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# Streaming Complex Plasmas: Ion Susceptibility for a Partially Ionized Plasma in Parallel Electric and Magnetic Fields

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The density response function for streaming ions in homogeneous, parallel electric and magnetic fields is derived self-consistently from kinetic theory. Ion-neutral collisions are treated with the Bhatnagar-Gross-Krook collision operator assuming a constant ion-neutral collision frequency. The result accounts for the non-Maxwellian distribution function of the ions and is valid in the full range from weak to strong magnetization. It provides the basis for various linear response calculations in the context of magnetized complex plasmas, where streaming ions interact with highly charged dust particles under the influence of a strong external magnetic field.



Magnetic fields play an important role in plasma physics because they allow one to confine and manipulate charged particles externally. Plasma properties such as wave spectra, particle diffusion, heat conduction, or plasma instabilities can be strongly modified when the plasma is magnetized. Strong magnetic fields of several Tesla are not only encountered in magnetic confinement or magnetic liner inertial fusion experiments [1] but have also become of interest for complex plasmas research [2, 3]. By adjusting the magnetic field strength it is possible to create plasma states where only the electrons, electrons and ions, or possibly all three charged particle species including the dust particles are magnetized [4]. Several new phenomena have been observed in experiments with magnetic fields, including a slow rotation of the entire dust cloud [5–9], the spinning of dust particles [10], and complicated flow patterns [11]. In recent experiments with strong magnetic fields on the order of a few Tesla, sufficient to magnetize not only the electrons but the ions as well, filaments appeared in the discharge [12]. The dynamics of a dust particle pair also showed a pronounced response to magnetic fields of this magnitude [13]. Experiments performed at the Magnetized Dusty Plasma Experiment (MDPX) at Auburn University [14] further demonstrate that ordered structures of a titanium mesh can be imposed on the dust particles under these conditions. Crystalline and strongly coupled fluid states [15,16] also occur naturally in dusty plasmas due to the strong Coulomb interaction between the grains. Even though the dust particles themselves are difficult to magnetize [17], their screened interaction, and thus, their collective behavior, can be affected significantly by an external magnetic field [18].

Dusty plasma experiments are often confronted with ion flows due to electric fields, especially in the sheath region. They affect the charging of the dust grains [19], their mutual interaction due to the formation of wake potentials [20, 21], and give rise to drag forces [22, 23]. Under the influence of an electric field the ion velocity distribution may considerably differ from a displaced Maxwellian due to collisions with the neutral gas. Various phenomena related to the interaction between ions and dust particles have been shown to be crucially affected by the different distribution, e.g., the ion-drag force [22, 23] or the ion-dust streaming instability [24]. The ion susceptibility is the basis for several ion-dust related linear response calculations, including the screened dust potential [18, 25, 26]. In particular, the susceptibility is well known for ions in an external homogeneous electric field [22, 27]. Extending these results, the ion susceptibility will be derived self-consistently in the presence of both an electric and a parallel magnetic field. Ion-neutral collisions are included via the Bhatnagar-Gross-Krook (BGK) collision operator. Crossed electric and magnetic fields with a finite flow along the magnetic field, introduced via a Doppler-shift of the frequency, were treated in Ref. [28]. The result presented here accounts for the non-Maxwellian ion velocity distribution and should be useful to extend the study of ion-dust streaming

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phenomena into the magnetized regime, which is expected to become of increasing interest in the coming years since several superconducting magnets in dusty plasma laboratories are now operational.

This paper is organized as follows. In Sec. 1 the kinetic equation used for the derivation of the susceptibility is linearized and solved for the ion distribution function. As a main result, two different representations of the response function are presented and discussed in Sec. 2. The paper concludes with a summary of the results in Sec. 3. Complementary mathematical details on the derivation of the response function are presented in the Appendices.

# 1 Derivation of density response function

## 1.1 Kinetic theory

For the derivation of the ion response function, the following kinetic equation is used,

$$\frac{\partial f_\mathrm{i}}{\partial t}+\frac{\partial f_\mathrm{i}}{\partial \vec{r}}\cdot\vec{v}+\frac{\partial f_\mathrm{i}}{\partial \vec{v}}\cdot\frac{q_\mathrm{i}}{m_\mathrm{i}}\left(\vec{E}+\vec{v}\times\vec{B}_0\right)=-\nu_\mathrm{in}\left(f_\mathrm{i}-n_\mathrm{i}\,\Phi_\mathrm{n}\right), \tag{1}$$

where the approximation of a constant ion-neutral collision frequency $\nu_\mathrm{in}$ has been made on the right-hand side, known as the BGK collision operator. Improvements are possible by allowing for a frequency-dependent collision frequency [29] or by using available (velocity-dependent) cross sections [30]. The velocity distribution of the neutral particles is assumed to be Maxwellian, $\Phi_\mathrm{n}(\vec{v})=\Phi_\mathrm{n}^z(v_z)\Phi_\mathrm{n}^\perp(v_\perp)$, with

$$\Phi_\mathrm{n}^z(v_z)=(2\pi v_\mathrm{th,n}^2)^{-1/2}\,\exp\left(-\frac{v_z^2}{2v_\mathrm{th,n}^2}\right),\qquad \Phi_\mathrm{n}^\perp(v_\perp)=(2\pi v_\mathrm{th,n}^2)^{-1}\,\exp\left(-\frac{v_\perp^2}{2v_\mathrm{th,n}^2}\right), \tag{2}$$

where $v_\mathrm{th,n}=(k_\mathrm{B}T_\mathrm{n}/m_\mathrm{i})^{1/2}$ is their thermal velocity. The distribution function, density, mass, and charge of the ions are denoted by $f_\mathrm{i}(\vec{r},\vec{v},t)$, $n_\mathrm{i}(\vec{r},t)$, $m_\mathrm{i}$, and $q_\mathrm{i}$, respectively.

The electric field $\vec{E}$ in Eq. (1) contains the field created by the plasma species and a DC electric field $\vec{E}_0=E_0\vec{e}_z$. It was shown [22, 30, 31] that the stationary ion distribution function, $f_\mathrm{i0}(\vec{v})=n_\mathrm{i0}\Phi_\mathrm{i0}(\vec{v})$, where $n_\mathrm{i0}$ is the unperturbed ion density, differs considerably from the usual assumption of a shifted Maxwellian:

$$\Phi_\mathrm{i0}(\vec{v})=\Phi_\mathrm{n}^\perp(v_\perp)\Phi_\mathrm{i0}^z(v_z),\qquad \Phi_\mathrm{i0}^z(v_z)=\int_0^\infty\exp(-x)\,\Phi_\mathrm{n}^z(v_z-x\,v_\mathrm{d})\,dx. \tag{3}$$

Compared with the distribution function perpendicular to the electric field, which is Maxwellian, the distribution in the streaming direction becomes significantly broader and highly asymmetric for Mach numbers $M_\mathrm{th}=v_\mathrm{d}/v_\mathrm{th,n}\gtrsim 1$, where $v_\mathrm{d}=qE_0/(m\,\nu_\mathrm{in})$ is the ion drift speed, see also Ref. [24]. The external magnetic field $\vec{B}_0$ is considered parallel to $\vec{E}_0$. Consequently, there is no $\vec{E}_0\times\vec{B}_0$ drift, and $f_\mathrm{i0}$ remains unaffected by the magnetic component of the Lorentz force.

## 1.2 Perturbed distribution function

The ion distribution will now be linearized around its stationary value, $f_\mathrm{i}\approx f_\mathrm{i0}+\delta f_\mathrm{i}$, where $\delta f_\mathrm{i}$ is a small perturbation. Similarly, we introduce small perturbations of the electric field, $\vec{E}\approx\vec{E}_0+\delta\vec{E}$. The perturbed density follows from $\delta n_\mathrm{i}=\int\delta f_\mathrm{i}\,d\vec{v}$. Dropping all terms of second order, one obtains from Eq. (1),

$$\frac{\partial \delta f_\mathrm{i}}{\partial t}+\frac{\partial \delta f_\mathrm{i}}{\partial \vec{r}}\cdot\vec{v}+\frac{\partial \delta f_\mathrm{i}}{\partial \vec{v}}\cdot\frac{q_\mathrm{i}}{m_\mathrm{i}}\left(\vec{E}_0+\vec{v}\times\vec{B}_0\right)=-\nu_\mathrm{in}\left[\delta f_\mathrm{i}-\delta n_\mathrm{i}\,\Phi_\mathrm{n}\right]-\frac{\partial f_\mathrm{i0}}{\partial \vec{v}}\cdot\frac{q_\mathrm{i}\,\delta\vec{E}}{m_\mathrm{i}}. \tag{4}$$

The solution of Eq. (4) can be found by the method of characteristics [32, 33]. For this purpose, we consider the equivalent equation

$$\left[\frac{d}{dt'}+\nu_\mathrm{in}\right]\,\delta f_\mathrm{i}(\vec{r}\,'(t'),\vec{v}\,'(t'),t')=c(\vec{r}\,'(t'),\vec{v}\,'(t'),t'), \tag{5}$$

where

$$c(\vec{r},\vec{v},t) = \nu_{\text{in}}\,\delta n_{\text{i}}(\vec{r},t)\,\Phi_{\text{n}}(\vec{v}) - \frac{\partial f_{\text{i}0}(\vec{v})}{\partial \vec{v}} \cdot \frac{q_{\text{i}}\,\delta\vec{E}(\vec{r},t)}{m_{\text{i}}}. \tag{6}$$

The trajectories $\{\vec{r}\,'(t'),\vec{v}\,'(t')\}$ satisfy the equations of motion

$$\dot{\vec{r}}\,'(t') = \vec{v}\,'(t'), \qquad \dot{\vec{v}}\,'(t') = \frac{q_{\text{i}}}{m_{\text{i}}}\left(\vec{E}_0 + \vec{v}\,'(t') \times \vec{B}_0\right), \tag{7}$$

with the condition $\vec{r}\,'(t'=t)=\vec{r}$ and $\vec{v}\,'(t'=t)=\vec{v}$. The explicit solutions of Eqs. (7) are given in Appendix A. The particular solution of Eq. (4) that vanishes for $t\to-\infty$ can now be written in the form

$$\delta f_{\text{i}}(\vec{r},\vec{v},t) = \int_{-\infty}^{t} \exp\left[-\nu_{\text{in}}(t-t')\right] c(\vec{r}\,'(t'),\vec{v}\,'(t'),t')\,dt', \tag{8}$$

corresponding to an integration along the unperturbed orbits. It is thereby assumed that $c(\vec{r},\vec{v},t)\to 0$ for $t\to-\infty$.

We analyze Eq. (8) for a single Fourier component, $\delta f_{\text{i}}(\vec{r},\vec{v},t) \sim \delta\hat{f}_{\text{i}}(\vec{k},\vec{v},\omega)\exp(i\vec{k}\cdot\vec{r}-i\omega t)$, where $\omega$ has a positive imaginary part, $\text{Im}(\omega)>0$. Corresponding expressions are used for the electric field and the density. Changing the integration variable from $t'$ to $\tau=t-t'$, the result for the Fourier coefficient becomes

$$\delta\hat{f}_{\text{i}}(\vec{k},\vec{v},\hat{\omega}) = \int_0^\infty d\tau \exp\left[i\Omega(\vec{k},\vec{v},\omega,\tau)\right]\left[\nu_{\text{in}}\,\delta\hat{n}_{\text{i}}(\vec{k},\omega)\Phi_{\text{n}}(\vec{v}\,'(t')) + i\frac{q_{\text{i}}}{m_{\text{i}}}\hat{\varphi}(\vec{k},\omega)\frac{\partial f_{\text{i}0}(\vec{v}\,'(t'))}{\partial\vec{v}\,'}\cdot\vec{k}\right]. \tag{9}$$

Here, the perturbed electric field has been written as $\delta\hat{\vec{E}}(\vec{k},\omega) = -i\vec{k}\,\delta\hat{\varphi}(\vec{k},\omega)$, where $\delta\hat{\varphi}$ denotes the Fourier component of the perturbed electrostatic potential. Choosing the orientation of the coordinate system such that $\vec{k}=(k_\perp,0,k_z)$ and employing the explicit expressions for the trajectories in Appendix A, the phase of the exponential term in Eq. (9) becomes (see also Ref. [32])

$$\Omega(\vec{k},\vec{v},\omega,\tau) = (\omega+i\nu_{\text{in}}-k_z v_z)\tau + \frac{k_z}{2}\frac{q_{\text{i}}E_0}{m_{\text{i}}}\tau^2 + \frac{k_\perp v_\perp}{\omega_{\text{ci}}}\left[\sin\phi - \sin(\phi+\omega_{\text{ci}}\tau)\right], \tag{10}$$

where $\omega_{\text{ci}} = q_{\text{i}}B_0/m_{\text{i}}$ is the ion cyclotron frequency.

### 1.3 Density response function

The susceptibility relates the induced ion density to the electrostatic potential and can be obtained from [22]

$$\chi_{\text{i}}(\vec{k},\omega) = -\frac{q_{\text{i}}}{\epsilon_0\,k^2}\frac{\delta\hat{n}_{\text{i}}(\vec{k},\omega)}{\delta\hat{\varphi}(\vec{k},\omega)}. \tag{11}$$

The density response is calculated from Eq. (9) as the integral over the velocities,

$$\delta\hat{n}_{\text{i}}(\vec{k},\omega) = \int \delta\hat{f}_{\text{i}}(\vec{k},\omega)\,d\vec{v} = \nu_{\text{in}}\,\delta\hat{n}_{\text{i}}(\vec{k},\omega)I_1(\vec{k},\omega) - \frac{q_{\text{i}}n_{\text{i}0}}{m_{\text{i}}}\delta\hat{\varphi}(\vec{k},\omega)I_2(\vec{k},\omega), \tag{12}$$

where the integrals $I_1$ and $I_2$ read

$$I_1 = \iint_0^\infty \Phi_{\text{n}}(\vec{v}\,')\exp\left[i\Omega(\vec{k},\vec{v},\omega,\tau)\right]d\tau d\vec{v}, \tag{13a}$$

$$I_2 = -i\,\vec{k}\cdot\iint_0^\infty \frac{\partial\Phi_{\text{i}0}(\vec{v}\,')}{\partial\vec{v}\,'}\exp\left[i\Omega(\vec{k},\vec{v},\omega,\tau)\right]d\tau d\vec{v}. \tag{13b}$$

Solving Eq. (12) for the density and using the result in Eq. (11), the ion susceptibility becomes

$$\chi_{\text{i}}(\vec{k},\omega) = \frac{\omega_{\text{pi}}^2}{k^2}\frac{I_2}{1-\nu_{\text{in}}I_1}, \tag{14}$$

where $\omega_{\text{pi}} = \sqrt{q_{\text{i}}^2 n_{\text{i}0}/(m_{\text{i}}\epsilon_0)}$ is the ion plasma frequency. The evaluation of the integrals, Eq. (13), can be found in Appendix B and Appendix C. In the next section, only the results will be presented and discussed.

## 2 Ion susceptibility

Equations (13a) and (13b) can be represented as (i) a sum over Bessel functions, known from the response function of a magnetized Maxwellian plasma [34,35] and an appropriately weighted plasma dispersion function [22], and (ii) in integral form without an infinite sum (Gordeev form), see Ref. [32]. These two representations will be discussed in the following.

### 2.1 Bessel series representation

The derivation of the series expansion can be found in Appendix B. Introducing $I_n(\eta)$ as a modified Bessel function of the first kind with argument $\eta = k_\perp^2 r_\mathrm{L}^2$ (Larmor radius $r_\mathrm{L} = v_\mathrm{th,n}/\omega_\mathrm{ci}$), the expansion reads

$$\chi_\mathrm{i}(\vec{k},\omega) = \frac{(k\lambda_\mathrm{in})^{-2}}{1+i/(l_E k_z)} \frac{1 + \sum\limits_{n=-\infty}^{\infty} I_n(\eta)\exp(-\eta)\left[\langle \xi_n Z(\xi_n)\rangle + \frac{n\omega_\mathrm{ci}\sqrt{1+i/(l_E k_z)}}{\sqrt{2}|k_z|v_\mathrm{th,n}}\langle Z(\xi_n)\rangle\right]}{1 + \sum\limits_{n=-\infty}^{\infty} \frac{i\nu_\mathrm{in}}{\omega + i\nu_\mathrm{in} - n\omega_\mathrm{ci}} I_n(\eta)\exp(-\eta)\xi_n(0)Z[\xi_n(0)]}. \tag{15}$$

where two length scales have been introduced, $l_E = m_\mathrm{i} v_\mathrm{th,n}^2/(q_\mathrm{i} E_0)$ and $\lambda_\mathrm{in} = v_\mathrm{th,n}/\omega_\mathrm{pi}$. The latter corresponds to the Debye length, but only in a stationary plasma ($E_0 = 0$)—the reason being the modified velocity distribution [31]. Note that in the perpendicular direction the ion distribution function is indeed Maxwellian, and the perpendicular ion temperature is equal to the temperature of the neutral gas. Therefore, the Larmor radius is well defined. The plasma dispersion function can be expressed in terms of the complementary error function,

$$Z(\xi) = i\sqrt{\pi}\,\exp\left(-\xi^2\right)\mathrm{erfc}(-i\xi), \qquad \xi_n(x) = \frac{\omega + i\nu_\mathrm{in} - n\omega_\mathrm{ci} - k_z v_\mathrm{d} x}{\sqrt{2}\,v_\mathrm{th,n}|k_z|\sqrt{1+i/(k_z l_E)}}. \tag{16}$$

The averages in Eq. (15) are performed in the same way as in Eq. (3), i.e.,

$$\langle f(x)\rangle = \int_0^\infty \exp(-x) f(x) dx, \tag{17}$$

see also Ref. [22].

Equation (15) extends the result for a magnetized Maxwellian plasma with ion-neutral collisions [35] and includes an external electric field that gives rise to a finite flow and the non-Maxwellian velocity distribution [22].

### 2.2 Integral representation

The convergence of the sums in Eq. (15) is very slow for $\eta = k_\perp^2 r_\mathrm{L}^2 \gtrsim 1$, i.e., when the perpendicular wavelength is much smaller than the Larmor radius, see Ref. [18]. However, there exists an alternative representation of the susceptibility, where the summation can be performed analytically, and the computation of $\chi_\mathrm{i}$ reduces to the evaluation of two integrals, see also Ref. [32]. Details on the derivation can be found in Appendix C.

The result for the integral form of the response function reads

$$\chi_\mathrm{i}(\vec{k},\omega) = \frac{1}{k^2\lambda_\mathrm{in}^2}\frac{1 + A(\vec{k},\omega)}{1 + B(\vec{k},\omega)}. \tag{18}$$

The functions $A$ and $B$ are given by the following integrals,

$$A(\vec{k},\omega) = \int_0^\infty \Lambda(\tau)\exp\left[\Psi(0,\tau)\right]\,d\tau, \qquad B(\vec{k},\omega) = -\nu_\mathrm{in}\int_0^\infty \exp\left[\Psi(0,\tau)\right]\,d\tau, \tag{19}$$

where the common argument of the exponential term is

$$\Psi(x,\tau) = i(\omega + i\nu_\mathrm{in} - k_z v_\mathrm{d} x)\tau - k_z^2 v_\mathrm{th,n}^2 \frac{\tau^2}{2}\left(1 + \frac{i}{l_E k_z}\right) + \eta[\cos(\omega_\mathrm{ci}\tau) - 1], \tag{20}$$

and the integrand of $A(\vec{k},\omega)$ is determined by

$$\Lambda(\tau) = \frac{i\left(\omega + i\nu_{\mathrm{in}} - k_z v_{\mathrm{d}}\nu_{\mathrm{in}}\tau\right)}{1 + ik_z v_{\mathrm{d}}\tau} - \frac{ik_z v_{\mathrm{d}}}{(1 + ik_z v_{\mathrm{d}}\tau)^2}. \tag{21}$$

In contrast to Eq. (15), the averaging procedure over the unperturbed ion distribution function has been performed analytically. A corresponding form was found in the unmagnetized limit [22, 31].

### 2.3 Discussion

The main result of this work is the ion susceptibility in parallel electric and magnetic fields, Eqs. (15) and (18). It is readily verified that for $k_\perp = 0$, i.e., in the direction of the external fields, the susceptibility is independent of the magnetic field and identical to the result for unmagnetized ions [22], see also Eq. (42). Only the $n = 0$ term in Eq. (15) yields a finite contribution to the susceptibility. In the perpendicular direction ($k_z = 0$), on the other hand, $\chi_{\mathrm{i}}$ is independent of $E_0$. It can thus be concluded that the complex interplay between the electric and magnetic field occurs under oblique angles, where both fields affect the susceptibility at the same time. An example is shown in Fig. 1.

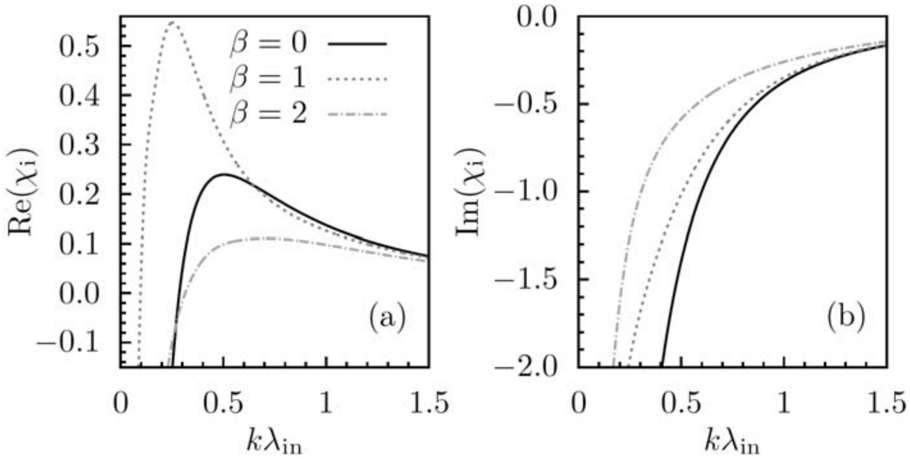


**Fig. 1** Real part (a) and imaginary part (b) of the static ($\omega = 0$) ion susceptibility as a function of $k = |\vec{k}|$ for $M_{\mathrm{th}} = 4$, $\nu_{\mathrm{in}}/\omega_{\mathrm{pi}} = 0.2$. The magnetization $\beta = \omega_{\mathrm{ci}}/\omega_{\mathrm{pi}}$ is indicated in the figure. The angle between the electric field $\vec{E}_0$ and the wave vector $\vec{k}$ is $\pi/3$.

Despite being a very simple approximation, the ion distribution function obtained with the BGK collision operator was shown to agree well with more complex calculations using the Boltzmann equation, at least in the small Mach number limit [30]. Thus, the present results should be best applicable in this regime. In addition, the ion subsystem can become unstable at high Mach numbers and at low ion-neutral damping [31]. A detailed investigation of the initial value problem [36] and a stability analysis of the ion system in the presence of a magnetic field is beyond the scope of this work.

## 3 Conclusion

In summary, the ion susceptibility has been derived for a situation, where the ions are subject to both an external electric field and a parallel magnetic field. It accounts for the non-Maxwellian velocity distribution and is applicable in a wide range of ion magnetization. Possible applications include the screening of a dust particle in the presence of streaming magnetized ions [18], the ion-dust streaming instability [37, 38], and other phenomena, where flowing ions interact with charged microparticles in a strong external magnetic field.

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# Appendix

## A Unperturbed orbits

Equation (7) describes the motion of a single ion in uniform parallel electric and magnetic fields. The trajectories without the influence of the former can be found in Ref. [32]. Adding the constant acceleration in the $z$-direction caused by the electric field, the unperturbed orbits are

$$x'(t') - x = \frac{v_\perp}{\omega_{\mathrm{ci}}}\left[\sin\phi - \sin(\phi + \omega_{\mathrm{ci}}\tau)\right], \qquad v'_x(t') = v_\perp \cos(\phi + \omega_{\mathrm{ci}}\tau),$$
$$y'(t') - y = -\frac{v_\perp}{\omega_{\mathrm{ci}}}\left[\cos\phi - \cos(\phi + \omega_{\mathrm{ci}}\tau)\right], \qquad v'_y(t') = v_\perp \sin(\phi + \omega_{\mathrm{ci}}\tau),$$
$$z'(t') - z = -v_z\tau + \frac{1}{2}\frac{q_{\mathrm{i}}E_0}{m_{\mathrm{i}}}\tau^2, \qquad v'_z(t') = v_z - \frac{q_{\mathrm{i}}E_0}{m_{\mathrm{i}}}\tau.$$

They satisfy the boundary conditions, $\vec{r}\,'(t' = t) = \vec{r}$ and $\vec{v}\,'(t' = t) = \vec{v}$. The polar angle of the velocity vector $\vec{v}\,'$ is denoted by $\phi$ and $\tau = t - t'$. The ion performs the usual cyclotron motion around the magnetic field and experiences the constant electric field force, which leads to the additional term in the $z$-component.

## B Susceptibility in terms of Bessel series and plasma dispersion function

In this appendix, the calculation of the integrals in Eq. (13) will be discussed, and it is shown how they can be written as a Bessel series involving the plasma dispersion function.

Equation (13a) is considered first. Using the identity $\exp(iz\sin\phi) = \sum_n J_n(z)\exp(in\phi)$, where $J_n(z)$ is a Bessel function of order $n$ [32], the phase factor [see Eq. (10)] can be written as

$$\exp(i\Omega) = \exp\left[i(\omega + i\nu_{\mathrm{in}} - n\omega_{\mathrm{ci}} - k_z v_z)\tau + \frac{k_z}{2}\frac{q_{\mathrm{i}}E_0}{m_{\mathrm{i}}}\tau^2\right] \cdot \sum_{n,n'} J_n\left(\frac{k_\perp v_\perp}{\omega_{\mathrm{ci}}}\right) J_{n'}\left(\frac{k_\perp v_\perp}{\omega_{\mathrm{ci}}}\right)\exp[i(n' - n)\phi]. \tag{22}$$

Noting that $\int_0^{2\pi}\exp[i(n' - n)\phi]d\phi = 2\pi\,\delta_{n,n'}$, the integral over the perpendicular velocity component in Eq. (13a) becomes [32]

$$\int_0^\infty J_n^2\left(\frac{k_\perp v_\perp}{\omega_{\mathrm{ci}}}\right)\frac{\exp[-v_\perp^2/(2v_{\mathrm{th,n}}^2)]}{2\pi v_{\mathrm{th,n}}^2} v_\perp dv_\perp = \frac{I_n(\eta)\exp(-\eta)}{2\pi}, \tag{23}$$

where Eq. (2) for the neutral particles' velocity distribution and the definition $\eta = k_\perp^2 v_{\mathrm{th,n}}^2/\omega_{\mathrm{ci}}^2$ have been used. Introducing

$$\Upsilon_n = (\omega + i\nu_{\mathrm{in}} - n\omega_{\mathrm{ci}} - k_z v_z)\tau + \frac{k_z}{2}\frac{q_{\mathrm{i}}E_0}{m_{\mathrm{i}}}\tau^2, \tag{24}$$

the remaining integrals over time ($\tau$) and the parallel velocity ($v_z$) in Eq. (13a) can be written as ($v'_z = v_z - q_{\mathrm{i}}E_0\tau/m_{\mathrm{i}}$)

$$\int_{-\infty}^\infty dv_z \int_0^\infty d\tau\,\frac{\exp[-(v'_z)^2/(2v_{\mathrm{th,n}}^2)]}{(2\pi v_{\mathrm{th,n}}^2)^{1/2}}\exp(i\Upsilon_n) = \int_0^\infty d\tau\exp\left\{-v_{\mathrm{th,n}}^2 k_z^2\frac{\tau^2}{2}\left[1 + \frac{i}{l_E k_z}\right] + i(\omega + i\nu_{\mathrm{in}} - n\omega_{\mathrm{ci}})\tau\right\} = \sqrt{\pi}\,\mathrm{erfc}[-i\,\xi_n(0)]\,\frac{\xi_n(0)\,\exp[-\xi_n^2(0)]}{\omega + i\nu_{\mathrm{in}} - n\omega_{\mathrm{ci}}} = -i\frac{\xi_n(0)\,Z[\xi_n(0)]}{\omega + i\nu_{\mathrm{in}} - n\omega_{\mathrm{ci}}}, \tag{25}$$

where the $v_z$ integration has been performed (Gaussian integral), and the $\tau$-integral has been written in terms of the complementary error function, $\mathrm{erfc}(z) = 1 - \mathrm{erf}(z)$. Further, the definition of the plasma dispersion function, Eq. (16), has been used. Collecting the results, Eq. (13a) now becomes

$$I_1 = -i \sum_{n=-\infty}^{\infty} I_n(\eta) \exp(-\eta) \frac{\xi_n(0)\, Z[\xi_n(0)]}{\omega + i\nu_{\text{in}} - n\omega_{\text{ci}}}. \tag{26}$$

The second integral [Eq. (13b)] can be split into a longitudinal and a transverse contribution, $I_2 = I_2^z + I_2^\perp$, where

$$I_2^z = -ik_z \iint_0^\infty \frac{\partial \Phi_{\text{i0}}}{\partial v_z} \exp(i\Omega) d\tau d\vec{v}, \quad I_2^\perp = \frac{ik_\perp}{v_{\text{th,n}}^2} \iint_0^\infty \Phi_{\text{i0}}(\vec{v}\,') \exp(i\Omega) v_\perp \cos(\phi + \omega_{\text{ci}}\tau) d\tau d\vec{v}. \tag{27}$$

For the perpendicular part, we have used the explicit form of the distribution function. After a partial integration with respect to $v_z$ and the same integration of the perpendicular velocity as for $I_1$, the longitudinal contribution becomes

$$\begin{aligned} I_2^z &= -ik_z \iint_0^\infty \frac{\partial \Phi_{\text{i0}}}{\partial v_z} \exp(i\Omega) d\tau d\vec{v} = k_z^2 \iint_0^\infty \Phi_{\text{i0}}(\vec{v}\,') \tau \, \exp(i\Omega) d\tau d\vec{v} \\ &= k_z^2 \sum_{n=-\infty}^{\infty} I_n(\eta) \exp(-\eta) \int_0^\infty d\tau \int_{-\infty}^{\infty} dv_z \Phi_{\text{i0}}^z(v_z') \exp(i\Upsilon_n)\tau. \end{aligned} \tag{28}$$

Inserting the explicit result for the ion distribution function in the streaming direction [Eq. (3)], the remaining integrals can be rewritten as [see Eq. (25)]

$$\begin{aligned} &\int_0^\infty d\tau \int_{-\infty}^{\infty} dv_z \, \Phi_{\text{i0}}^z \left( v_z - \frac{q_{\text{i}} E_0}{m_{\text{i}}} \tau \right) \exp(i\Upsilon_n)\tau \\ &= \int_0^\infty d\tau \int_0^\infty dx \exp\left\{ -v_{\text{th,n}}^2 k_z^2 \frac{\tau^2}{2} \left[ 1 + \frac{i}{l_E k_z} \right] + i(\omega + i\nu_{\text{in}} - n\omega_{\text{ci}} - k_z v_{\text{d}} x)\tau \right\} \tau \exp(-x). \end{aligned} \tag{29}$$

With the relation $-2b \int_0^\infty \exp(a\tau + b\tau^2)\tau d\tau = 1 + a \int_0^\infty \exp(a\tau + b\tau^2) d\tau$ (with $a$ and $b$ such that convergence of the integrals is assured), Eq. (29) can be further simplified to

$$\begin{aligned} &\frac{1}{v_{\text{th,n}}^2 k_z^2 \left[1 + i/(l_E k_z)\right]} \left[ 1 + i \int_0^\infty dx \, \exp(-x)(\omega + i\nu_{\text{in}} - n\omega_{\text{ci}} - k_z v_{\text{d}} x) \right. \\ &\left. \int_0^\infty d\tau \exp\left\{ -v_{\text{th,n}}^2 k_z^2 \frac{\tau^2}{2} \left[ 1 + \frac{i}{l_E k_z} \right] + i(\omega + i\nu_{\text{in}} - n\omega_{\text{ci}} - k_z v_{\text{d}} x)\tau \right\} \right] \\ &= \frac{1}{v_{\text{th,n}}^2 k_z^2 \left[1 + i/(l_E k_z)\right]} \left[ 1 + \int_0^\infty dx \exp(-x) \xi_n(x) Z(\xi_n(x)) \right] = \frac{1 + \langle \xi_n(x) Z(\xi_n(x)) \rangle}{v_{\text{th,n}}^2 k_z^2 \left[1 + i/(l_E k_z)\right]}. \end{aligned} \tag{30}$$

The integral in the first line involving the plasma dispersion function is equivalent to the integral in Eq. (25). In the last line, Eq. (17) for the average $\langle \ldots \rangle$ has been used.

The perpendicular component of the integral $I_2$ will be considered next, see Eq. (27). The steps are similar to those above. With the identity [32]

$$\frac{1}{2\pi} \int_0^{2\pi} \exp\{iz\left[\sin\phi - \sin(\phi + \omega_{\text{ci}}\tau)\right]\} \cos(\phi + \omega_{\text{ci}}\tau) d\phi = \sum_{n=-\infty}^{\infty} \frac{n\, J_n^2(z)}{z} \exp(-in\omega_{\text{ci}}\tau) \tag{31}$$

 

for the angular ($\phi$) integration and Eq. (23) for the $v_\perp$ integral, one finds

$$\begin{aligned} I_2^\perp &= \frac{ik_\perp}{v_{\text{th,n}}^2}\int_0^\infty d\tau \int_{-\infty}^\infty dv_z \int_0^\infty dv_\perp\, v_\perp^2 \exp\left[i(\omega+i\nu_{\text{in}}-k_z v_z)\tau + \frac{k_z}{2}\frac{q_{\text{i}}E_0}{m_{\text{i}}}\tau^2\right] \\ &\quad\cdot \frac{\exp[-v_\perp^2/(2v_{\text{th,n}}^2)]}{2\pi v_{\text{th,n}}^2}\Phi_{\text{i0}}^z(v_z')\int_0^{2\pi} d\phi \cos(\phi+\omega_{\text{ci}}\tau)\, \exp\left\{\frac{ik_\perp v_\perp}{\omega_{\text{ci}}}\left[\sin\phi - \sin(\phi+\omega_{\text{ci}}\tau)\right]\right\} \\ &= \frac{i\omega_{\text{ci}}}{v_{\text{th,n}}^2}\int_0^\infty d\tau \int_{-\infty}^\infty dv_z \sum_n 2\pi n \int_0^\infty dv_\perp v_\perp \frac{\exp[-v_\perp^2/(2v_{\text{th,n}}^2)]}{2\pi v_{\text{th,n}}^2} J_n^2\left(\frac{k_\perp v_\perp}{\omega_{\text{ci}}}\right)\Phi_{\text{i0}}^z(v_z')\exp(i\Upsilon_n) \\ &= \frac{i\omega_{\text{ci}}}{v_{\text{th,n}}^2}\sum_n I_n(\eta)\exp(-\eta) n \int_0^\infty d\tau \int_{-\infty}^\infty dv_z\, \Phi_{\text{i0}}^z(v_z')\exp(i\Upsilon_n) \\ &= \frac{\omega_{\text{ci}}}{v_{\text{th,n}}^2}\sum_n I_n(\eta)\exp(-\eta)\frac{n\,\langle Z(\xi_n(x))\rangle}{\sqrt{2}v_{\text{th,n}}|k_z|\sqrt{1+i/(l_E k_z)}}. \end{aligned} \tag{32}$$

Summarizing the results, Eq. (13b) can be written as

$$I_2 = \frac{1}{v_{\text{th,n}}^2\left[1+i/(l_E k_z)\right]}\sum_n I_n(\eta)\exp(-\eta)\left[1+\langle \xi_n Z(\xi_n)\rangle + \frac{n\omega_{\text{ci}}\sqrt{1+i/(l_E k_z)}}{\sqrt{2}|k_z|v_{\text{th,n}}}\langle Z(\xi_n)\rangle\right]. \tag{33}$$

Equation (15) in the main text now follows from Eqs. (14), (26), and (33) and the identity $\sum_{n=-\infty}^{\infty} I_n(z) = \exp(z)$ [32].

## C Integral form of susceptibility

The steps that lead to the integral form of the response function, Eq. (18), are detailed in the following. For this purpose one returns to Eqs. (26), (28) and (32) and performs the summation of the infinite series of Bessel functions analytically.

For the integral $I_1$ [see Eqs. (25) and (26)], we use $\sum_n I_n(\eta)\exp(-in\omega_{\text{ci}}\tau) = \exp[\eta\cos(\omega_{\text{ci}}\tau)]$ [32] to obtain

$$\begin{aligned} I_1 &= \sum_n I_n(\eta)\exp(-\eta)\int_0^\infty \exp\left\{i(\omega+i\nu_{\text{in}}-n\omega_{\text{ci}})\tau - v_{\text{th,n}}^2 k_z^2\frac{\tau^2}{2}\left[1+\frac{i}{l_E k_z}\right]\right\}d\tau \\ &= \int_0^\infty \exp[\Psi(0,\tau)]d\tau, \end{aligned} \tag{34}$$

where $\Psi$ is defined in Eq. (20). The second integral can be rewritten in a similar fashion. From Eqs. (28) and (30) one gets

$$\begin{aligned} v_{\text{th,n}}^2\left[1+i/(l_E k_z)\right] I_2^z &= \sum_n I_n(\eta)\exp(-\eta)\left[1+i\int_0^\infty dx\, \exp(-x)(\omega+i\nu_{\text{in}}-n\omega_{\text{ci}}-k_z v_{\text{d}}x)\right. \\ &\quad\left.\cdot\int_0^\infty d\tau \exp\left\{-v_{\text{th,n}}^2 k_z^2\frac{\tau^2}{2}\left[1+\frac{i}{l_E k_z}\right]+i(\omega+i\nu_{\text{in}}-n\omega_{\text{ci}}-k_z v_{\text{d}}x)\tau\right\}\right] \\ &= 1+\int_0^\infty dx\, \exp(-x)\int_0^\infty d\tau\left[i(\omega+i\nu_{\text{in}}-k_z v_{\text{d}}x)-\eta\,\omega_{\text{ci}}\,\sin(\omega_{\text{ci}}\tau)\right]\exp[\Psi(x,\tau)], \end{aligned}$$

 

where the same summation formula as above and $\sum_n n\, I_n(\eta)\exp(-in\omega_{\rm ci}\tau) = -i\,\eta\,\sin(\omega_{\rm ci}\tau)\exp[\eta\cos(\omega_{\rm ci}\tau)]$ for the sum involving the factor $n$ have been employed [32]. Using Eq. (32) and performing similar manipulations, the perpendicular component of $I_2$ can be expressed as

$$
\begin{aligned}
I_2^{\perp} &= i \sum_{n=-\infty}^{\infty} I_n(\eta)\exp(-\eta)\frac{n\omega_{\rm ci}}{v_{\rm th,n}^2}\int_0^\infty d\tau \int_0^\infty dx\, \exp(-x) \\
&\quad\cdot \exp\left\{ i(\omega + i\nu_{\rm in} - n\omega_{\rm ci} - k_z v_{\rm d} x)\tau - v_{\rm th,n}^2 k_z^2 \frac{\tau^2}{2}\left[1 + \frac{i}{l_E k_z}\right]\right\} \\
&= \frac{\eta\,\omega_{\rm ci}}{v_{\rm th,n}^2}\int_0^\infty dx\, \exp(-x)\int_0^\infty d\tau\, \sin(\omega_{\rm ci}\tau)\exp[\Psi(x,\tau)]. 
\end{aligned} \tag{35}
$$

One now combines the results for $I_2^z$ and $I_2^{\perp}$ to obtain

$$
\begin{aligned}
I_2 &= \frac{1}{v_{\rm th,n}^2\,[1 + i/(l_E k_z)]}\left\{1 + \int_0^\infty dx\, \exp(-x)\int_0^\infty d\tau\, i\left[\omega + i\nu_{\rm in} - k_z v_{\rm d} x + \frac{\eta\,\omega_{\rm ci}}{l_E\, k_z}\sin(\omega_{\rm ci}\tau)\right]\right. \\
&\qquad\qquad \left.\cdot \exp[\Psi(x,\tau)]\right\}, \\
&= \frac{1}{v_{\rm th,n}^2}\left\{1 + \int_0^\infty dx\, \exp(-x)\int_0^\infty d\tau\, [i(\omega + i\nu_{\rm in} - k_z v_{\rm d}(x + \nu_{\rm in}\tau))]\exp[\Psi(x,\tau)]\right\},
\end{aligned} \tag{36}
$$

which yields Eqs. (18) and (19) upon performing the $x$ integral.

## D Limiting cases

In this Appendix it is briefly shown how the results for the susceptibility can be reduced to (i) the case of magnetized ions with a Maxwellian velocity distribution for $E_0 \to 0$ (i.e., no streaming [32, 35]), and (ii) the susceptibility of unmagnetized, non-Maxwellian (streaming) ions for $B_0 = 0$ [22].

### D.1 Stationary ions: $E_0 \to 0$

In the limit $E_0 \to 0$, where the ions become stationary, one may drop the averages in Eq. (15) to obtain

$$
\chi_{\rm i}(\vec{k},\omega; E_0 \to 0) = \frac{1}{k^2\lambda_{\rm in}^2}\,\frac{1 + \sum\limits_{n=-\infty}^{\infty}\dfrac{\omega + i\nu_{\rm in}}{\omega + i\nu_{\rm in} - n\omega_{\rm ci}} I_n(\eta)\exp(-\eta)\zeta_n Z(\zeta_n)}{1 + \sum\limits_{n=-\infty}^{\infty}\dfrac{i\nu_{\rm in}}{\omega + i\nu_{\rm in} - n\omega_{\rm ci}} I_n(\eta)\exp(-\eta)\zeta_n Z(\zeta_n)},
$$

where the argument of the plasma dispersion function is

$$
\zeta_n = \frac{\omega + i\nu_{\rm in} - n\omega_{\rm ci}}{\sqrt{2}\, v_{\rm th,n}|k_z|}. \tag{37}
$$

This is the result for a magnetized Maxwellian plasma with ion-neutral collisions taken into account [18, 35].

### D.2 Unmagnetized ions: $B_0 \to 0$

For the analysis of the limit $B_0 \to 0$, the integral form [Eq. (18)] will be employed. Taking the limit $\omega_{\rm ci} \to 0$, the cosine term in Eq. (20) simplifies, $k_\perp^2 v_{\rm th,n}^2/\omega_{\rm ci}^2[\cos(\omega_{\rm ci}\tau) - 1] \to -k_\perp^2 v_{\rm th,n}^2 \tau^2/2$, and one obtains

$$
\Psi(x,\tau; B_0 \to 0) = i(\omega + i\nu_{\rm in} - k_z v_{\rm d} x)\tau - k^2 v_{\rm th,n}^2 \frac{\tau^2}{2}\left[1 + \frac{i\,k_z}{k^2\, l_E}\right]. \tag{38}
$$

 

With this result at hand, the integrals $I_1$ and $I_2$ [Eqs. (13a) and (13b)] can be rewritten, thereby starting from Eqs. (34) and Eq. (36). Equation (34) becomes

$$I_1 = \int_0^\infty \exp[\Psi(0,\tau;B_0 \to 0)]d\tau = \frac{-i\,\alpha(0)Z[\alpha(0)]}{\omega + i\nu_{\text{in}}}, \tag{39}$$

where the parameter $\alpha$ reads

$$\alpha(x) = \frac{\omega + i\nu_{\text{in}} - k_z v_{\text{d}} x}{\sqrt{2}\, v_{\text{th,n}} k \sqrt{1 + \dfrac{i\,k_z}{k^2 l_E}}}. \tag{40}$$

The integral can be evaluated by comparing Eqs. (38) and (39) with Eq. (25). From Eq. (36), one gets

$$\begin{aligned} v_{\text{th,n}}^2\, I_2 &= 1 + \int_0^\infty dx\, \exp(-x) \int_0^\infty d\tau\, [i(\omega + i\nu_{\text{in}} - k_z v_{\text{d}}(x + \nu_{\text{in}}\tau))] \exp[\Psi(x,\tau;B_0 \to 0)] \\ &= k^2 v_{\text{th,n}}^2 \int_0^\infty dx \exp(-x) \int_0^\infty d\tau\, \tau\, \exp[\Psi(x,\tau;B_0 \to 0)] \\ &= \left(1 + \frac{i\,k_z}{k^2 l_E}\right)^{-1} [1 + \langle \alpha(x) Z(\alpha(x)) \rangle]\,. \end{aligned} \tag{41}$$

A comparison with Eqs. (29) and (30), where an equivalent integral occurs, yields the last line.

The susceptibility for unmagnetized ions is finally obtained in the form

$$\chi_{\text{i}}(\vec{k},\omega;B_0 \to 0) = \frac{(k\lambda_{\text{in}})^{-2}}{1 + \dfrac{i\,k_z}{k^2 l_E}} \frac{1 + \langle \alpha Z(\alpha) \rangle}{1 + \dfrac{i\nu_{\text{in}}}{\omega + i\nu_{\text{in}}} \alpha(0) Z[\alpha(0)]}, \tag{42}$$

which is the result given in Ref. [22]. If we perform the $x$-integration in the second line of Eq. (41) and combine this result with Eq. (39), we obtain the integral form of $\chi_{\text{i}}$ [22].

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