75] reproduce the collective linear response of a quantum plasma quantitatively correctly only in the high-frequency limit, e.g. [81, 82] of one-dimensional system. Other applications require phenomenological adjustments of the coefficients of the Bohm potential and the Fermi pressure [81, 83]), for details see Sec. III.

This complicated behavior of the QHD equations is not surprising, given the complexity of the underlying many-particle quantum system. On the other hand, this means that, when applying QHD models to real quantum plasmas, careful applicability tests and comparisons to more accurate approaches, such as quantum kinetic theory, DFT or quantum Monte Carlo, are indispensable. However, many papers that applied QHD models to quantum plasmas neither include a careful validity analysis nor comparisons to experiments or more accurate theories. So it does not come as a surprise that numerous predictions have been made that are either highly speculative or even in conflict with results that are well established in other fields. For this reason these results have not been picked up by researchers outside the QHD-quantum plasma community except for occasional critical comments, e.g. Refs. [78, 84, 85]. For example, Vranies *et al.* concluded (we quote from the abstract of Ref. [84]): “The quantum plasma theory has flourished in the past few years without much regard to the physical validity of the formulation or its connection to any real physical

system. It is argued here that there is a very limited physical ground for the application of such a theory.”

A prominent example of untested claims was the prediction of “novel attractive forces” between protons in an equilibrium quantum plasma by Shukla and Eliasson (known as SE potential) [86]. These predictions were proven wrong by *ab initio* density functional theory simulations in Ref. [78] where also the limitations of linearized QHD were pointed out, see also Refs. [79, 87]. The failure of the SE potential is also confirmed by quantum kinetic theory [88]. More details on QHD and its validity range are discussed in Sec. III.

A second prominent example of unjustified claims was the prediction of unphysically high spin polarization in quantum plasmas [89], the notion of “spinning quantum plasmas” [89] and the invention of “spin lasers” [90] that were criticized in Refs. [85, 91] where also further references are given. Interestingly, some of these papers received high citation numbers in the QHD-quantum plasma community. However, although spin effects are being actively studied in condensed matter physics for decades in the context of spintronics, e.g. [92, 93] and references therein, the above papers received no feedback in that community.

In 2005 application of quantum hydrodynamics lead to another discovery [94]: “quantum dusty plasmas” (QDP). This can be considered one of the most spectacular predictions of QHD for quantum plasmas. According to this article, cooling of a dusty plasma will give rise not only to *quantum electrons and ions but also to quantum dust particles* (details are discussed in the supplementary material [95]). The paper [94] has had a high impact in the QHD-quantum plasma community and was followed by many investigations of novel oscillations and waves in “quantum dusty plasmas”. As done in Ref. [94], the follow up papers typically claimed that their QDP results are of importance for many diverse systems that include white dwarf stars, neutron stars, supernovae, semiconductor plasmas, metallic structures and more. Unfortunately, for none of those strong claims any proof or, at least, convincing arguments have been given. This is not surprising because a fairly simple analysis and consistency check of parameters is sufficient to understand that “quantum dusty plasmas” cannot exist in any of the mentioned physical systems. Moreover, they are non-existing in any other system. It is one of the goals of this article to present this elementary analysis.

One may wonder how the claims about relevance of the results for compact stars, on the one hand, and for solid state systems, on the other, are being justified in the QHD-quantum plasma papers. This is a crucial question for this field in general that goes far beyond the particular case of “quantum dusty plasmas”. The answer is simple: there is no careful justification, neither of the validity of the model nor of the relevance for specific types of quantum plasmas. The typical “justification” is references to earlier QHD-quantum plasma papers where the chosen parameters were introduced before. The consequence of

this style of research is a flood of papers of questionable validity and importance containing a large number of astonishing predictions, examples of which were listed above. This is not restricted to “quantum dusty plasmas”, but they are a particularly disturbing case and will, therefore, be discussed in some detail. Let us stress that these problematic predictions have nothing to do with the QHD approach which we believe to be powerful, if applied within its range of validity.

This paper is organized as follows: in Sec. II we recall the main parameters of quantum plasmas and the relevant temperature and density range. Section III presents a systematic re-derivation of the QHD equations, starting from density operator theory. There we derive *microscopic QHD equations* that allow for a clear analysis of the approximations made in QHD and of its applicability limits. We also present a comparison to other approaches such as Bohmian quantum mechanics and TD-DFT. In Sec. IV we critically discuss the transfer of results from one plasma to another pointing out the limitations of such an approach. After this, in Sec. V we discuss in some detail the concept of “quantum dusty plasmas” including often used examples and parameters and examine the stability conditions of a dust particle in a quantum plasma. We conclude in Secs. VI and VII with an outlook for quantum plasma theory where we summarize the main challenges the field is facing, in general, and the scientific contributions, QHD is capable to make, in particular.

II. QUANTUM PLASMA PARAMETERS

Let us recall the basic parameters of quantum plasmas [39]:

1. the electron degeneracy parameters $\theta_e = k_B T_e / E_{Fe}$ and $\chi_e = n_e \Lambda_e^3$, where $\Lambda_e = h / \sqrt{2\pi m_e k_B T_e}$, is the thermal DeBroglie wave length, and the Fermi energy of electrons is

$$E_{Fe} = \frac{\hbar^2}{2m_e} (3\pi^2 n_e)^{2/3}, \quad (1)$$

where n_e is the electron density and T_e the electron temperature (note that Eq. (1) applies only to a 3D Fermi gas of spin 1/2 particles, more cases are discussed in the appendix),

2. the ion degeneracy parameter $\chi_i = n_i \Lambda_i^3$, where $\Lambda_i = h / \sqrt{2\pi m_i k_B T_i}$. Obviously the ion degeneracy parameter is a factor $(m_e T_e / m_i T_i)^{3/2}$ smaller than the one of the electrons;
3. the classical coupling parameter of ions $\Gamma_i = Q_i^2 / (a_i k_B T_i)$, where Q_i is the ion charge, and a_i is the mean inter-ionic distance.

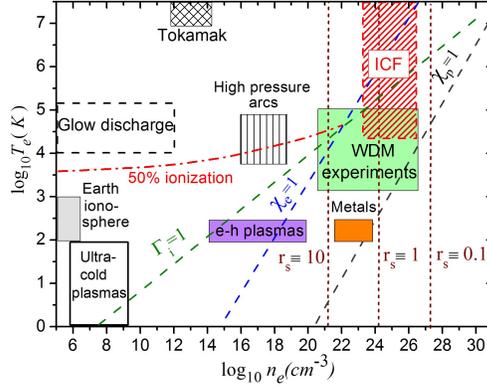


FIG. 1. Density-temperature plain with examples of plasmas and characteristic plasma parameters. ICF and WDM denote inertial confinement fusion and warm dense matter, respectively. Metals (semiconductors) refers to the electron gas in metals (electron-hole plasma in semiconductors). Strong ionic coupling is found below the line $\Gamma_i = 1$. Electronic (ionic) *quantum effects* are observed to the right of the line $\chi_e = 1$ ($\chi_p = 1$). The coupling strength of quantum electrons increases with r_s (with decreasing density). Atomic ionization due to thermal effects is dominant above the red line (for equilibrium hydrogen plasmas [96]). Ionization in the case of quantum electrons (pressure ionization) is considered in Fig. 2. Note that the values of χ_p and r_s refer to the case of hydrogen.

- the quantum coupling parameter (Brueckner parameter) of electrons in the low-temperature limit,

$$r_s = \frac{a_e}{a_B}, \quad a_B = \frac{\hbar^2 \epsilon_b}{m_r Q_i e}, \quad (2)$$

where $a_e = (4/3\pi n_e)^{-1/3}$ denotes the mean distance between two electrons, a_B is the first Bohr radius, and $m_r = m_e m_i / (m_e + m_i)$ and ϵ_b are the reduced mass and background dielectric constant, respectively, for hydrogen $m_r \approx m_e$, $\epsilon_b = 1$, and $a_B = 0.529 \text{ \AA}$;

- the degree of ionization of the plasma: the ratio of the number of free electrons to the total (free plus bound) electron number, $\alpha^{ion} = n_e/n_{tot}$, determines how relevant plasma properties are compared to neutral gas or fluid effects.

The parameters χ_e , χ_i and Γ_i are shown in Fig. 1 where we indicate where these parameters equal one. We also plot three lines for constant values of r_s . Note that the classical coupling parameter increases with density whereas the quantum coupling parameter decreases with n_e . We underline that the parameters a_B , E_{Fe} , Λ_e , θ_e contain the density of free electrons and Γ_i , χ_i the density of free ions. This means the lines of constant Γ_i , χ_e , χ_i , r_s

shown in Fig. 1 refer to the free electron (ion) density. In cases when the plasma is only partially ionized the free electron density has to be replaced by $n_e \rightarrow \alpha^{ion} \times n_e$.

The degree of ionization decreases when the temperature is lowered, according to the Saha equation, $\alpha^{ion} \sim e^{-|E_b|/k_B T}$, where E_b denotes the binding energy of the atom, and in Fig. 1 we indicate the line where a classical hydrogen plasma has a degree of ionization of 0.5. The extension to a dense quantum plasma, i.e. to the right of the line $\chi_e = 1$, where ionization occurs due to compression will be discussed below, in Fig. 2.

III. QUANTUM HYDRODYNAMICS

Quantum hydrodynamics has been reviewed in many text books and articles, e.g. [74, 80, 83, 97, 98], therefore, we here concentrate on aspects that are of relevance for quantum plasma applications.

A. N-particle Schrödinger equation. Amplitude-Phase representation

We consider a non-relativistic many-particle quantum system described by the spin-independent hamiltonian

$$\hat{H} = \sum_{i=1}^N \left(-\frac{\hbar^2}{2m} \nabla_i^2 + V(\mathbf{r}_i) \right) + \frac{1}{2} \sum_{i \neq j} w_{ij}(\mathbf{R}), \quad (3)$$

where $\mathbf{R} = (\mathbf{r}_1, \sigma_1; \mathbf{r}_2, \sigma_2; \dots; \mathbf{r}_N, \sigma_N)$, \mathbf{r}_i are the particle coordinates and σ_i their spin projections. Assuming first as pure state, the dynamics of the system are governed by the N-particle Schrödinger equation

$$i\hbar \frac{\partial \Psi(\mathbf{R}, t)}{\partial t} = \hat{H} \Psi(\mathbf{R}, t), \quad \Psi(\mathbf{R}, t_0) = \Psi_0(\mathbf{R}), \quad (4)$$

that is supplemented by an initial condition and the normalization $\sum_{\sigma_1 \dots \sigma_N} \int d^3N R |\Psi(\mathbf{R}, t)|^2 = N$. For particles with spin s , there are $g_s = 2s + 1$ different spin projections, and each spin sum gives rise to a factor g_s (in the following, we will not write the spin arguments and spin sums explicitly).

The Schrödinger equation has been investigated in great detail and has many equivalent representations, including Schrödinger's, Heisenberg's and Feynman's path integral picture. An alternative representation was derived by Madlung [99], Bohm [100], and others by making an ansatz in terms of two real functions – the amplitude A and the phase S ,

$$\Psi(\mathbf{R}, t) = A(\mathbf{R}, t) e^{\frac{i}{\hbar} S(\mathbf{R}, t)}. \quad (5)$$

Inserting this ansatz into the Schrödinger equation (4) we obtain two coupled equations for the phase and the

squared amplitude

$$\frac{\partial A^2}{\partial t} + \sum_{i=1}^N \nabla_i (\mathbf{v}_i \cdot A^2) = 0, \quad \mathbf{v}_i \equiv \frac{1}{m} \nabla_i S, \quad (6)$$

$$\frac{\partial S}{\partial t} + \sum_{i=1}^N \left\{ V(\mathbf{r}_i) + Q(\mathbf{r}_i) + \frac{1}{2} \sum_{j \neq i} w_{ij} + \frac{p_i^2}{2m} \right\} = 0, \quad (7)$$

where we denoted $\nabla_i \equiv \partial/\partial \mathbf{r}_i$. Obviously, equation (6) is nothing but the continuity equation for the N -particle probability density, $A^2(\mathbf{R}, t) = |\Psi(\mathbf{R}, t)|^2$, with a $3N$ -dimensional probability current density vector with the components $v_{i\alpha} A^2$ with $\alpha = x, y, z$. On the other hand, Eq. (7) has the form of a classical Hamilton-Jacobi equation where the momenta are defined as $\mathbf{p}_i = \nabla_i S$. Also, this equation contains an additional potential $Q(\mathbf{r}_i)$ that arises from the quantum kinetic energy,

$$Q(\mathbf{r}_i) = -\frac{\hbar^2}{2m} \frac{\nabla_i^2 A}{A}. \quad (8)$$

B. Bohmian quantum mechanics

Identifying in Eq. (7) the effective hamilton function H via $\partial S/\partial t + H = 0$ allows one to formulate effective classical equations of motion where the forces are produced by the total potential V^{tot}

$$\frac{d\mathbf{p}_i}{dt} = -\frac{\partial H}{\partial \mathbf{r}_i} = -\nabla_i V^{\text{tot}}(\mathbf{R}), \quad i = 1 \dots N, \quad (9)$$

$$V^{\text{tot}} = \sum_{i=1}^N \left\{ V(\mathbf{r}_i) + Q(\mathbf{r}_i) + \frac{1}{2} \sum_{j \neq i} w_{ij} \right\}. \quad (10)$$

Thus the N -particle quantum system is mapped onto a system of N coupled classical trajectories (9). The main difference to a classical system is that the initial quantum state is not given by deterministic values for all particle coordinates and momenta but by an initial probability density $|\Psi_0(\mathbf{R})|^2$. This can be taken into account by frequently repeating the dynamics starting from many independent initial coordinates and momenta which are sampled randomly from this probability density.

Let us return to the assumption of an initial pure state. A precise (including accurate phase) specification of an initial wave function is only possible for a limited number of cases including low temperature isolated quantum systems such as cold atoms in optical lattices or Bose-Einstein condensates, cf. Sec. III D. For plasma applications, instead, the system is always coupled to the environment which leads to an incoherent superposition of wave functions Ψ^α with real probabilities p_α , see Sec. III C. For the Bohm trajectory approach inclusion of mixed states does not pose a fundamental problem because such a state can be taken into account by using a modified initial sampling probability $\sum_\alpha p_\alpha |\Psi_0^\alpha(\mathbf{R})|^2$.

Solutions of quantum problems with Bohm trajectories have been frequently attempted. An example are applications in semiconductor quantum transport, by Barker *et al.*, e.g. [101, 102] and references therein. For a more general overview, see the text book by Wyatt [97]. Finally, we mention recent applications of the trajectory approach to dense quantum plasmas where Coulomb interaction effects are important: Gregori *et al.* proposed a new method how to sample the probability amplitude A^2 from the pair distribution function to treat the dynamic properties of equilibrium warm dense matter in linear response [103].

C. Many-body systems in a mixed state. Density operators

If the many-body system (3) is coupled to the environment – as is typically the case in plasmas – a description in terms of wave functions and the Schrödinger equation (4) is no longer adequate. Instead, the system is described by an incoherent superposition of wave functions (“mixed state”). This can be taken into account, by replacing the N -particle wave function by the N -particle density operator [39],

$$\hat{\rho}(t) = \sum_a p_a |\Psi^a(t)\rangle \langle \Psi^a(t)|, \quad \text{Tr } \hat{\rho}(t) = 1, \quad (11)$$

where the sum runs over projection operators on all solutions of the Schrödinger equation (4), and p_a are real probabilities, $0 \leq p_a \leq 1$, with $\sum_a p_a = 1$. Here we used a general representation-independent form of the quantum states. It is directly related to the wave functions if the coordinate representation is being applied: $\langle \mathbf{R} | \Psi^a(t) \rangle = \Psi^a(\mathbf{R}, t)$ [$|\mathbf{R}\rangle$ are eigenstates of the coordinate operator in N -particle Hilbert space]. The previous case of a pure state is naturally included in definition (11) by setting $p_k = 1$ and all $p_{a \neq k} = 0$. The second relation (11) is the normalization condition where the trace denotes the sum over the diagonal matrix elements of $\hat{\rho}$, see below.

The method of density operators is well established in quantum many-body theory, and the Wigner representation – where the density operator transforms into the Wigner function (that is frequently being used in quantum plasma theory) – is one of many, though not the most efficient, representations. There is no room to go into the formal details here, see e.g. the text book [39] or the reviews [45, 80]. Here we concentrate on a few aspects that are of particular relevance for the QHD equations for quantum plasmas.

The equation of motion of $\hat{\rho}$ follows from the Schrödinger equation (4) and is given by

$$i\hbar \frac{\partial}{\partial t} \hat{\rho} - [\hat{H}, \hat{\rho}] = 0, \quad (12)$$

$$\hat{\rho}(t_0) = \sum_a p_a |\Psi_0^a\rangle \langle \Psi_0^a|, \quad (13)$$

which is the von Neumann equation supplemented by the initial condition. From the N -particle density operator all time-dependent properties of a quantum system can be obtained. However, in many cases simpler quantities are sufficient such as reduced s -particle density operators, including the single-particle density operator (which is related to the distribution function or Wigner function) [39]:

$$\hat{F}_1(t) \equiv N \text{Tr}_{2\dots N} \hat{\rho}(t), \quad \text{Tr}_1 \hat{F}_1 = N. \quad (14)$$

The equation of motion for \hat{F}_1 follows straightforwardly from Eq. (12) and will be given below, cf. Eq. (21).

D. Quantum hydrodynamics for Bose-Einstein condensates

Instead of the trajectory approach the Bohmian representation of the Schrödinger equation, Eqs. (6, 7), can be used to derive equations that resemble classical hydrodynamic equations for collective variables (fields) such as the particle density $n(\mathbf{r}, t)$ and the mean momentum $\mathbf{p}(\mathbf{r}, t)$. This is easily seen for the case of a single particle, $N = 1$, where interactions are absent, $w_{ij} \equiv 0$. Then the equations (6, 7) can be rewritten identically as coupled equations for the density, $n(\mathbf{r}, t) = A^2(\mathbf{r}, t)$, and momentum field $\mathbf{p}(\mathbf{r}, t) = m\mathbf{v}(\mathbf{r}, t) = \nabla S(\mathbf{r}, t)$, as already noticed by Madelung [99],

$$\frac{\partial n}{\partial t} + \nabla \cdot (n\mathbf{v}) = 0, \quad (15)$$

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v} \nabla \cdot \mathbf{p} = -\nabla(V + Q), \quad (16)$$

where again, the momentum evolution is driven by gradients of the external potential V and the quantum potential Q , Eq. (8), with $A \rightarrow \sqrt{n}$. Note that these equations are equivalent to the original Schrödinger equation and are, therefore, exact and valid on all length and time scales (as long as Eq. (4) is valid).

Let us now discuss the extension of this set of fluid-like quantum equations to many-particle systems, such as quantum plasmas. Returning to the original N -particle system requires to restore all pair interactions w_{ij} as well as the effects of finite temperature and the coupling to the environment (mixed state description). Finally, since the hydrodynamic equation contain single-particle quantities, a suitable procedure is required that reduces the N -particle quantities to one-particle fields.

Before considering quantum plasmas we note that, for weakly interacting bosons at low temperature, a very simple solution of the original Schrödinger equation (4) exists: using a basis of eigenfunctions ϕ_i of the single-particle hamiltonian, $\hat{h}_i = -\frac{\hbar^2}{2m}\nabla_i^2 + v(\mathbf{r}_i)$, i.e. $\hat{h}_i\phi_i = \epsilon_i\phi_i$, which are assumed to be complete and orthonormal, $\langle \phi_i | \phi_j \rangle = \delta_{ij}$, the N -particle wave function can be

expressed in terms of all possible products

$$\Psi(\mathbf{R}, t) = \sum_{\{\alpha\}} C_{\{\alpha\}}(t) \phi_{\alpha_1}(\mathbf{r}_1) \dots \phi_{\alpha_N}(\mathbf{r}_N) + \Psi^{\text{cor}}(\mathbf{R}, t), \quad (17)$$

where $\phi_{\alpha_k}(\mathbf{r}_k)$ denotes the orbital occupied by particle k , α_k stands for any of the eigenfunctions of \hat{h} , and the sum runs over all possible products of eigenfunctions. Further Ψ^{cor} is due to correlations and is negligible if the system is weakly interacting, which we will assume. Finally, at low temperature, the sum contains only a single product, corresponding to the ground state of the system. In the case of bosons in the condensate each particle occupies the same lowest energy orbital ϕ_0 ,

$$\phi_{\alpha_1} \equiv \phi_{\alpha_2} \equiv \dots \equiv \phi_{\alpha_N} \equiv \phi_0 = A_0 e^{\frac{i}{\hbar} S_0}. \quad (18)$$

In order to derive the Schrödinger equation and the quantum fluid equations for a Bose-Einstein condensate a convenient approach is to use the method of density operators that was introduced in Sec. III C. We make further progress by introducing a decoupling of the N -particle density operator (first line) and of the two-particle density operator (second line) that is analogous to Eq. (17):

$$\hat{\rho}(t) = \hat{F}_1(t) \hat{F}_2(t) \dots \hat{F}_N(t) + \hat{\rho}^{\text{cor}}(t), \quad (19)$$

$$\hat{F}_{12} = \hat{F}_1 \hat{F}_2 + \hat{g}_{12}, \quad (20)$$

where the first term is a product of N single particle density operators and corresponds to independent particles whereas correlation effects are accounted for by the second operator. The equation of motion for \hat{F}_1 follows from the von Neumann equation (12):

$$i\hbar \frac{\partial \hat{F}_1}{\partial t} - [\hat{H}_1, \hat{F}_1] = \text{Tr}_2[\hat{w}_{12}, \hat{g}_{12}] \equiv \hat{I}_1, \quad (21)$$

$$\hat{\tilde{H}}_1(t) = \hat{H}_1 + \hat{H}_1^{\text{H}}(t), \quad \hat{H}_1^{\text{H}}(t) \equiv \text{Tr}_2 \hat{w}_{12} \hat{F}_2(t), \quad (22)$$

which is the general operator form of a quantum kinetic equation where the collision integral \hat{I} involves the pair correlation operator \hat{g}_{12} . Equation (22) defines an effective single-particle hamiltonian that contains, in addition to kinetic and potential energy (first term), the Hartree mean field \hat{H}^{H} . The physical meaning of this term will become clear below, in the context of Eqs. (23) and (28).

Since we have assumed that the particles are weakly interacting, we neglect correlation effects, i.e. $\hat{\rho}^{\text{cor}} \rightarrow 0$, $\hat{g}_{12} \rightarrow 0$, as we did in Eq. (18). Then collisions are absent ($\hat{I}_1 = 0$), and equation (21) reduces to the time-dependent Hartree equation. In the Wigner representation this becomes the familiar quantum Vlasov equation, and the one-particle density operator becomes the Wigner function, $\hat{F}_1(t) \rightarrow f(\mathbf{r}, \mathbf{p}, t)$. Here it is more convenient to use, instead of the Wigner representation, the coordinate representation. Using eigenstates $|\mathbf{r}\rangle$ of the coordinate operator, \hat{r} , the density operator becomes the density matrix (we do not explicitly write the spin arguments), $\langle \mathbf{r}' | \hat{F}_1(t) | \mathbf{r}'' \rangle = f(\mathbf{r}', \mathbf{r}'', t)$. The coordinate

representation of Eq. (21) is given by [39]

$$i\hbar \frac{\partial}{\partial t} f(\mathbf{r}', \mathbf{r}'', t) = -\frac{\hbar^2}{2m} (\nabla_{\mathbf{r}'}^2 - \nabla_{\mathbf{r}''}^2) f(\mathbf{r}', \mathbf{r}'', t) + \{U^{\text{eff}}(\mathbf{r}', t) - U^{\text{eff}}(\mathbf{r}'', t)\} f(\mathbf{r}', \mathbf{r}'', t) \quad (23)$$

$$U^{\text{eff}}(\mathbf{r}, t) = U(\mathbf{r}) + U^{\text{H}}(\mathbf{r}, t),$$

$$U^{\text{H}}(\mathbf{r}, t) = g_s \int d\bar{r} w(r - \bar{r}) f(\bar{r}, \bar{r}, t), \quad (24)$$

where the diagonal element of the density matrix is the density, $f(\mathbf{r}, \mathbf{r}, t) = n(\mathbf{r}, t)$. The interactions give rise to an induced potential, $U^{\text{H}}(\mathbf{r}, t)$, the Hartree (or quantum Vlasov) mean field [the matrix element of Hartree mean field operator \hat{H}^{H} , Eq. (22)]. Obviously, U^{H} is the solution of Poisson's equation,

$$\Delta \phi^{\text{H}}(\mathbf{r}, t) = -4\pi \rho^{\text{ind}}(\mathbf{r}, t), \quad (25)$$

where the Hartree potential and induced charge density are given by $e\phi^{\text{H}} = U^{\text{H}}$, and $\rho^{\text{ind}}(\mathbf{r}, t) = eN|\phi_0(\mathbf{r}, t)|^2$, respectively [note that in the case of cold atoms, the interaction w_{ij} is not Coulombic but short-range].

From Eq. (23) we readily obtain the equation of the condensate wave function ϕ_0 . Similarly as $\hat{\rho}$, Eq. (11), the one-particle density operator is the projection operator on one-particle states, which follows from Eq. (14), the pure state assumption and the product ansatz of the wave function (17),

$$\hat{F} = N \text{Tr}_{2\dots N} |\psi_0(1)\rangle \dots |\psi_0(N)\rangle \langle \psi_0(1)| \dots \langle \psi_0(N)| = N |\psi_0(1)\rangle \langle \psi_0(1)|. \quad (26)$$

This yields the following density matrix:

$$f(\mathbf{r}', \mathbf{r}'', t) = N \langle \psi_0(1) | \psi_0(1) \rangle |\mathbf{r}'\rangle \langle \mathbf{r}''| = N \phi_0(\mathbf{r}', t) \phi_0^*(\mathbf{r}'', t), \quad (27)$$

which becomes a product of two condensate wave functions. Inserting this into Eq. (23) we immediately identify that its solution is given by the following equation for the condensate wave function

$$i\hbar \frac{\partial}{\partial t} \phi_0(\mathbf{r}, t) = \left\{ -\frac{\hbar^2}{2m} \nabla_{\mathbf{r}}^2 + V(\mathbf{r}) + U_{B0}^{\text{H}}(\mathbf{r}, t) \right\} \phi_0(\mathbf{r}, t) \quad (28)$$

$$U_{B0}^{\text{H}}(\mathbf{r}, t) = N \int d\mathbf{r}_2 w(\mathbf{r} - \mathbf{r}_2) |\phi_0(\mathbf{r}_2, t)|^2, \quad (29)$$

Thus, ϕ_0 is the solution of a nonlinear Schrödinger equation or of the coupled Schrödinger-Poisson system (28, 25) and is nothing but the Gross-Pitaevskii equation [70, 71]. Since this is an effective single-particle Schrödinger equation, it can again be mapped onto coupled equations for amplitude and phase or, alternatively, on the hydrodynamic equations (15, 16) with the only replacements $V(\mathbf{r}) \rightarrow V(\mathbf{r}) + e\phi^{\text{H}}(\mathbf{r}, t)$ and $n(\mathbf{r}, t) = NA^2(\mathbf{r}, t)$. Finally, we note that here we allowed for an external potential $V(\mathbf{r})$ that is able to trap the particles, as is the case for many experiments with ions or ultracold atoms. Thus, a system of N weakly interacting quantum particles in a Bose-Einstein condensate behaves essentially like a singly particle subject to an additional effective mean field U_{B0}^{H} .

E. Schrödinger equation for dense plasmas. Comparison to density functional theory

The example of a Bose-Einstein condensate (BEC) is instructive to understand how the reduction of the N -particle Schrödinger equation (or von Neumann equation for the density operator) to an effective single-particle Schrödinger equation (28), on one hand, and to quantum hydrodynamic equations for single-particle quantities, on the other, works, cf. Eqs. (30, 31). At the same time, a quantum plasma (in particular, the electron component) differs in several important points from a BEC:

1. The confinement potential V is not required and is usually absent.
2. Plasmas typically contain (at least) two oppositely charged components and are overall neutral.
3. Due to the Fermi statistics two particles cannot occupy the same orbital ϕ_0 (Pauli principle).
4. Particles in a plasma are usually in a mixed state.

We start by dealing with the first two issues. To this end, we now drop the external potential and assume that stability and confinement of the electrons is provided by charge neutrality, i.e. by the compensating ionic background of mean density n_0 . In that case, the role of the confinement potential V is taken over by the modified induced electronic charge density, in Eq. (25) which is replaced by $\rho^{\text{ind}} - en_0$. In that case the hydrodynamic equations (15, 16) become

$$\frac{\partial n}{\partial t} + \nabla(\mathbf{v}n) = 0, \quad (30)$$

$$\frac{\partial \mathbf{p}}{\partial t} + \mathbf{v} \nabla \mathbf{p} = -\nabla(U_0^{\text{H}} + Q), \quad (31)$$

$$Q(\mathbf{r}) = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n(\mathbf{r})}}{\sqrt{n(\mathbf{r})}}, \quad (32)$$

$$U_0^{\text{H}}(\mathbf{r}, t) = \int d\mathbf{r}_2 w(\mathbf{r} - \mathbf{r}_2) \{N|\phi_0(\mathbf{r}_2, t)|^2 - n_0\}. \quad (33)$$

We now turn to point 3. and analyze the effect of the Fermi statistics on the quantum hydrodynamic equations. But first we consider how the effective single-particle Schrödinger equation (28) is modified in the case of fermions. Starting with an analysis of the ground state (neglecting mixed state effects), the N -particle wave function can again be sought as a product of single-particle orbitals $|\phi_i\rangle$ (cf. second line),

$$|\Psi\rangle = |1, 1, \dots, 1, 0, 0, \dots\rangle \quad (34)$$

$$= |\phi_1\rangle |\phi_2\rangle \dots |\phi_N\rangle + \text{permutations}, \quad (35)$$

where each one is occupied by a single electron (or by two, in case we perform a spin-resolved analysis, and the energies are spin-independent), where the corresponding single particle energies are understood to be ordered, $\epsilon_1 \leq \epsilon_2 \leq \dots \leq \epsilon_N$.

1. Ground state results

The ground state corresponds to the N energetically lowest orbitals being occupied, as indicated by Eq. (34). Note that the correct ground state wave function is a Slater determinant (it corresponds to all $N!$ possible ways to distribute N particles over these orbitals). This is written in the first line in occupation number representation: the notation means that the N lowest orbitals are exactly occupied by $f_i = 1$ particle each, whereas all higher orbitals are empty, $f_i = 0$, $i > N$.

From these arguments it is clear that the single-particle density operator has the form

$$\begin{aligned} \hat{F}_1 &= N \text{Tr}_{2\dots N} |\Psi\rangle\langle\Psi| \\ &= \frac{N}{N} \{|\phi_1\rangle\langle\phi_1| + \dots + |\phi_N\rangle\langle\phi_N|\}, \end{aligned} \quad (36)$$

corresponding to the N orbitals particle “1” can occupy with equal probability $1/N$. From this we again obtain the density matrix by multiplying with coordinate eigenstates $\langle\mathbf{r}'|$ and $|\mathbf{r}''\rangle$:

$$f(\mathbf{r}', \mathbf{r}'', t) = \phi_1(\mathbf{r}', t)\phi_1^*(\mathbf{r}'', t) + \dots + \phi_N(\mathbf{r}', t)\phi_N^*(\mathbf{r}'', t).$$

We now insert this ansatz into the equation of motion (23). It is easy to verify that this equation is solved when each orbital fulfills the following single-particle Schrödinger equation ($i = 1 \dots N$)

$$i\hbar \frac{\partial}{\partial t} \phi_i(\mathbf{r}, t) = \left\{ -\frac{\hbar^2}{2m} \nabla_{\mathbf{r}}^2 + U_{F0}^H(\mathbf{r}, t) \right\} \phi_i(\mathbf{r}, t), \quad (37)$$

$$U_{F0}^H(\mathbf{r}, t) = \int d\mathbf{r}_2 w(\mathbf{r} - \mathbf{r}_2) [g_s n(\mathbf{r}_2, t) - n_0], \quad (38)$$

where the fermionic Hartree mean field contains the densities of all occupied orbitals, $n(\mathbf{r}, t) = \sum_{i=1}^N |\phi_i(\mathbf{r}, t)|^2$. Equations (37) and (38) are the time-dependent Hartree equations for weakly interacting fermions (interactions are taken into account approximately, only via this mean field). Note that, while the equations for the individual orbitals all have the same form, they are in fact coupled via the Hartree mean field. Further, all orbitals differ due to the Pauli principle which is assured by enforcing the orthonormality condition, $\langle\phi_i|\phi_j\rangle = \delta_{ij}$, and the ground state is obtained from minimization of the total energy. The missing exchange and correlation contributions can be taken into account approximately by adding the exchange-correlation potential $V^{\text{xc}}(\mathbf{r}, t)$ [and, in general, also an external potential $V(\mathbf{r})$] to the Hartree potential, as we discuss in the next section.

2. Including exchange, correlations and finite temperature effects

In fact, with V^{xc} added, equations (37, 38) are the time-dependent Kohn-Sham equations—the basic equations of time-dependent density functional theory (TD-DFT) [104]. A particular strength of this theory is

its solid theoretical foundation on the Runge-Gross theorem [104] and the corresponding theorems for time-independent DFT [105]. The basic statement is that a system of N interacting fermions can be mapped exactly on a system of N non-interacting particles with the same density $n(\mathbf{r}, t)$ where all interactions are lumped into an effective single-particle potential that is a direct generalization of the Hartree potential (38),

$$U_{F0}^H(\mathbf{r}, t) \rightarrow U_F^H[n(\mathbf{r}, t)] + V^{\text{xc}}[n(\mathbf{r}, \tilde{t}); \beta, \mu], \quad (39)$$

$$U_F^H[n(\mathbf{r}, t)] = \int d\mathbf{r}_2 w(\mathbf{r} - \mathbf{r}_2) [g_s n(\mathbf{r}_2, t) - n_0], \quad (40)$$

$$n(\mathbf{r}, t; \beta, \mu) = \sum_{i=0}^{\infty} f_i(\beta, \mu) |\phi_i(\mathbf{r}, t)|^2. \quad (41)$$

The first remarkable property of these equations is that, both, the mean field and the additional exchange-correlation potential do not explicitly depend on the individual orbital wave functions but only on the total density, so also the coordinate dependence is only implicit, via the functional $n(\mathbf{r}, t)$.

Second, we indicated that the ground state result is directly extended to equilibrium systems at finite temperature $k_B T = \beta^{-1}$ and given chemical potential $\mu = \mu(n, T)$ (grand ensemble). This extension is realized in a very simple form, by replacing the above occupation numbers f_i by temperature and density-dependent numbers. In equilibrium these numbers are known and given by the Fermi function $f_i(\beta, \mu) = [\exp\{\beta(\epsilon_i - \mu)\} + 1]^{-1}$. The f_i have to be understood as mean orbital occupation numbers that, in general, deviate from zero and one which requires to extend the sum in (41) to infinity. This extension of DFT to finite temperatures is originally due to Mermin [106] and is now widely used in dense plasma and warm dense matter simulations, e.g. [107]. In Eq. (40) we also indicated that, at finite temperature, V^{xc} carries a temperature dependence, and recently, first *ab initio* results for the local density approximation (LDA) and for the generalized gradient approximation (GGA) have been reported, e.g. [108–110], and references therein.

Even though the theorems of DFT prove the existence of the functional V^{xc} , its exact form is, in most cases, not known. The power of the method is due to the fact that high quality approximations for V^{xc} are available, that are constantly improve by a large community, for an overview see Ref. [111, 112]. A particular problem of time-dependent DFT is that the exact functional V^{xc} does not only depend on the current density $n(\mathbf{r}, t)$ but, in general the dependence is also on the density profile at earlier times, $n(\mathbf{r}, \tilde{t})$, $0 \leq \tilde{t} \leq t$. While most current implementations neglect this “memory” effect and use an adiabatic approximation (e.g. adiabatic LDA, ALDA), $\tilde{t} \rightarrow t$, presently extensive research is devoted to finding efficient improved functionals that go beyond that limit.

To conclude this discussion, we underline that the accuracy of DFT and TD-DFT is rooted in the simultaneous access to all time-dependent orbitals ϕ_i which also

allows for an efficient treatment of partially ionized plasmas and the crossover the condensed matter and plasma phases upon heating or compression. In contrast, QHD aims at transferring the many-body problem to coupled equations for a single mean density and a single mean velocity. Therefore, in general, information is lost, and the accuracy is reduced [78]. In the following, we discuss how this transition to hydrodynamic equations can be systematically achieved for fermions, starting from Eqs. (37) and (38).

F. Quantum hydrodynamics for dense plasmas

We start from the time-dependent Hartree equations (37) and (38) and convert each solution, $\phi_i(\mathbf{r}, t) = A_i(\mathbf{r}, t)e^{\frac{i}{\hbar}S_i(\mathbf{r}, t)}$, into an individual pair of amplitude and phase equations [76],

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (\mathbf{v}_i n_i) = 0, \quad (42)$$

$$\frac{\partial \mathbf{p}_i}{\partial t} + \mathbf{v}_i \nabla \mathbf{p}_i = -\nabla (U_{F0}^H + Q_i), \quad (43)$$

$$Q_i(\mathbf{r}) = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n_i(\mathbf{r})}}{\sqrt{n_i(\mathbf{r})}}, \quad (44)$$

where $n_i = A_i^2$ and $\mathbf{p}_i = \nabla S_i$. This system can be understood as “microscopic QHD equations (MQHD)” and is fully equivalent to TD-DFT and quantum kinetic theory (QKT) – the differences depend on the treatment of correlation effects via an exchange correlation potential, V^{xc} , that was discussed above, or collision integrals, in the case of QKT. We show in the appendix that the linear response properties of the present microscopic QHD (i.e. without exchange and correlation corrections) are, in fact, equivalent to the random phase approximation (RPA). In particular, they yield the correct plasmon spectrum and the correct screening of a test charge – in contrast to the standard QHD (see below). Finally, we note that the MQHD equations are closed, i.e. they are not coupled to additional equations of higher moments of the distribution function f_i , as in the case of standard fluid theory.

1. Averaging over the orbitals

To convert these microscopic equations into a single pair of density and momentum equations (QHD), a suitable averaging over the orbitals is necessary which we

denote by a “bar”:

$$\bar{n}(\mathbf{r}, t) = \frac{1}{N} \sum_{i=1}^{\infty} f_i n_i(\mathbf{r}, t), \quad (45)$$

$$\bar{\mathbf{p}}(\mathbf{r}, t) = \frac{1}{N} \sum_{i=1}^{\infty} f_i \mathbf{p}_i(\mathbf{r}, t), \quad (46)$$

$$\bar{Q}(\mathbf{r}, t) = -\frac{\hbar^2}{2mN} \sum_{i=1}^{\infty} f_i \frac{\nabla^2 \sqrt{n_i(\mathbf{r})}}{\sqrt{n_i(\mathbf{r})}}. \quad (47)$$

Here we introduced the statistical weights f_i (mean occupation numbers) of the orbitals and extended the sum to infinity, as done in Eq. (41). In thermodynamic equilibrium they are given by a Fermi distribution, which in the ground state (pure state) reduces to the Fermi step function $f_i = \Theta(E_F - \epsilon_i)$. (Note that, if this function is converted to a function of momentum or velocity, correlation effects will lead to a deviation from a Fermi function). We note that the same averaging procedure was considered before. In Ref. [75] the authors assumed that all orbital amplitudes are equal [cf. Eq. (55)] whereas, in Ref. [76], it was assumed that one can substitute $\sum_{i=1}^{\infty} f_i \frac{\nabla^2 \sqrt{n_i(\mathbf{r})}}{\sqrt{n_i(\mathbf{r})}} \rightarrow \frac{\nabla^2 \sqrt{n(\mathbf{r})}}{\sqrt{n(\mathbf{r})}}$. Here we avoid any assumption but present a systematic derivation that allow to understand the applicability limits of these assumptions and to derive corrections.

As an illustrative example we consider a uniform electron gas in three dimensions ($0 \leq x, y, z \leq L$) where the ideal orbitals are plane waves,

$$\phi_{\mathbf{m}}(\mathbf{r}, t) = L^{-3/2} e^{-i(E_{\mathbf{m}}t - \mathbf{k}_{\mathbf{m}}\mathbf{r})/\hbar}, \quad (48)$$

$$\mathbf{k}_{\mathbf{m}} = \frac{2\pi}{L} \mathbf{m}, \quad m_{\alpha} = \pm 1, \pm 2, \dots, \quad \alpha = x, y, z,$$

with the orbital energies $E(\mathbf{k}) = \hbar^2 \mathbf{k}^2 / 2m$ [80]. Thus, in this case, the orbital index “ i ” is replaced by the multi-index (m_x, m_y, m_z) . The plane waves (48) form a complete set of wave functions and are solutions of the system (37) in the case of $w \rightarrow 0$ (i.e. without the Hartree potential), and in the ground state ($T = 0$) all orbitals with momenta up to the Fermi momentum $\hbar k_F$ are occupied. If the Hartree mean field is included, the solutions are not given by the system (48) but can be constructed as a linear superposition:

$$\phi_{\mathbf{m}}^H(\mathbf{r}, t) = \sum_{\mathbf{m}'} C_{\mathbf{m}'}^{\mathbf{m}} \phi_{\mathbf{m}'}(\mathbf{r}, t), \quad C_{\mathbf{m}'}^{\mathbf{m}} \in \mathbb{C}. \quad (49)$$

To proceed from the microscopic QHD equations (42, 43, 44) to the QHD equations, we express each of the orbital quantities (n_i , \mathbf{p}_i , Q_i) in terms of their averages and fluctuations: $n_i = n + \delta n_i$, and so on. This allows us to perform an orbital average of the microscopic equations (42, 43, 44) taking into account that the average of products of two orbital quantities is given by

$$\overline{a_i b_i} = \bar{a} \cdot \bar{b} + \overline{\delta a_i \delta b_i},$$

$$\frac{\partial \bar{n}}{\partial t} + \frac{1}{m} \nabla \cdot (\bar{\mathbf{p}} \cdot \bar{n}) = -\frac{1}{m} \nabla \cdot \overline{\delta \mathbf{p}_i \delta n_i}, \quad (50)$$

$$\frac{\partial \bar{\mathbf{p}}}{\partial t} + \frac{1}{m} \bar{\mathbf{p}} \cdot \text{div} \bar{\mathbf{p}} = -\nabla \cdot (U_{F0}^H + \bar{Q}) - \frac{1}{m} \overline{\delta \mathbf{p}_i \text{div} \delta \mathbf{p}_i}, \quad (51)$$

$$\bar{Q} = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\bar{n}}}{\sqrt{\bar{n}}} + Q^\Delta, \quad (52)$$

$$Q^\Delta \approx \frac{\hbar^2}{2m\bar{n}} \overline{\delta A_i \cdot \nabla^2 \delta A_i} + O\left(\left(\frac{\delta A_i}{A}\right)^2\right). \quad (53)$$

We underline that this averaging over “i” is associated with an averaging over the individual orbitals $\phi_i(\mathbf{r}, t)$ which inevitably causes a loss of spatial and temporal resolution [78], as in classical hydrodynamics.

Let us discuss this very general result. First, the correlator of the momentum fluctuations can be rewritten in coordinates as ($\alpha, \beta = x, y, z$, summation over repeated Greek indices is implied)

$$\begin{aligned} \frac{1}{m} \overline{\delta p_{i\alpha} \delta p_{i\beta}} &= \frac{1}{2m} \partial_\alpha \overline{\delta p_{i\alpha}^2} + \frac{1}{m} \partial_\gamma \overline{\delta p_{i\alpha} \delta p_{i\gamma}}, \quad \gamma \neq \alpha \\ &= \frac{1}{3} \partial_\alpha \overline{E_{\text{kin}}} + \frac{1}{n} \partial_\gamma \overline{\sigma_{\alpha\gamma}} \\ &\approx \frac{1}{5} \partial_\alpha \overline{E_F}, \quad \text{ideal Fermigas, } T = 0. \end{aligned} \quad (54)$$

The first (diagonal) term is nothing but the average kinetic energy per particle, arising from the momentum variance which is non-zero, even in a pure state. For the ideal Fermi gas (48) in the thermodynamic limit ($N \rightarrow \infty$, $L \rightarrow \infty$, $n = N/L^3 = \text{const}$) at $T = 0$ this is directly related to the pressure of a three-dimensional quantum plasma via $\overline{P_{F0}^{\text{id}}}(\bar{n}) = \frac{2}{5} \bar{n} E_F(\bar{n})$. The second term contains non-diagonal elements of the pressure tensor, i.e. the mean shear stress. Here we will focus on weakly nonideal Fermi systems where the shear stress is negligible (third line). Thus, for a 3D ideal Fermi gas, the last term in Eq. (51) becomes $-\frac{1}{2\bar{n}} \nabla \cdot \overline{P^{\text{id}}}(\bar{n})$. The results for 1D and 2D are given in the appendix. We will return to the question of possible closures of the QHD equations at the end of this section.

Second, for an ideal Fermi gas at $T = 0$ all amplitudes are given by Eq. (48): $n_i = A_i^2 = L^{-3} \equiv \bar{n}$. Thus, all density fluctuations vanish exactly, $\delta n_i = 0$, and the r.h.s. of Eq. (50) and also Q^Δ vanish. The same result would also be obtained if the system is in a weak external potential $V(\mathbf{r})$ so that the orbitals become weakly space-dependent but remain equal,

$$A_1(\mathbf{r}) = \dots = A_N(\mathbf{r}), \quad (55)$$

what still enforces vanishing of the density fluctuations. With this we recover the result of Manfredi and Haas [75] that they derived for a one-dimensional ideal Fermi gas at $T = 0$ from the assumption (55). Finally, we note that the above result is naturally generalized to finite temperatures by using the corresponding statistical weights f_i that are given by a Fermi function instead of the step function [76, 83].

2. Linearization. Application to plasma oscillations

Let us briefly discuss some consequences of the equations (50), (51) and (52). Without an external field the system is spatially homogeneous, $\bar{n} = \text{const}$, and $\bar{\mathbf{p}} = \bar{Q} = 0$. The stability condition is then given by $U_{F0}^H = -\overline{P_{F0}^{\text{id}}}/\bar{n}$. Next we investigate the spectrum of collective modes of the system. To this end we apply a weak monochromatic perturbation to the Hartree mean field, $\phi_1(\mathbf{r}, t) \sim e^{-i\omega t + i\mathbf{q}\mathbf{r}}$. After linearization and Fourier transformation, we obtain the following spectrum of plasma oscillations [for details see the appendix],

$$\omega(q)^2 = \omega_p^2 + \frac{1}{3} v_F^2 q^2 + \frac{\hbar^2}{4m^2} q^4. \quad (56)$$

This agrees with the plasmon dispersion of an ideal three-dimensional Fermi gas, except for the coefficient of the q^2 term which is known from kinetic theory to be equal 3/5. Thus the present QHD model yields a coefficient that is by a factor 9/5 too small, as was noticed in Ref. [83]. Thus, while here the Bohm potential Q is correct (it produces the correct coefficient of the q^4 term) the Fermi pressure term is not. We underline that, for an ideal 3D Fermi gas at $T = 0$, the Fermi pressure is tied to the Fermi energy as $\frac{2}{5} n E_F$ without any freedom of choice. In contrast, the microscopic QHD equations (43, 44) yield the correct result, as is shown in the appendix. Interestingly, in a 1D model the coefficient of the q^2 term is v_F^2 , which coincides with the quantum kinetic (MQHD) result, however, in 2D, the coefficients of the q^2 and q^4 terms are incorrect [in all cases a 3D Coulomb potential has been used, cf. the appendix].

In fact, the discrepancies between QHD and microscopic QHD are more far-reaching. Aside from the inaccuracy of the prefactor of the pressure, $\overline{P^{\text{id}}}$, in general, also the prefactor of the Bohm term Q in Eq. (52) requires adjustments. This term depends on the frequency and the wave number of the excitation, ω and k : for example, for small ω , there appears an additional factor 1/9 [81]. This is crucial, e.g. for the correct description of screening of a test charge in a quantum plasma and for the dispersion of ion-acoustic modes. A comprehensive analysis of the ω - and k -dependence of these coefficients has been performed in Ref. [83]. To summarize, our analysis shows that the assumption of identical orbital amplitudes A_i , Eq. (55), holds strictly only for a zero temperature ideal Fermi gas for which the orbitals are given by Eq. (48) when the mean field is neglected [with U^H taken into account, the solutions are given by Eq. (49) and not necessarily fulfill condition (55)]. This leads to the correct plasmon dispersion in 1D and may be an indication why QHD models have been found to work well for some high-frequency excitations of electrons in metals or metallic nanostructures [113], although there slightly different model equations are being used, e.g. [33, 72, 73].

3. Discussion of the QHD equations and possible improvements

Let us try to understand the origin of the problems of the QHD equations that were mentioned above. This is easily seen by comparing step by step the linearization procedure of the QHD and the MQHD equations. While in QHD the microscopic equations (42, 43 and 44) are first averaged over the orbitals (over “ i ”) and then linearized, in MQHD, in contrast, linearization is done first and averaging second. Of course the order is crucial and the approach of MQHD (and, equivalently, quantum kinetic theory) is more accurate than the QHD procedure. The difference becomes already clear from an analysis of the linearized continuity equation after Fourier transformation [details are given in the appendix]:

$$\text{QHD : } \mathbf{q} \cdot \bar{v}_1(\omega, \mathbf{q}) = \frac{\bar{n}_1(\omega, \mathbf{q})}{n_0} \omega, \quad (57)$$

$$\text{MQHD : } \overline{\mathbf{q} \cdot \bar{v}_{i1}(\omega, \mathbf{q})} = \frac{\bar{n}_{i1}(\omega, \mathbf{q})}{n_{i0}} (\omega - \mathbf{q} \cdot \mathbf{v}_{i0}). \quad (58)$$

While the left hand side of the MQHD, after averaging, coincides with the l.h.s. of the QHD, averaging of the r.h.s. of the MQHD result does not reduce to the QHD because the averaging is over a product of three i -dependent quantities. Most importantly, in the MQHD the unperturbed velocities enter which are non-zero in general. For example, for an ideal Fermi gas [cf. Eq. (48)], $v_i \sim \nabla S_i \sim k_i$, which are densely packed in the range of $[-k_F, k_F]$. This is a consequence of Fermi statistics and of the Pauli principle. In contrast, in QHD only the mean unperturbed velocity enters which, for a field-free system is zero, $\mathbf{v}_0 = 0$. Similar behavior is observed in the momentum balance.

Based on this analysis we conclude this section with a brief summary and outlook on QHD:

1. The equation of motion for the single-particle density operator, Eqs. (21) and (23), is a convenient and general starting point for the strict derivation of TD-DFT and of the QHD equations for bosons and fermions.
2. Introducing a Hartree product ansatz for the N -particle wave function and the occupation number representation (35) directly yields microscopic QHD equations [76] with orbital resolved densities and momenta that allow for a systematic and intuitive derivation of the QHD equations for dense plasmas, as well as for future improvements.
3. The comparison of Eqs. (57) and (58) indicates that in the linearized Fourier transformed QHD equations the Fermi pressure and the Bohm potential must have prefactors that depend on frequency, wave number and dimensionality D ,

$$\bar{P}_F \rightarrow \alpha(\omega, k, D) \bar{P}_F, \quad (59)$$

$$\bar{Q} \rightarrow \gamma(\omega, k, D) \bar{Q}. \quad (60)$$

The values of α and γ for the important limiting cases of high and low frequency, as well as high and low wave number are known analytically, even at finite temperature [83]. Thus for these situations reliable simulations are possible.

4. The term Q^Δ as well as the r.h.s. of Eq. (50) were neglected in the derivation of Ref. [75], which was justified by the assumption (55). However, when the orbital amplitudes are not identical both terms become relevant, and it will be very interesting to study their consequences.
5. If interactions in the Fermi gas are treated more accurately (beyond the Hartree mean field approximation), also interaction contributions to the Fermi pressure and the shear stress $\bar{\sigma}_{\alpha\beta}$ become important. Here one can either derive an equation of motion for $\bar{\sigma}_{\alpha\beta}$ and find a proper closure of the corresponding extended QHD equations. Alternatively, if the orbital occupations f_i are known, $\bar{\sigma}_{\alpha\beta}$ can be computed explicitly.
6. Adding exchange and correlations is an obvious improvement of the theory, and a simple ansatz for V^{xc} was suggested in Ref. [77]. Advanced approximations can be obtained by following the experience from TD-DFT, cf. the discussion of the time-dependent Kohn-Sham equations (37). At the same time, inclusion of V^{xc} cannot compensate for deficiencies in the treatment of Fermi pressure and Bohm potential. Moreover, the contributions from V^{xc} should be consistent with the closure of the QHD equations (see above).
7. The present approach of the microscopic QHD equations can be straightforwardly extended to explicitly include the spin degrees of freedom and spin dynamics.

From this summary it is clear that there is plenty of room for substantial further improvements of the QHD for fermions. We will return to a discussion of the QHD equations and their perspectives for quantum plasmas below, in Secs. VID and VII.

IV. TRANSFER OF PLASMA PHYSICS RESULTS FROM ONE SYSTEM TO ANOTHER

There is a successful tradition in plasma physics: the dominant role of the strong and long-range Coulomb interaction between charged particles allows to transfer results that were obtained for one type of plasma to another one. This applies, in particular, to high-frequency plasma oscillations (Langmuir waves) in the long wavelength limit which have a universal value, $\omega(k \rightarrow 0) = \omega_{pl} = (4\pi n_e e^2 / m_e)^{1/2}$. The electron plasma frequency depends on the environment of the electrons: for example, in a solid, the effective electron mass depends

on the band structure and is, in general, different from the free electron mass, i.e. $m_e \rightarrow m_e^{\text{eff}}$. Furthermore, in a semiconductor, the Coulomb interaction is screened by the surrounding lattice, and $e^2 \rightarrow e^2/\epsilon_b$, where the background dielectric constant is typically in the range of 10...20, see Eq. (2). Thus, the plasma frequency is the same function in any system with Coulomb interaction, $\omega_{pl} = \omega_{pl}[m_e^{\text{eff}}, \epsilon_b]$, and knowledge of the effective mass and background dielectric constant is sufficient to compute it.

Note that, while the long wavelength limit of the dispersion, ω_{pl} , depends only on material parameters, the dispersion

$$\omega^2(k) = \omega_{pl}^2 + a_2 k^2 + a_4 k^4 + \dots, \quad (61)$$

at finite k may be very different in different plasmas, strongly depending on the correlations in the system, i.e. on the value of r_s , see e.g. Ref. [114]. The same applies to the plasmon damping. Thus, there is no universal way to transfer the results for the coefficients a_2, a_4, \dots from an electron-ion plasma to other plasmas. Just taking the results for a_2, a_4, \dots for an electron-ion plasmas and replacing the densities, masses and dielectric function by those of the new system, as it is often done in the QHD-quantum plasma publications, will generally lead to incorrect results. Furthermore, the low-frequency acoustic modes are strongly differing in different two-component plasmas, depending on the material details.

A. Plasmas in solid state systems

A well-known example of transfer of plasma physics results are condensed matter systems such as metals and semiconductors. Plasma oscillations in metals were investigated, in analogy to gas plasmas, already by Bohm and Pines [40, 41]. This was the first example of a quantum plasma, cf. Fig. 1.

The second example is the electron-hole plasma in semiconductors that – to some extent – can be modeled as a two-component plasma where the (positive) holes (h) take over the role of the ions. Then, the results from an electron-ion plasma are readily transferred to an e-h plasma by properly rescaling the particle masses and background dielectric constant, as discussed above. In particular, the traditional fluid approach developed for e-i plasmas to compute plasma oscillations, waves, and instabilities, has been applied to e-h plasmas already in the 1960s and 1970s, e.g. [115] and references therein.

However, semiconductor physics has made dramatic progress during the last four decades, and it is clear that a two-fluid description is way too simple for most problems (hydrodynamic models are still being applied occasionally, mostly to transport problems). Most importantly, electrons in solids do not have a parabolic energy dispersion as in a plasma, i.e.

$$\frac{p^2}{2m} \rightarrow \epsilon_\alpha(p), \quad \alpha = 1, 2, \dots, \quad (62)$$

and, moreover, occupy different energy bands α . Only in the limit of very small momenta a parabolic momentum dependence is observed where one can use the concept of an effective “mass” given by $1/m_\alpha^{\text{eff}} \sim d^2 \epsilon_\alpha / dp^2$. This means that, only for very weak excitation will the electron intraband dynamics and plasma oscillations be governed by a quadratic dispersion. For strong excitation and, in particular, for nonlinear modes, the deviation from the plasma-like quadratic dispersion may be dramatic.

While plasma oscillations in metals are typically carried by electrons occupying the highest energy band (conduction band, “c”), the situation in semiconductors is different. Here, the uppermost two bands (the conduction band and the valence band, “v”) are separated by an energy gap (band gap), $E_{\text{gap}} = \epsilon_c(0) - \epsilon_v(0)$, [this is written for the case of a direct gap] that is on the order of 0.5...3eV. At normal conditions, typically the conduction band is empty [an exception are doped semiconductors that require a separate more involved analysis]. By applying an external excitation, such as a laser pulse, electrons can be transferred from the valence band to the conduction band. The electron that is now “missing” in the valence band behaves like a positive particle (a hole) with an effective mass that depends on the curvature of the valence band, cf. Eq. (62). Intense laser excitation then creates an “electron-hole plasma”. The density of this plasma depends on the laser intensity (photon number) and may correspond to a classical or quantum plasma, cf. Fig. 1.

Note that this plasma may be far from equilibrium, if the photon energy exceeds the band gap, $\hbar\omega > E_{\text{gap}}$, e.g. [32, 39]. Another difference from electron-ion plasmas is the finite life time of this e-h-plasma: electrons in the conduction band will ultimately spontaneously return to the valence band (recombine with a hole). This occurs on times on the order of $\tau_{\text{rec}} \sim 1\text{ps}$. So any plasma oscillation that has a period $T_{\text{osc}} \sim 2\pi/\omega_{\text{osc}}$ that is longer than the e-h-life time, τ_{rec} , will be not observable.

Thus we conclude that the transfer of results for plasma oscillations from e-i-plasmas to condensed matter systems is only possible for a very limited number of cases. In particular, nonlinear oscillations, such as solitons, cannot be transferred, due to the different energy dispersion (62). The same is true for low-frequency oscillations that will not be observable in semiconductors due to the finite life time of e-h-pairs. This does not mean that such modes cannot exist in condensed matter plasmas. However, they have to be derived from a realistic model of the solid. Their properties can definitely not be derived from a two-component e-i-plasma model (fluid or kinetic) by simply rescaling masses and dielectric constants. We stress that this has nothing to do with the type of plasma model that is being used. At the same time, it is the large number of QHD-quantum plasma papers that have applied exactly such a purely formal “rescaling” procedure that leads to “semiconductor plasmas” that have nothing to do with reality.

An improved description of condensed matter plasmas that goes beyond fluid models, based on quantum kinetic theory within the random phase approximation, has already been developed long ago, e.g. [116–118], and references therein. The modern theoretical description of electron-hole plasmas, e.g. [31, 32], includes details of the band structure, disorder, Coulomb correlations and bound states (excitons, trions, biexcitons [119, 120]), the effect of the lattice (phonons) and kinetic effects that are important, in particular, for nonresonant laser excitation of semiconductors. So, although fluid models are still occasionally used, state of the art approaches in the field of semiconductors and metals as well as DFT, time-dependent DFT and quantum kinetic theory.

To summarize, only a quite limited number of properties of quantum (or classical) plasmas in solids can, in some cases, be approximately described by fluid models. Thus, when transferring (classical or quantum) hydrodynamic results from e-i-plasmas to semiconductors or metals, in each case, a very careful analysis of the validity of a fluid description is mandatory. This requires to first perform a state of the art theoretical analysis and then prove that it is justified to reduce it to a fluid model, for the considered application. Furthermore, additional effects such as finite lifetime, dissipation and deviation from a parabolic energy dispersion or from thermal equilibrium have to be analyzed.

B. Dusty plasmas

A field to which results from electron-ion plasmas have been transferred successfully are dusty plasmas that have been actively studied for more than 25 years. The addition of micrometer-size “dust” particles to a low-temperature plasma leads to a broad range of novel phenomena, e.g. [121–123]. They include strong correlation effects, due to the high charge (on the order of several thousand elementary charges) of these particles giving rise to large classical coupling parameters Γ_d , of the dust component, e.g. [124], and interesting transport and wave phenomena [125]. Dusty plasmas have successfully been described by transferring results from multi-component e-i-plasmas by treating the dust particles like (negative) macro-ions. A well-known example is the prediction of dust acoustic waves, in analogy to ion-acoustic waves, by Rao, Shukla and Yu [126] that were experimentally verified by Barkan, Merlino, and D’Angelo [127]. At the same time, despite the similarity with electron-ion plasmas, there are important differences that have to be taken into account and limit a direct transfer of results. This includes the low degree of ionization, as well as the anisothermal and nonequilibrium character of the plasma. The relevance of these effects can be directly tested in experiments.

Since we will be interested in the extension to “quantum dusty plasmas”, cf. Sec. V, it is instructive to consider the parameters of conventional dusty plasmas in

some detail. Typical densities and temperatures of dusty plasmas are sketched in Fig. 2. The line “50 % ionization” divides the ranges where neutral particles dominate (inside the line) from the highly ionized case. For definiteness, this line is shown for hydrogen, but its location only slightly differs for other plasmas, see also Sec. V C. In addition to the dimensionless plasma parameters that were introduced in Sec. IV, the presence of dust particles gives rise to the following additional parameters:

6.: the parameters of individual dust particles: the particle radius and charge number, a_D, Z_D , the atomic species “A” and mass density ρ_D , and the binding energy E_D of atoms to the dust particle;

7.: the Havnes parameter $P = |Z_d|n_D/n_e$, that is the ratio of the charge density concentrated on the dust particles to the free electron density.

8.: the dimensionless dust particle surface potential,

$$\phi_s = -Z_d e^2 / a_D k_B T_e, \quad (63)$$

for which the simplest approximation is given by the orbital motion limited theory (OML) [124]. Equating the fluxes of electrons and ions, ϕ_s can be evaluated from the equation

$$\sqrt{\frac{m_i T_e}{m_e T_i}} e^{-\phi_s} = \left(1 + \phi_s \frac{T_e}{T_i}\right), \quad (64)$$

where electron release from the dust particle surface due to radiation is neglected. In the special case of an isothermal plasma with $T_e = T_i$, ϕ_s depends only on the mass ratio. Then, for the example of a hydrogen (helium) plasma, $\phi_s \simeq 2.5$ ($\phi_s \simeq 3$). In general, for any atomic mass, $\phi_s \lesssim 5$. Note that for the non-isothermal case, the value of ϕ_s weakly depends on T_e/T_i . For instance, at $T_e/T_i = 100$, ϕ_s is approximately two times smaller compared to the isothermal case.

Furthermore, the effect of collisions is to decrease the dust particle surface potential [128, 129]. Also, ϕ_s is restricted from above by the tensile strength of the dust material. Fracturing will occur if the electrostatic stress, inside of a dust particle due to charging, $S \sim (\phi_s k_B T_e / e a_D)^2$, exceeds the tensile strength [130]. Additionally, the grain surface potential is limited by electron (ion) field emission [130]. In general ϕ_s may depend on many additional factors including the plasma particle temperatures and densities, dust particle material, size and shape, external fields (electrostatic, magnetic, radiation). Nevertheless, the majority of studies on dust particle charging in different plasma environments clearly show that the dimensionless dust particle surface potential, $|\phi_s|$, is on the order of 1. Specifically, this is true for dust charging by external radiation [131], which is important for consideration of dust in astrophysical contexts.

These results have also frequently been directly applied to “Quantum Dusty Plasmas” which has to be questioned. The correct extension to quantum plasmas will be discussed in Sec. V A.

V. “QUANTUM DUSTY PLASMAS”

It is certainly an interesting idea to combine dusty plasmas and quantum plasmas by “adding” dust particles to a quantum plasma or by cooling a dusty plasma to low temperatures, giving rise to a new field of “quantum dusty plasma” (QDP). As in the case of dusty plasmas, this promises many novel physical phenomena. In fact, this idea was put forward in 2005 by Shukla and Ali [94]. That paper has had, and continues to have, a high impact in the field (it collected more than 90 and the follow-up paper [132] more than 130 citations), so it is fair to quote from it to describe the concept of QDP: “...when a dusty plasma is cooled to an extremely low temperature...ultracold dusty plasma behaves like a Fermi gas...” More details on this paper are discussed in the supplement [95].

The work [94] has been followed by a large number of papers on QDP that were dedicated to important plasma physics phenomena, such as the Jeans instability [132–137], different types of dust ion acoustic waves [134, 138–159], the dust-lower-hybrid instability [160, 161], new low-frequency oscillations [162], plasma waves and instabilities under a gravitational field [163, 164], screening effects [165], and nonlinear ion acoustic waves [166]. These and related studies demonstrated that the inclusion of charged dust particles leads to a broad set of waves and oscillations that is much richer than in “ordinary” electron-ion quantum plasmas (see e.g. Refs. [148, 159, 162]). In addition, it was shown that the presence of the charged dust particles significantly affects the parameters of the acoustic waves and ionic solitons [166] in quantum plasmas due to modification of the charge balance between ions and electrons. On the other hand, the results that are well-known from classical (dusty) plasmas get modified due to incorporation of quantum corrections. For example, quantum effects have been reported to stabilize the Jeans instability [132, 133, 164, 167], and dust acoustic waves are modified due to quantum effects [134, 143, 144, 147].

A. Basic parameters of QDP

In the above mentioned papers quantum dusty plasmas with a broad range of parameters were considered, a few characteristic examples are listed in table I. The authors considered a huge range of temperatures – from cryogenic ($T \sim 100$ K) to extremely hot ($T = 10^8$ K), whereas the plasma density was typically assumed to be very high, $n_e \gtrsim 10^{23}$ cm $^{-3}$. At the same time, the parameters describing the dust particles such as their material, geometry and size, are often omitted (this is indicated by the question marks in the table) suggesting that the results are independent of the dust particle properties. This is difficult to understand since the particle radius is crucial, e.g. for the particle charge, surface potential, coupling parameter and degeneracy parameter (see

Reference	Plasma density n_e [cm $^{-3}$]	Temperature T [K]	Dust parameters $A : a_D$ [μ m]
[94]	?	?	? : ?
[160]	$10^{25} - 10^{27}$	10^5	C : ?
[165]	10^{24}	300	? : ?
[161]	10^{24}	?	? : 10^{-2}
[164]	10^{27}	10^8	? : ?
[147]	5×10^{23}	100	? : ?
[134]	5.9×10^{22}	6.4×10^4	? : ?
[139]	10^{23}	$10^4 - 10^5$? : ?
[167]	?	?	? : ?
[142]	5×10^{29}	100	? : ?

TABLE I. Parameters of quantum dusty plasmas that were used in the references listed in the left column which are typical examples. “A” denotes the chemical element and a_D the radius of the dust particles. Missing information is indicated by question marks.

below). Moreover, the missing information makes it difficult to reproduce the results of the theoretical analysis or, at least, verify their physical significance. This is critical since QDP studies have been motivated by their authors by claiming importance of dust particles for nearly all quantum plasmas, including such diverse objects as white dwarf stars, the outer envelope of neutron stars, as well as metals and micro- and nano-electromechanical devices.

However, to date not a single experiment with dust particles in a quantum plasma has been reported. This is very surprising considering the strong effect dust particles are known to have on the properties of classical plasmas and the achieved good agreement of experiments with the theoretical predictions in that field, e.g. [121, 168, 169] which indicates an overall good understanding of these systems. The answer to the question why there are no QDP experiments is simple: as we will show in Secs. V C, these systems do not exist in reality.

Let us start by giving a definition of a quantum dusty plasma. Following the concept of Ref. [94] that a quantum dusty plasma is produced by cooling a classical dusty plasma (see above), we formulate three requirements:

- I.:** The plasma should be quantum degenerate. Compared to Ref. [94] we relax this condition by requiring only that the electrons are quantum degenerate.
- II.:** The plasma should contain stable dust particles of micrometer (or at least nanometer) diameter that are clearly distinct from atoms, ions and molecules by their (significantly bigger) size and charge. To be specific we will require that a dust particle contains at least $N_A = 10^3$ atoms [this number is not critical for our results obtained in Sec. V C which are valid even for $N_A = 1$].
- III.:** The dust particles should be stable in the presence of the quantum electron component.

An additional parameter characterizing QDP is the dust degeneracy parameter, $\chi_D = n_D \Lambda_D^3$, which is a factor

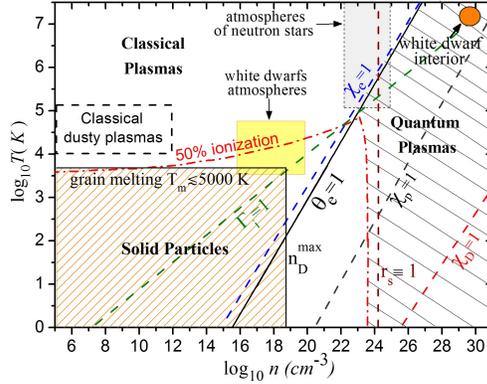


FIG. 2. Typical parameters of classical dusty plasmas and predicted occurrences of “quantum dusty plasmas”. *Dust particles can only exist* inside the orange shaded area: to the left of the line n_D^{\max} , Eq. (73), [shown for carbon and particles containing $N_A = 10^3$ atoms], and below the grain melting line. “Classical dusty plasmas” refers to the (nonequilibrium) electron component in RF discharges in the case of full (single) ionization. The free electron density follows by multiplying with the degree of ionization, α^{ion} . Inside the line “50% ionization” the plasma is predominantly neutral. *Dust quantum effects* set in below the line, $\chi_D = 1$, inside the orange shaded area, i.e. for $T \lesssim 10^{-4}$ K, see Sec. V D B).

$(m_e T_e / m_D T_D)^{3/2}$ smaller than the one of the electrons. For an isothermal plasma and carbon dust of minimal size (cf. II.) we have $\chi_D / \chi_e \approx 10^{-9}$. The line $\chi_D = 1$ is shown in Fig. 2, and the area of degenerate dust is below this line and at densities below n_D^{\max} that is given by Eq. (73), see below.

Another parameter that changes in a QDP is the surface potential ϕ_s , Eq. (64). In particular, one has to use Fermi-Dirac statistics for the description of electrons, instead of Boltzmann statistics, with the result $\phi_s = -Z_D e^2 / a_D E_{Fe}$. It turns out (details are given in the supplementary material [95]) that $|\phi_s|$ is, indeed, on the order of 1 even in the case of degenerate electrons.

B. Predicted occurrences of QDP

Three examples that have been commonly used in the context of “quantum dusty plasmas” are white dwarf stars, neutron stars, and micro- and nano-devices that are “contaminated” by dust particles. Before we will consider in detail the plasma parameters in these objects which are also depicted in Fig. 2, we analyze the requirements I.–III. that were formulated above.

To fulfill the requirement I. of quantum degenerate electrons it is necessary that the plasma is below (to the right of) the lines $\chi_e = 1$, $\theta_e = 1$ in Fig. 2. However,

this is not sufficient to realize quantum degeneracy effects because the parameters depend only on the number of free electrons (ions). For vanishing ionization degree, α^{ion} , the system is neutral. Thus the sufficient condition for electron degeneracy is that $\chi_e^* = n_e \alpha^{\text{ion}} \Lambda_e^3 > 1$ where the line $\chi_e^* = 1$ is a factor α^{ion} to the right of the line $\chi_e = 1$. Since isochoric cooling of a plasma gives rise to the formation of atoms and molecules ($\alpha^{\text{ion}} \rightarrow 0$) it is impossible to achieve quantum degeneracy of electrons this way, at least at low densities.

Of course, cooling will ultimately lead to a quantum state of matter when the DeBroglie wave length of atoms or molecules is comparable to their interparticle distance. For the example of hydrogen atoms this will happen below the line $\chi_p = 1$ in Fig. 2. Even though the behavior of dust particles immersed into a neutral quantum Fermi or Bose gas (or fluid) is certainly interesting, this violates condition I., as this is not a plasma.

Thus, for a QDP we have to concentrate on parameters where $\chi_e^* \geq 1$. In an isothermal plasma, ionization is observed for densities exceeding

$$n_e^{\text{ion}} \sim 5 \cdot 10^{23} \text{ cm}^{-3}, \quad (65)$$

For illustration, the line where 50% ionization is observed is depicted in Fig. 2, for the case of hydrogen. The estimate (65) will be derived in Sec. V C.

Let us now turn to condition II. for the dust component. The existence of stable dust particles requires that their temperature does not exceed their melting temperature, which is shown in Fig. 2 by the horizontal line “grain melting”. This line gives an estimate from above for known materials, a detailed discussion is given in the supplement. Furthermore, the interparticle distance of the dust component cannot be smaller than the particle diameter which leads to a maximum density n_D^{\max} , Eq. (73). The resulting area of existence of dust particles is shown in Fig. 2 by the orange shading.

Thus, we conclude that, for a “quantum dusty plasma” to exist, the dust and electron components have to fulfill, the conditions II. and I., respectively, that were derived above. Finally, we have to require that both conditions are fulfilled simultaneously, i.e. that the presence of quantum electrons does not destroy the dust particles (condition III.). As it turns out, this is impossible, as we will show in detail in Sec V C. But before doing this, we critically analyze the parameters of the three common QDP candidates that have been suggested in the literature.

- a. One of the motivations for studying “quantum dusty plasmas” is the observation of certain white dwarf atmospheres that are “contaminated” by dust particles [170]. As observations indicate, the presence of metals within cool white dwarf atmospheres is due to external sources (circumstellar and interstellar). Dust particles can sustain in the photosphere of white dwarfs through radiative levitation if $T_{\text{eff}} > 20,000 \text{ K}$ (30,000 K), for hydro-

gen (helium) atmospheres [171], where the effective temperature T_{eff} is determined by Stefan's law [172]. At lower temperatures, white dwarfs develop convection zones which enhance the gravitational settling of the heavy elements. One of the explanations of the atmospheric pollutions is the accretion from a dusty disk which is created by the destruction of a minor body within the tidal radius of the star [170, 173].

It is important, however, to recall that the plasma parameters of the atmosphere and the interior of white dwarfs are strikingly different. The atmosphere of a cool white dwarf is a dense gas or liquid at $\sim 10^3 - 10^4$ K containing non-degenerate classical electrons with a number density $n_e \lesssim 10^{19} \text{ cm}^{-3}$ [174] (with the atmosphere mass density $\rho \lesssim 3 \text{ g/cm}^3$). In contrast, the white dwarf interior contains degenerate dense electrons with $n_e > 10^{27} \text{ cm}^{-3}$. This difference is due to the separation of heavy elements (e.g. carbon and oxygen) from light elements (e.g. hydrogen and helium) by a strong gravitational field. In Fig. 2, we included the parameters of the white dwarf's atmosphere and interior. It is clearly seen that the white dwarf's atmosphere is outside of the quantum regime, whereas the white dwarf's interior is in a quantum plasma state. Therefore, we exclude the white dwarf's atmosphere from our analysis of quantum plasmas. On the other hand, the interior of white dwarfs appears to be way too hot and too dense for dust particles to survive, see Sec. V C.

- b. The atmosphere (outer envelope) of a neutron star has an electron density in the range $10^{22} \text{ cm}^{-3} \lesssim n_e \lesssim 10^{25} \text{ cm}^{-3}$ (the mass density is $\rho \gtrsim 10 \text{ g/cm}^{-3}$) and temperature, $T \gtrsim 10^5 \text{ K}$ [15]. The atmosphere of a cold neutron star has a thickness of about $\sim 1 \text{ cm}$ and can be as thin as few millimeters when the surface temperature is of the order of $\sim 10^5 \text{ K}$. Even cooler neutron stars have a solid surface without an atmosphere. Due to the high density, the atmosphere of a neutron star can be a degenerate or partially degenerate plasma. This is illustrated in Fig. 2. The parameters of the interior of neutron stars are not relevant to the current analysis. It should be noted that no sign of dust particles has been detected in the atmosphere of neutron stars.
- c. "Contaminated" micro- and nano-devices have been put forward as an example of quantum dusty plasmas in the original paper by Ali and Shukla [94] in analogy to classical dusty plasmas. The plasma in gas discharge experiments containing dust particles [124, 175, 176] is non-degenerate with electrons having temperatures of a few electron volts and densities on the order of $\sim 10^{10} \text{ cm}^{-3}$, cf. Fig. 2. Ions and neutrals are, as a rule, at room temperature. Even in the gas discharge at cryogenic condi-

tions [177–180], with heavy particle temperatures $\sim 10 \text{ K}$, ions and electrons remain too hot for manifestation of quantum effects [181, 182]. The dust particles that form inside a gas discharge plasma eventually fall down on the surface of a plate (which is usually metallic in industrial plasma reactors) [183, 184]; thereby "contaminating" the micro- or nano-device.

The authors of Ref. [94] suggested that further cooling of a dusty plasma will give rise to a "quantum dusty plasma" where even the dust particles behave quantum mechanically. We will show in Sec. V C that this is impossible. Reference [94] is discussed in more detail in the supplement.

- d. Some authors suggested that quantum dusty plasmas can exist in "metallic devices". At first sight this is reasonable because there, indeed, the electrons typically behave as a quantum degenerate moderately correlated Fermi gas [4], see Fig. 1. Indeed, such predictions have been made in a number of references, e.g. [185, 186]. For example, the authors of Ref. [185] state "The present results should be helpful for understanding the nonlinear electrostatic structures in metallic nanostructures involving charged particles in nanomaterials." However, the electron gas resides inside an ionic lattice with a spacing of a few angstrom, see Sec. IV A. It is impossible to "embed" a dust particle in this electron gas without destroying or strongly modifying the ionic lattice, see also the supplementary material.

From this discussion and Fig. 2 it is clear that a "quantum dusty plasma" cannot exist in the condensed matter phase (examples c. and d.) but, potentially, only in the gas phase. Now we proceed further and show that, also in the gas phase examples of compact stars (examples a. and b.), a dust particle is not able to survive in a quantum degenerate electron plasma.

C. Impossibility of dust grain existence in a quantum plasma

There are several simple arguments which show that a dust particle cannot exist in a quantum plasma. Here we concentrate on static arguments that are based on an analysis of the kinetic energy of degenerate electrons in a quantum plasma. While this is completely sufficient to rule out the existence of quantum dusty plasmas there are also dynamic arguments that are related to extremely high particle and energy fluxes that would immediately destroy any microparticle in a quantum plasma, this analysis can be found in the supplementary material.

Let us analyze under what conditions a dust particle can form in a quantum plasma. The first necessary step is, obviously, that a neutral atom can form from the electron-ion plasma. Only after this has happened and the density of atoms is sufficiently high, aggregation of

atoms into dimers and larger complexes and, eventually into a “dust particle”, can occur.

For the formation of the seed neutral atom of type “A” it is necessary that the binding energy of the dust atoms (electron-ion binding energy) exceeds the kinetic energy of the electron to be captured by the seed ion,

$$E_D^{(1)}(A) > E_{\text{kin}}^e. \quad (66)$$

Consider first a classical plasma in thermal equilibrium. Then condition (66) amounts to requiring $E_D^{(1)}(A) > \frac{3}{2}k_B T_e$. For the example of hydrogen, this temperature is of the order of 10^5 K. The next step – the formation of a dimer (e.g. H_2 molecule) – requires already a much lower threshold for the electron temperature on the order of 30,000 K due to the lower molecule binding energy. With increasing cluster size the binding energy of the next atom converges to a value on the order of $E_D^{(N_A)}(A) \sim 1...5$ eV, (where $N_A \geq 100$.) corresponding to $T_e \sim 11,500...60,000$ K, for most materials. This value for the electron temperature exceeds the melting temperature of the dust particles, $T_m \lesssim 5,000$ K which, therefore, sets the upper threshold for the existence of dust particles, in a classical dusty plasma, cf. Fig. 2.

Let us now turn to a quantum plasma. In the extreme case of strong electron degeneracy, $T_e = 0$, the mean kinetic energy is given by the Fermi energy times 3/5, and condition (66) for bound state formation changes to $E_D^{(1)}(A) > \frac{3}{5}E_{Fe}$. At finite temperature the expression can be corrected via a Sommerfeld expansion. Retaining terms of lowest order in θ_e we obtain:

$$E_D^{(1)}(A) > \frac{3}{5}E_{Fe} \left\{ 1 + \frac{5\pi^2}{12}\theta_e^2 \right\}, \quad \theta_e \leq 0.1. \quad (67)$$

For example, for a hydrogen atom, $E_D^{(1)}(H) \equiv E_H = 13.6$ eV, and it is straightforward to compute the maximal density a hydrogen atom can withstand in a low-temperature quantum plasma ($\theta_e = 0$). From Eq. (67) one readily finds the critical value of the Brueckner parameter, $r_s^{cr}(T=0; H) \approx 1.5$ which corresponds to an electron density $n_e^{cr}(T=0; H) \simeq 5 \times 10^{23} \text{ cm}^{-3}$ which is in reasonable agreement with quantum Monte Carlo results [187].

At finite temperature we obtain from Eq. (67),

$$r_s^{cr}(\theta_e; A) \approx \frac{1.5}{\alpha^{1/2}(A)} \sqrt{1 + \frac{5\pi^2}{12}\theta_e^2}, \quad (68)$$

$$n_e^{cr}(\theta_e; A) \approx \frac{5\alpha^{3/2}(A)}{\left[1 + \frac{5\pi^2}{12}\theta_e^2\right]^{3/4}} 10^{23} \text{ cm}^{-3}, \quad (69)$$

where $\alpha(A) = E_D^{(1)}(A)/E_H$ is the ratio of the binding energy of the considered ion (atom) of type “A” to the hydrogen binding energy. Thus, Eq. (69) defines the maximum electron density an atom of type “A” can withstand at a temperature $\theta_e \lesssim 0.1$. For larger temperatures, instead of the Sommerfeld expansion, the

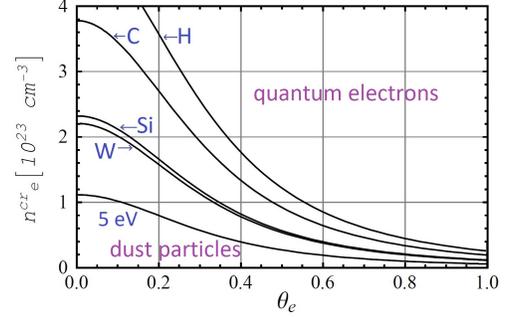


FIG. 3. Critical electron density calculated according to Eqs. (69) and (70), vs. electronic degeneracy parameter. In a quantum plasma hydrogen, carbon, silicon and tungsten atoms can only exist *below* the respective lines (H: $\alpha = 1$, C: $\alpha = 0.83$, Si: $\alpha = 0.6$, W: $\alpha = 0.58$). Dust particles have binding energies not exceeding 5eV and exist only *below* the line “5 eV”. For electrons in a hydrogen plasma to be quantum degenerate the electron density has to be *above* the line “H” and similar for other plasmas. For details see text.

full expression for the kinetic energy of the Fermi gas ($\beta = 1/k_B T$ and μ denotes the chemical potential)

$$E_{kin}(r_s, \theta) = \frac{3}{2}k_B T \frac{I_{3/2}(\beta\mu)}{I_{1/2}(\beta\mu)}, \quad (70)$$

has to be used.

The maximum plasma density at which a seed atom can form, Eq. (69), is plotted in Fig. 3 for three different types of atoms: carbon, silicon and tungsten. As discussed above, for the stability of a dust particle consisting of e.g. $N_A = 100$ atoms, in condition (67) we have to replace $E_D^{(1)}(A) \rightarrow E_D^{(100)}(A) \lesssim 5$ eV, as we estimated above. At the same time, we have to require that the electrons are quantum degenerate. As was discussed above, quantum degeneracy of electrons is only possible if α^{ion} is not vanishingly small. This can be expressed using the above condition (67), but now applied for vanishing of plasma atoms: the binding energy $E_D^{(1)}(P)$ should be smaller than the electronic kinetic energy,

$$E_D^{(1)}(P) < \frac{3}{5}E_{Fe} \left\{ 1 + \frac{5\pi^2}{12}\theta_e^2 \right\} \quad \theta_e \leq 0.1. \quad (71)$$

Combining the results of Eqs. (67) and (71) we conclude that a dust particle of size N_A can only exist in a low-temperature quantum plasma if

$$E_D^{(1)}(P) < E_D^{(N_A)}(A). \quad (72)$$

This inequality could be only fulfilled for special combinations of dust material and plasma gas, and only for $N_A = 1$. But this would not be a dust “particle” as discussed above. Thus for any realistic dust particles and

plasmas, condition (72) cannot be satisfied, i.e. solid particles and quantum degenerate electrons cannot coexist which is a violation of condition III. of Sec. V A. For illustration, in Fig. 3 we also show the critical densities corresponding to hydrogen atoms and to dust particles (using the estimate for the binding energy, $E_D = 5$ eV).

From this analysis it is clear that, in a quantum plasma (at $\theta_e < 1$), no dimers of neutral atoms C and Si, which are often considered to be building blocks of the dust particles, can form. The same conclusion is also valid for W, which has the highest melting point of all elements. The line (69) is also plotted in Fig. 2, for the example of hydrogen and provides the approximate boundary between neutral and ionized hydrogen (labeled “50% ionization”).

For completeness we mention that, for heavier elements with the nuclear charge Z , the binding energy of the first electron to the nucleus equals $Z^2 \times 13.6$ eV, giving rise to a $(Z - 1)$ -fold charged ion. However, with the capture of each subsequent electron the binding energy rapidly decreases until it reaches values on the order of the hydrogen binding energy, for the last electron. For example, for the important case of carbon, the binding energy of the first electron (formation of the C^{5+} -ion) is approximately 490 eV whereas for the neutral atom it is 11.26 eV. As discussed above, only after a neutral atom has been formed, eventually, particle growth can set in.

D. Test of “quantum dusty plasma” parameters used in the literature

In the following, we critically review the parameter values and assumptions that have been used in the QDP literature, see Tab. I.

- i. Many papers, e.g. [134, 139, 160, 164, 188] consider high plasma temperatures, way above the stability limits of any material as we demonstrated in Sec. V C. Some examples have been listed in table I.
- ii. Many papers consider very strongly degenerate electrons with $\theta_e \ll 1$, e.g. [94, 133–166]. As we have shown above, in such plasmas dust particles cannot survive because the electron kinetic energy exceeds the binding energy of dust particles.

On the other hand, some papers considered plasmas with $\theta_e \gg 1$, but this has nothing to do with a quantum plasma no matter if dust particles are present or not. For example, Ref. [188] considered an astrophysical plasma with $n_e \sim 10^{19} \text{ cm}^{-3}$ and $T_e = 10^5$ K which is way outside the quantum plasma range, cf. Fig. 2.

- iii. Often dust parameters are used that are clearly incompatible with each other. For example, Ref. [161] used the following (quoting) “typical parameters ... for the interiors of the neutron stars, the magnetars, and the white dwarfs ...”:

a dust density of 10^{18} cm^{-3} and a dust radius $a_D \sim 10^{-5}$ cm. One readily verifies that, at this density, the mean nearest neighbor distance of two dust particles is less than 10^{-6} cm, i.e., more than an order of magnitude smaller than the dust radius. This is clearly impossible without destroying the dust particles and is outside the dust existence area, cf. Fig. 2 and Eq. (73).

- iv. Following Ref. [94], in many works dust particles are treated as quantum degenerate fermions with the dust degeneracy parameter $\theta_D = k_B T_D / E_F^D \ll 1$, e.g. [134–138, 142, 145–147, 152, 156, 160, 161, 165], where T_D is the characteristic temperature corresponding to the chaotic motion of the dust particles which is different from their surface temperature T_s and from the Fermi energy (1) of the dust component.

To consider the most favorable condition for dust quantum effects we assume again a very small dust particle size of $N_A = 10^3$ atoms [cf. condition III. in Sec. IV] and estimate the temperature needed for an ensemble of dust particles to become quantum degenerate. A carbon particle of this size has a radius $a_D \sim 15 \text{ \AA} \approx 30 a_B$, so the mean interparticle distance between two dust particles cannot be smaller than $60 a_B$. The maximum density such a gas of dust particles can reach is, obviously,

$$n_D^{max} = \frac{3}{4\pi(2a_D)^3} \sim 5 \cdot 10^{18} \text{ cm}^{-3}, \quad (73)$$

see Fig. 2. A degeneracy parameter $\chi_D = 1$ is reached below a temperature $T_D^{max} \sim 0.0001$ K. Thus, only below temperatures of one hundred microkelvin and dust densities below $\sim 5 \cdot 10^{18} \text{ cm}^{-3}$ quantum effects of extremely small dust particles can be reached if they are maximally densely packed. For more realistic dust particles of bigger size and lower density, the degeneracy temperature will be even smaller.

For the often claimed applications in microelectronics, e.g. [94, 159] such low temperatures are highly unrealistic. Also the close packing of dust particles with distances of 30 \AA are not realistic. In microelectronics contamination is typically due to a few dust particles.

Moreover, the assumption that these “quantum dust” particles form a Fermi gas cannot be justified as is shown in the supplement.

- v. In some papers, in addition to the assumption $\theta_e \ll 1$, the parameter $\mu = (|Z_d|n_D) / (|Z_i|n_i)$ for the negatively charged dust particles was assumed to vary in the range from zero up to one [142, 144, 146, 147, 153, 155] (see the note [189]). This assumption contradicts the condition $\theta_e \ll 1$ used in these works. Indeed, $\mu \rightarrow 1$ corresponds to

vanishing electron density and thus not to a quantum plasma.

As this discussion has shown, it does not require a complicated analysis to realize that many of the above assumptions are incorrect or inconsistent. In the supplementary material [95] we consider two particular publications on “quantum dusty plasmas” where the fundamental problems of these papers can be clearly understood.

VI. QUO VADIS QHD FOR QUANTUM PLASMAS?

The fictitious nature of “quantum dusty plasmas” that has been demonstrated in Sec. V by very simple arguments raises the question why so many scientific articles have been (and continue to be) published on this subject. As we have shown, many papers even fail to choose proper parameters that assure that electrons are quantum degenerate and dust particles can exist at all (requirements I. and II.). Moreover, the obvious requirement that dust particles and quantum electrons have to co-exist (requirement III.) is ignored in all of these articles. There is no room to analyze these papers here, to illustrate the problem we considered two typical examples in the supplementary material [95]. We underline that the problems that are common to quantum dusty plasma papers are *not related to the QHD approach*, they are independent of the theoretical tools that are being used. At the same time, it is interesting to observe that all papers that are devoted to quantum dusty plasmas have been using QHD.

A. Scientific standards have to be enforced again

Aside from the QDP topic, there are serious more general problems with many of the QHD-based quantum plasma papers. As discussed in the introduction and illustrated on the examples in the supplement, in many papers plasma physics results are “extended” to other systems, in particular, semiconductors. For example, many QHD papers are devoted to nonlinear oscillations and waves that are known to exist in electron-ion plasmas. A simple rescaling of system parameters then leads to the prediction of solitons in semiconductors, e.g. in Ref. [190]. In fact, solitons and acoustic pulses have been observed in semiconductor experiments, e.g. [191, 192], which seems to motivate a QHD based theoretical approach. However, an elementary analysis of the experimental papers reveals that those solitons are caused by completely different physics: they are related to lattice distortions [191, 192] and have nothing to do with plasma effects that could, possibly, be captured by QHD. For more details on this example see the supplementary material.

The example of Ref. [190] is not an exception but rather typical for QHD-based quantum plasma papers.

Many of these papers derive results for linear and nonlinear oscillations and waves that are subsequently “applied” or “extended” to plasmas in metals, semiconductors, white dwarf stars, neutron stars and so on. This “extension” is done by inserting the values of masses, dielectric constants, densities and temperatures, the authors believe to be relevant for those systems. However, such a simple rescaling of parameters in a fluid model was shown to miss, in many situations, essential properties of quantum plasmas in real systems, in particular in condensed matter, cf. Secs. IV and supplementary material. This does not mean that fluid models are not applicable in these areas, however they have to be derived from more accurate models of these fields. In contrast, the QHD-based quantum plasma approach avoids such a derivation and robust validity and consistency tests against the modern literature in those fields. Instead, many of these papers contain bold and unproven claims of the significance of their results for the mentioned fields. This is in striking contradiction to elementary scientific standards.

We underline once more that this recent “scientific” tradition in the field of QHD-based quantum plasmas is by no means caused by the quantum hydrodynamics approach. The mentioned deficiencies have to be rejected, regardless of the method that is being used by the authors. At the same time, apparently, the relative simplicity of QHD, as compared to more advanced theoretical concepts used in quantum plasmas, cf. Sec. I, has contributed to the illusion that this approach is sufficient to describe these systems and that the results are universally applicable to quantum plasmas in the laboratory and in the universe.

This has led to a stream of hundreds of QHD papers, including close to one hundred papers on “quantum dusty plasmas”, that claim to make important contributions to condensed matter physics, astrophysics and microelectronics. The only justification (if any) for that importance that is given is that previous QHD-quantum plasma papers used the same parameters. It is characteristic that these claims have not been published in condensed matter physics journals (and only very few in astrophysics journals) where experts in those fields would critically assess the validity and importance of the results for their area. Instead, these results were typically published in plasma physics journals, as reflected by the reference list of the present article. So how could it happen that so many papers on – obviously non-existing (QDP) – systems or grossly oversimplified models found their way into highly respected journals? How to explain that plasma physics journals publish unproven claims that refer to areas of physics outside the scope of the journals? Clearly, plasma physics journals have to critically re-assess their manuscript handling and to reinforce their quality standards, as done for example in Ref. [193].

B. Requirements to QHD-based quantum plasma theory papers

Let us be clear: of course, linear and nonlinear oscillations and waves in charged particle systems are an interesting subject and deserve a thorough mathematical analysis, no matter whether the analysis is based on QHD or any other approach. However, if there is no application of the mathematical results to real plasmas then a mathematics journal could be the right address. If, on the other hand, the focus is on the application to plasmas, semiconductors, metals, or compact stars, the applicability to these systems has to be convincingly demonstrated. This, first of all, requires profound knowledge of those areas. Moreover, it is the responsibility of the authors to demonstrate that their results are of relevance (significant as compared to competing effects) and detectable in experiments or by observations. This appears to be a triviality that is also part of the acceptance criteria of many journals, but it has been clearly ignored in the context of hundreds of QHD-quantum plasma papers over the past 15 years.

In the particular case of QHD-quantum plasma papers another elementary requirement has been ignored over and over: application of a any model should always include basic tests of its validity and applicability limits. As was pointed out in a number of papers, cf. e.g. Refs. [78, 83] and demonstrated in Sec. III, the currently used QHD models have severe limitations concerning the spin statistics effects (Pauli blocking), the accessible coupling strength r_s , and the spatial and temporal resolution. In addition, most papers directly follow the original model of Ref. [75] and neglect finite temperature effects or the dependence of the Bohm potential and the pressure on frequency and wave number of the excitation [83] and on the dimensionality of the plasma [194], cf. Sec. III. This appears to be not critical for the special case of nearly ideal 1D plasmas and frequencies in the range of (or above) the electron plasma frequency and where fluid models (although often different ones compared to the one of Ref. [75]) are successfully being applied to metallic systems and nanoplasmonics, e.g. [33, 72, 73, 113].

At the same time, the model of Ref. [75] continues to be uncritically applied to quantum plasmas of different dimensionality or frequencies, beyond the validity limits of the model. In particular, for low-frequency excitations the Bohm potential of Ref. [75] turns out to be an order of magnitude too big [81, 88]. As a consequence, until recently, all QHD papers that were devoted to acoustic oscillations and statically screened ion potentials that used the model of Ref. [75] produced wrong results. This concerns the results for ion-acoustic modes in quantum plasmas, solitons, magneto-acoustic modes and so on. A particularly striking example of unphysical results was the prediction of an attractive ion potential in an equilibrium quantum plasma [86] – an effect that immediately vanishes if the correct Bohm potential is being used [88].

Thus, despite its relative formal simplicity, QHD has to be applied to quantum plasmas with great care. The current QHD models have to be regarded as unreliable with *a-priori* unknown accuracy. The agreement in the case of weak external excitation (linear response) is quite limited, and there is no reason to expect that the accuracy will be higher in the nonlinear regime. Therefore, new effects or “discoveries” (we refer to Sec. I for examples) made on the basis of current QHD models are certainly exciting, but they have a high risk of being not a physical effect but rather a consequence of deficiencies of the model. In such cases, a careful proof of the validity of the model for the chosen parameters is crucial. Moreover, comparisons to experiments or to more accurate and well tested theoretical methods, such as steady functional theory or quantum kinetic theory [77, 78, 88], can provide strong support of the results.

C. Towards an improved QHD for fermions

As we have demonstrated in Sec. III F, QHD for plasmas can be rigorously derived from the many-particle Schrödinger equation, for pure states, or from reduced density operators, for general situations. The result are microscopic QHD equations that essentially coincide with the basic equations of quantum kinetic theory or time-dependent DFT. This connection allows one to track all the simplifications made on the road to the final QHD equations and, at the same time, to remove some of the simplifications to improve the model. The starting point has to be an as accurate as possible description of the ideal (or weakly nonideal) Fermi gas. Here interactions are treated on the Hartree level (corresponding to the Poisson equation for the induced potential), and quantum kinetic theory within the random phase approximation provides a rigorous benchmark for the pressure and Bohm term in the QHD [83]. In Sec. III F we have listed a number of possible improvements to the QHD for fermions which will not be repeated here. Instead, we provide a few further considerations.

If the limiting case of a weakly nonideal Fermi gas is correctly built into the QHD-model, one can move on to quantum plasmas with stronger interactions. A first, phenomenological, way is to add new terms to the equations – such as an exchange-correlation potential V_{xc} , borrowed from DFT in the local density approximation [77]) that has been used in a number of subsequent papers, e.g. in Ref. [141]. This procedure is formally justified since V_{xc} does not depend on the individual orbital wave functions, cf. Sec. III E 2. However, the corresponding correction terms will also carry a frequency and wave number dependence (as we found for the Fermi pressure and Bohm potential) that is currently unknown. Therefore, there is no guarantee that the resulting model is generally more accurate than without these terms. Here again tests against DFT simulations, or against quantum kinetic theory beyond RPA, are needed.

A second strategy towards improved QHD models is the combination with input from other independent approaches. For example, DFT can provide the microscopic wave functions ϕ_i (the Kohn-Sham orbitals) which would allow to perform the orbital averaging directly, at least for some model cases. Similarly, for a mixed state the orbital occupations f_i can be computed *ab initio* from quantum Monte Carlo simulations, e.g. [57, 195]. Also, the case of very strong coupling where the electrons exhibit liquid or even Wigner crystal behavior can be approached by using the experience from quantum Monte Carlo simulations, e.g. [36, 196] which yields benchmark data for thermodynamic quantities (e.g. energy, equation of state), the density distribution and pair correlations. Another useful test is to consider an electron gas in a harmonic confinement potential and compute its normal modes which replace the plasma oscillations in an infinite system. A good candidate to test the model is the monopole oscillation (breathing mode) the frequency of which, in a quantum system, depends on the interaction strength [197], on the spin statistics and on the dimensionality [198–200]. Moreover, in addition to the monopole mode that is due to the pair interaction between particles and that exists in classical systems as well, a quantum system supports a second purely quantum monopole mode that arises from the quantum kinetic energy (Bohm potential) [198, 201].

A third strategy is to resort to quantum kinetic theory. As we have shown in Sec. III, QHD follows directly from the random phase (or quantum Vlasov) approximation, correlation corrections to QHD can be taken by starting from kinetic equations with collision integrals included. Possible approximations were outlined in Ref. [83] and include collision integrals in relaxation-time approximation [202] or dielectric approximations involving static local field corrections. Finally, we note valuable alternative attempts to derive an improved quantum kinetic theory that may, possibly, allow for a better description of quantum plasmas, e.g. [203] and references therein, and to improve QHD [204]. Yet it is mandatory to put these developments in the perspective of the existing extensive theoretical and numerical literature on quantum kinetic equations, e.g. [32, 38, 39] and recover important known limits. This also concerns the developments in “spin-QHD” that have, in part, led to unphysical predictions and interpretations (such as “spinning quantum plasma”), that were criticized in Ref. [85]. While coherent spin dynamics in realistic dense laboratory plasmas are highly unlikely due to dissipation and dephasing, such effects are known to occur in Bose gases and fluids as well as in certain condensed matter systems. There exist well established theoretical approaches that successfully describe current experiments, e.g. in spintronics or skyrmion physics [92, 93] and references therein that should be used as benchmarks. The situation may be different in dense astrophysical plasmas with ultrastrong magnetic fields such as in neutron stars or magnetars where spin equilibration may be (partially) suppressed

by the field, for an overview, see Ref. [205] and references therein. Here, spin-QHD may, eventually, be an effective approach.

In any case, best progress will be made if the QHD results can be tested against independent analytical or simulation methods. Here one can also use results from semiclassical molecular dynamics [206] or Bohmian trajectory simulations [cf. Sec. III B] that are conceptually close to QHD. This will reliably map out the applicability range of QHD together with existing problems.

D. Current topics in quantum plasma theory

As discussed in the introduction, quantum plasmas is a rapidly developing field, both, for astrophysical applications and for modern laboratory experiments, cf. Fig. 1. The frontier areas and key topics in dense plasma research have been reviewed in various recent reports, e.g. [207, 208] and text books [1]. These are a useful starting point to map out future research directions. Open questions of current interest include the thermodynamic properties and equation of state of dense quantum plasmas, transport and optical properties (e.g. electrical and heat conductivity, optical absorption and reflectivity, Thomson scattering signal). Also, the properties of partially ionized plasmas such as the degree of ionization and the ionization potential depression are of importance. These questions are summarized in table II together with the theoretical approaches that are being applied or are promising candidates to tackle them. Also, the time evolution of dense quantum plasmas following an external excitation is becoming of increasing interest. This includes the thermalization of the electron distribution function as well as the temperature equilibration in electron-ion plasmas [209–211].

Without doubt, the QHD approach can play an important role in modern quantum plasma research. An example is the application to plasmonics in metallic systems and nanostructures, where QHD has been successfully tested against DFT and applied to experiments, for an overview see Ref. [113]. On the other hand, we are not aware of recent applications of QHD to semiconductors that have been noticed and regarded as important contributions by researchers in that field — in striking contrast to the often high number of citations these papers receive within the QHD-based quantum plasma community.

At the same time, QHD can contribute to the question of plasma oscillations and collective modes in quantum plasmas. These topics, including nonlinear plasmon damping [64] and negative plasmon dispersion [114], are of high current interest in warm dense matter research, in particular as a possible experimental diagnostic for the plasma density, degree of ionization and temperature in Thomson scattering with X-rays at free electron laser facilities [24].

However, to make successful contributions that will be noticed outside the QHD community, in all these cases,

Theory	Quantity, topic	Reference	System
BO-DFT	el. conductivity	[62, 63]	WDM, SC
	opt. absorption	[64, 212]	WDM, SC
	Thermodynamics	[62]	WDM, SC
	Thomson scattering	[63]	WDM
LRT	Plasmon damping	[64]	WDM
	plasmon spectrum	[4]	UEG, M
QKT	thermalization	[39]	WDM, SC
	plasmon spectrum	[213]	UEG, SC, M
QMC	el. conductivity	[38]	WDM, SC, M
	opt. absorption	[32, 214]	WDM, SC, M
	Thermodynamics	[54, 108]	WDM, M
	Plasmon spectrum	[114]	WDM, M
QMD	Thermodynamics	[215, 216]	WDM
	Transport	[59, 216]	WDM
TD-DFT	Thomson scattering	[65]	WDM, M
QHD	Plasmon dispersion	[75, 76]	Ideal Plasma
	nonlinear plasmons solitons	[34, 205]	M, NP, USMF
		[34]	M, NP ?

TABLE II. Theoretical and simulation approaches applied to quantum plasmas including examples of relevant references. BO-DFT: Born-Oppenheimer DFT (DFT-MD). LRT: quantum linear response theory, QKT: Quantum Kinetic Theory and Nonequilibrium Green Functions, QHD: Quantum Hydrodynamics, QMD: quantum molecular dynamics, TD-DFT: time-dependent DFT, UEG: uniform electron gas, WDM: warm dense matter, SC: semiconductors, M: metals, NP: metallic nanoparticles. In the bottom part we list QHD applications to ideal plasmas, metallic systems and astrophysical plasmas with ultrastrong magnetic fields (USMF). The relevance of nonlinear QHD results to realistic quantum plasmas remains to be demonstrated.

it is crucial to seek the contact with experimental groups in dense plasmas and warm dense matter, in order to understand all details of the specific system, of its parameters and of the experiment. Then chances are good that a meaningful experiment-theory comparison can be performed which will be valuable for both sides. On the other hand, QHD-based simulations should be compared to alternative theoretical approaches to quantum plasmas in order to verify the accuracy of the former and to firmly establish its place in the broad area of quantum plasma physics.

Such a closer integration into the rapidly developing broad fields of quantum plasma physics and astrophysics provides the unique chance for QHD to systematically verify and improve the method. On the other hand, it is well possible that QHD can provide a more efficient approach to some complex quantum plasma problems than *ab initio* methods. This will certainly be welcomed by the community once the method, eventually, has demonstrated its reliability and predictive power.

In the past years a large part of QHD-based plasma theory papers concentrated on developing solution methods for the nonlinear QHD equations, and on incremental extensions of the model via the addition of new terms and exploration of new conditions and parameter combi-

nations. This has led to a tremendous experience in nonlinear oscillations and wave phenomena that still waits to be applied to relevant dense plasma problems. QHD-based plasma physicists should be more ambitious and try to explain real experiments and to predict new phenomena in real quantum plasmas. There is an immense body of questions, in particular in astrophysics and warm dense matter where the QHD experience can make a difference.

VII. CONCLUSIONS

We have presented a critical assessment of recent activities in QHD-based quantum plasma theory. The related papers are mostly devoted to linear and nonlinear oscillations and waves. While the mathematical analysis appears to be formally correct, the “application” of many of the results to quantum plasmas in astrophysical objects or condensed matter systems was shown to often miss any convincing arguments about the relevance to these systems.

We considered in some detail a particular example – “quantum dusty plasmas” – and showed that the recent high activity in this area rests on unjustified assumptions and unrealistic plasma parameter combinations. That research is typically motivated by unrealistic examples including neutron stars, white dwarf stars or microelectronics devices. It was demonstrated that dust particles cannot survive or form neither in a dense quantum plasma in general, nor in the mentioned example systems, in particular. The reason is that quantum electrons produce a pressure that is so high that it unavoidably destroys any micrometer and even nanometer size particle. Therefore, dust particles are restricted to the regime of classical plasmas with temperatures below the grain melting temperature. It is obvious that, under these circumstances, the discussion of quantum degeneracy and Fermi statistics of the dust particles themselves [94] is meaningless and detached from reality, cf. Fig. 2. It is certainly of interest to analyze what happens to classical dusty plasmas when they are cooled to low temperatures. But, as we have shown, this will not produce quantum degenerate electrons, and thus, this will not be a “quantum dusty plasma”.

Certainly, “quantum dusty plasmas” are an extreme case of unwarranted claims made in QHD-based plasma physics publications. However, similar unjustified statements about the importance of certain formal mathematical results for other fields including condensed matter physics and astrophysics are frequently encountered in the QHD-based plasma physics literature. As we discussed in this article, this transfer of results to other fields is often based on oversimplified models that miss current developments and, thus, have no chance to make a (positive) impact on those fields. We have made suggestions how to overcome this unsatisfactory situation—the key being, to actively seek the contact with these fields.

Our analysis of the above problems revealed that they have nothing to do with the QHD approach. In particular, there is no basis to conclude that QHD is not suitable for modeling quantum plasmas. In contrast, once its limitations and area of applicability are reliably established, QHD can turn into a powerful tool that is complementary to current *ab initio* approaches. Similar as in classical plasmas, a fluid description can be much more efficient than a kinetic approach. At the same time, the applicability range of the former are being carefully established by comparisons with the latter, e.g. [68], or with molecular dynamics, e.g. [217, 218].

A similarly successful fluid theory for fermions, and electrons in quantum plasmas, in particular, is still missing. QHD for fermions is well capable to make important contributions here. Here we have re-analyzed the approach of Manfredi and Haas [75] and Manfredi [76] and pointed out its limitations. Using density operator theory and the occupation number representation, we re-derived the microscopic QHD equations and showed that they are directly related to time-dependent DFT and quantum kinetic theory. Presenting a systematic averaging procedure of the MQHD equations, we discussed how to derive the QHD equations for fermions. We are confident that this will allow for additional improvements of QHD and important new contributions to quantum plasmas in the near future.

SUPPLEMENTARY MATERIAL

This supplement contains additional information on 1.) the stability of dust particles in a quantum degenerate plasma and 2.) a brief discussion of representative examples of QHD articles with applications to “quantum dusty plasmas” and to semiconductors.

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APPENDIX: LINEARIZATION OF THE QHD EQUATIONS

In this appendix we present a few details on the microscopic QHD and standard QHD for fermions. In par-

ticular, we derive the linearized versions and discuss the plasmon dispersion.

A. Microscopic QHD equations

Recall the microscopic QHD equations (42, 43, 44),

$$\frac{\partial n_i}{\partial t} + \nabla \cdot (\mathbf{v}_i n_i) = 0, \quad (\text{A1})$$

$$\frac{\partial \mathbf{p}_i}{\partial t} + \mathbf{v}_i \text{div} \mathbf{p}_i = -\nabla (U^H + Q_i + V^{\text{xc}}), \quad (\text{A2})$$

$$U^H(\mathbf{r}, t) = \int d\mathbf{r}_2 w(\mathbf{r} - \mathbf{r}_2) [g_s n(\mathbf{r}_2, t) - n_0], \quad (\text{A3})$$

$$Q_i(\mathbf{r}, t) = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{n_i(\mathbf{r}, t)}}{\sqrt{n_i(\mathbf{r}, t)}}. \quad (\text{A4})$$

Here $n(\mathbf{r}, t) = \sum_{i=1}^{\infty} f_i \cdot |\phi_i(\mathbf{r}, t)|^2$, and $g_s = 2s + 1$. The exchange-correlation potential V^{xc} takes into account exchange and correlation effects, if the MQHD equations are derived from time-dependent density functional theory. If, on the other hand, the starting point is a quantum kinetic equation, V^{xc} would be replaced by a collision integral I_i that is different for each orbital, cf. Eq. (21) of the main text. Assuming a nearly ideal ($V^{\text{xc}} \rightarrow 0$, but $U^H \neq 0$) Fermi gas at $T = 0$, the stability condition is given by $\partial_t n_{i0} = \partial_t \mathbf{p}_{i0} = 0$, further $\mathbf{v}_{i0} \text{div} \mathbf{p}_{i0} = -\nabla (U_0^H + Q_{i0})$, and, in the absence of external fields, there is no mean velocity, $\bar{\mathbf{v}}_{i0} = 0$.

Now we investigate the response of the system to a weak monochromatic perturbation, $V_1^{\text{ext}}(\mathbf{r}, t) = \bar{V}_1^{\text{ext}} e^{-i\hat{\omega}t + i\mathbf{q}\mathbf{r}}$, where $\hat{\omega} = \omega + i\epsilon$, $\epsilon > 0$. This gives rise to weak perturbations, $|n_{i1}| \ll n_{i0}$ and so on,

$$\begin{aligned} n_{i0} &\rightarrow n_{i0} + n_{i1}, \\ \mathbf{v}_{i0} &\rightarrow \mathbf{v}_{i0} + \mathbf{v}_{i1}, \\ U_0^H &\rightarrow U_0^H + U_1^H, \\ Q_{i0} &\rightarrow Q_{i0} + Q_{i1}, \end{aligned}$$

and we introduce an effective potential $U_1^{\text{eff}} = V_1^{\text{ext}} + U_1^H$. Linearization of the microscopic QHD equations yields the following first order equations

$$\frac{\partial n_{i1}}{\partial t} + \nabla \cdot (\mathbf{v}_{i0} n_{i1}) + \nabla \cdot (\mathbf{v}_{i1} n_{i0}) = 0, \quad (\text{A5})$$

$$\frac{\partial \mathbf{p}_{i1}}{\partial t} + \mathbf{v}_{i0} \text{div} \mathbf{p}_{i1} + \mathbf{v}_{i1} \text{div} \mathbf{p}_{i0} \quad (\text{A6})$$

$$= -\nabla (U_1^{\text{eff}} + Q_{i1}), \\ Q_{i1}(\mathbf{r}, t) \approx -\frac{\hbar^2}{2m} \frac{1}{2n_{i0}(\mathbf{r})} \nabla^2 n_{i1}(\mathbf{r}, t). \quad (\text{A7})$$

Introducing the Fourier-Laplace transform of all first order quantities which we denote by “tilde”, we obtain

$$\mathbf{q}\tilde{\mathbf{v}}_{i1} = \frac{\tilde{n}_{i1}}{n_{i0}} (\hat{\omega} - \mathbf{q}\mathbf{v}_{i0}), \quad (\text{A8})$$

$$\frac{m\tilde{n}_{i1}}{n_{i0}} (\hat{\omega} - \mathbf{q}\mathbf{v}_{i0})^2 = \tilde{U}_1^{\text{eff}} + \frac{\hbar^2 q^4 \tilde{n}_{i1}}{4mn_{i0}}. \quad (\text{A9})$$

Solving for \tilde{n}_{i1} and averaging with the occupation numbers f_i , we obtain the perturbation of the mean density

$$\tilde{n}_1(\mathbf{q}, \hat{\omega}) = \frac{1}{N} \sum_{i=1}^{\infty} f_i \tilde{n}_{i1}(\mathbf{q}, \hat{\omega}) \quad (\text{A10})$$

$$= \tilde{U}_1^{\text{eff}}(\mathbf{q}, \hat{\omega}) \tilde{\Pi}_1^R(\mathbf{q}, \hat{\omega})$$

$$\tilde{\Pi}_1^R(\mathbf{q}, \hat{\omega}) = \frac{1}{N} \sum_{i=1}^{\infty} \frac{f_i}{(\hat{\omega} - \mathbf{q}\mathbf{v}_{i0})^2 - \frac{\hbar^2 q^4}{4m^2}}. \quad (\text{A11})$$

The result (A11) is nothing but the longitudinal polarization function in random phase approximation (RPA), e.g. [39]. It is related to the retarded dielectric function via $\epsilon(\mathbf{q}, \hat{\omega}) = 1 - \tilde{w}(\mathbf{q}) \tilde{\Pi}_1^R(\mathbf{q}, \hat{\omega})$, where $\tilde{w}(\mathbf{q}) = 4\pi e^2/q^2$, and the dispersion of collective excitations follows from $\text{Re } \epsilon = 0$ (for weak damping). For example, for $T = 0$, the optical plasmon dispersion is found to be

$$\omega^2(q) = \omega_{pl}^2 + \frac{3}{D+2} v_F^2 q^2 + (1 - \delta_{2,D}) \frac{\hbar^2}{4m^2} q^4, \quad (\text{A12})$$

for a system of dimensionality $D = 1, 2, 3$, and the Kronecker symbol indicates that, in 2D, the term $\sim q^4$ vanishes [194].

In Eq. (A12) the Fermi velocity is always defined by $E_F = \frac{m}{2} v_F^2$, where the Fermi energy depends on the dimensionality, particle spin and spin polarization:

$$E_F^{1D} = \frac{(2\pi\hbar)^2}{2m} n_{1D}^{*2},$$

$$E_F^{2D} = \frac{2\pi\hbar^2}{m} n_{2D}^*,$$

$$E_F^{3D} = \frac{\hbar^2}{2m} (6\pi^2 n_{3D}^*)^{2/3},$$

where $n_D^* \equiv n_D/g_s$ is the D -dimensional particle density per spin projection ($g_s = 2s + 1$) of fermions with spin s and has the dimension of particle number per L^D with L being the system length. These expressions correspond to the paramagnetic case where all spin projections occur with the same probability. Otherwise, g_s has to be modified, for example, in the ferromagnetic limit $g_s = 1$. Note that in the analysis above we considered, for all dimensions, the case of a 3D Coulomb potential \tilde{w} . For the case of a strictly 2D or 1D potential [39, 219, 220] the plasmon dispersion would be different (acoustic).

B. (Macroscopic) QHD equations

Let us now compare the result for the microscopic QHD equations to the equations of Manfredi *et al.* [75] that we obtained in the main text by averaging the microscopic equations over the orbital occupations, cf.

Eqs. (50, 51, 52)

$$\frac{\partial \bar{n}}{\partial t} + \frac{1}{m} \nabla \cdot (\bar{\mathbf{p}} \cdot \bar{\mathbf{n}}) = -\frac{1}{m} \nabla \cdot \overline{\delta \mathbf{p}_i \delta n_i}, \quad (\text{B1})$$

$$\frac{\partial \bar{\mathbf{p}}}{\partial t} + \frac{1}{m} \bar{\mathbf{p}} \cdot \text{div} \bar{\mathbf{p}} = -\nabla (U^{\text{H}} + \bar{Q} + V^{\text{xc}}) - \frac{1}{n} \partial_\beta \bar{P}_{\alpha\beta} \quad (\text{B2})$$

and

$$\frac{1}{n} \partial_\beta \bar{P}_{\alpha\beta} = \frac{1}{n} \nabla \bar{P}_F + \frac{1}{n} \partial_\gamma \bar{\sigma}_{\alpha\gamma} \quad \gamma \neq \alpha, \quad (\text{B3})$$

$$\bar{Q} = -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\bar{n}}}{\sqrt{\bar{n}}} + Q^\Delta, \quad (\text{B4})$$

$$Q^\Delta \approx \frac{\hbar^2}{2m\bar{n}} \overline{\delta A_i \cdot \nabla^2 \delta A_i} + O\left(\left(\frac{\delta A_i}{A}\right)^2\right), \quad (\text{B5})$$

where summation over repeated Greek indices is implied. Here the mean pressure tensor $\bar{P}_{\alpha\beta}$ is created by the momentum fluctuations, $\frac{1}{m} \delta p_{i\alpha} \partial_\beta \delta p_{i\beta} = \frac{1}{n} \partial_\beta \bar{P}_{\alpha\beta}$. In the following, again an ideal Fermi gas at $T = 0$ is considered where the shear stress and exchange correlation corrections vanish $\bar{\sigma}_{\alpha\gamma} \rightarrow 0$, $V^{\text{xc}} \rightarrow 0$, whereas the relation between kinetic energy fluctuations (diagonal part of the tensor) and pressure in a D -dimensional Fermi gas is given by

$$\frac{1}{m} \delta \mathbf{p}_i \text{div} \delta \mathbf{p}_i = \frac{1}{D} \nabla \delta E_{\text{kin}} = \frac{m}{2(D+2)} \nabla v_F^2 = \frac{1}{2\bar{n}} \nabla \bar{P}^{\text{id}}.$$

Similarly as in the previous case, the stability condition of an ideal Fermi gas without external fields at $T = 0$ becomes, $\partial_t \bar{n}_0 = \partial_t \bar{\mathbf{p}}_0 = 0$, $\bar{v}_0 = 0$ and $U_0^{\text{H}} = -\bar{P}_0^{\text{id}}/\bar{n}_0$. Turning again on a weak monochromatic potential V_1^{ext} (which is assumed to vanish rapidly) the average quantities are weakly perturbed,

$$\bar{n}_0 \rightarrow \bar{n}_0 + \bar{n}_1,$$

$$\bar{\mathbf{v}}_0 \rightarrow \bar{\mathbf{v}}_0 + \bar{\mathbf{v}}_1,$$

$$U_0^{\text{H}} \rightarrow U_0^{\text{H}} + U_1^{\text{H}},$$

$$\bar{Q}_0 \rightarrow \bar{Q}_0 + \bar{Q}_1 \approx -\frac{\hbar^2}{2m} \frac{\nabla^2 \sqrt{\bar{n}_0}}{\sqrt{\bar{n}_0}} + \frac{\hbar^2}{4m} \nabla^2 \sqrt{\bar{n}_1},$$

$$\bar{P}_0^{\text{id}} \rightarrow \bar{P}_0^{\text{id}} + \bar{P}_1^{\text{id}} \approx \frac{2}{D+2} \bar{n}_0 E_F(\bar{n}_0) + \frac{2}{D} \frac{\bar{n}_1}{\bar{n}_0} E_F(\bar{n}_0),$$

and the linearized equations for the perturbations of the average quantities obtain the form

$$\frac{\partial \bar{n}_1}{\partial t} + \nabla \bar{\mathbf{v}}_1 \bar{n}_0 = 0, \quad (\text{B6})$$

$$\frac{\partial \bar{\mathbf{p}}_1}{\partial t} = -\nabla \left(U_1^{\text{H}} + \frac{\hbar^2}{4m} \nabla^2 \sqrt{\bar{n}_1} \right) - \frac{2}{\bar{n}_0 D} \nabla \frac{\bar{n}_1}{\bar{n}_0} E_F(\bar{n}_0).$$

After Fourier transformation the two equations become

$$\frac{1}{m} \bar{p}_1 \cdot \mathbf{q} = \frac{\bar{n}_1}{\bar{n}_0} \omega,$$

$$-i\omega \bar{p}_1 = -i\mathbf{q} \bar{U}_1^{\text{H}} - i\mathbf{q} \frac{\hbar^2}{4m} q^2 - i \frac{\mathbf{q}}{\bar{n}_0} \frac{2}{D} \frac{\bar{n}_1}{\bar{n}_0} E_F(\bar{n}_0).$$

Multiplying the second equation by \mathbf{q} and using the result of the first, we obtain for the plasmon dispersion

$$\omega^2(q) = \omega_{pl}^2 + \frac{1}{D}v_F^2q^2 + \frac{\hbar^2}{4m^2}q^4, \quad (\text{B7})$$

which agrees with the microscopic QHD result (A12) in 1D, but exhibits an incorrect coefficient of the q^2 term in 3D and the incorrect coefficients of the q^2 and q^4 terms in 2D, see above. Note that we again used a 3D Coulomb potential in all cases, as for the microscopic QHD, Eq. (A12).

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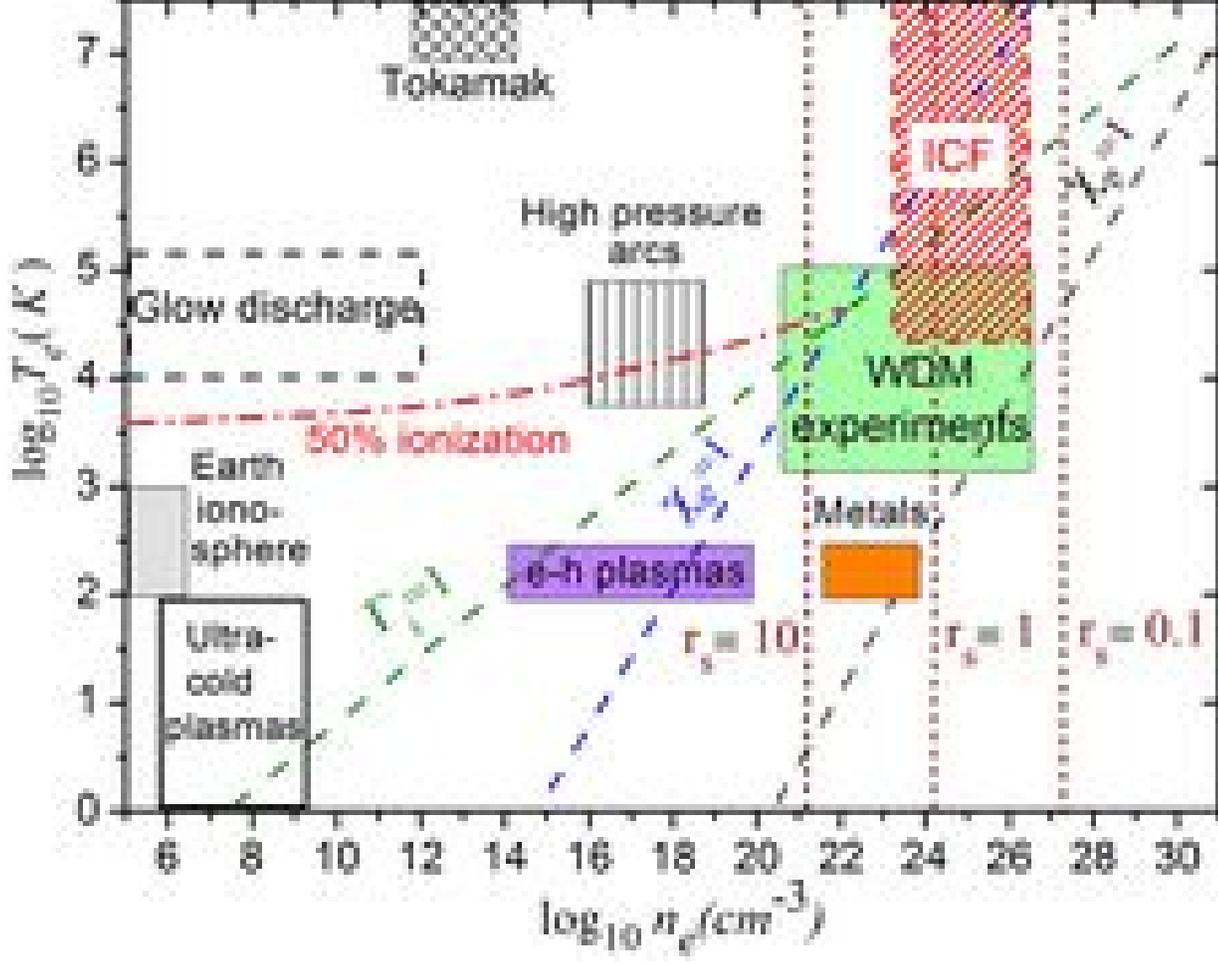
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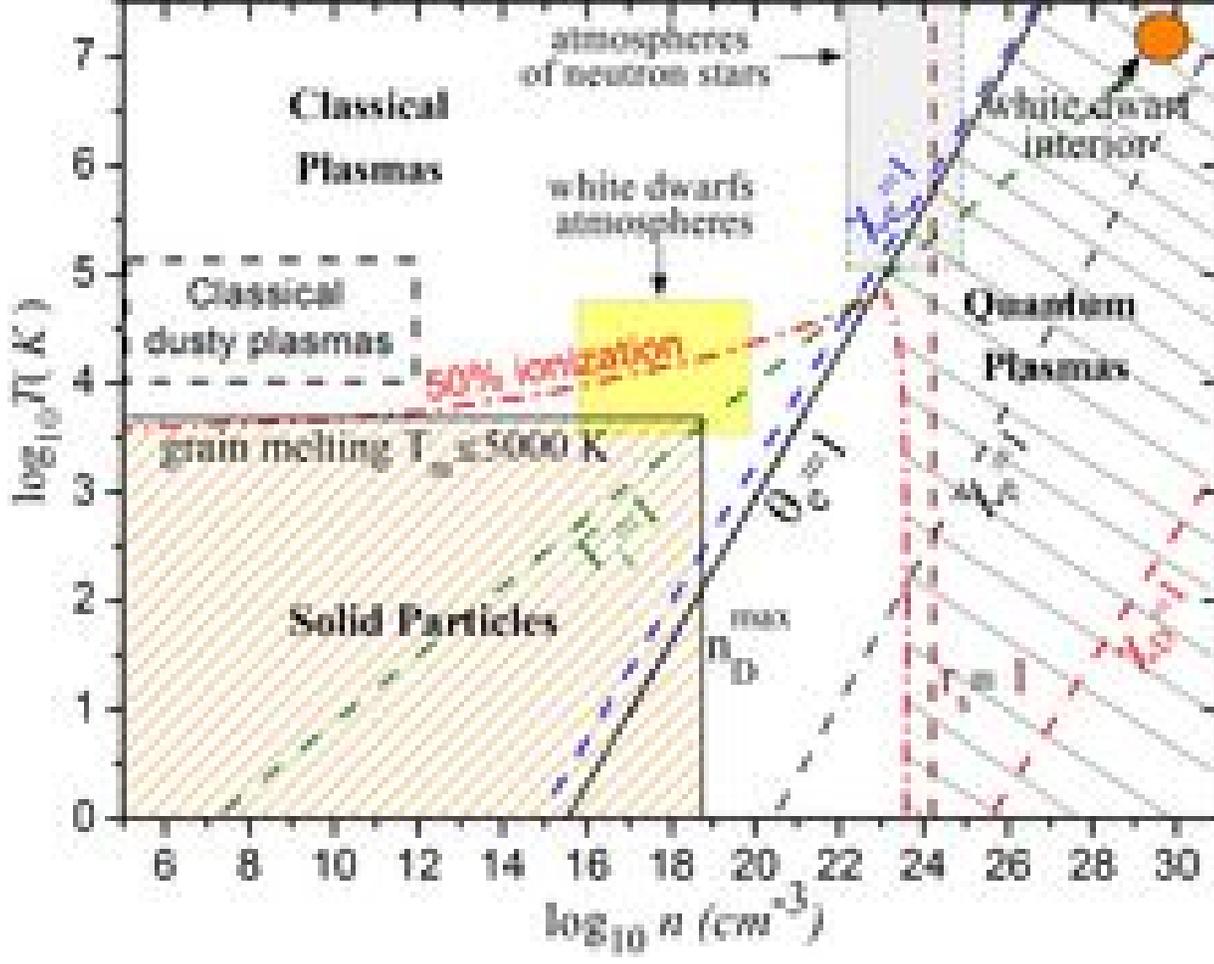
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