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Kinetic Theory for Quantum Plasmas

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Abstract. Quantum plasmas have been studied theoretically for more than four decades. Important early applications include the electron gas in metals and the electron-hole plasma in semiconductors and dimensionally reduced nanostructures. Experiments with lasers and ion beams are now making dense quantum plasmas of electrons and (classical) ions accessible in the laboratory giving rise to increased theoretical activities. In this article we review the basic concepts of quantum kinetic theory in the frame of reduced density operators as well as second quantization which have been very successfully applied to condensed matter systems, nuclear matter and dense plasmas.

Keywords: quantum plasmas, quantum kinetic theory, quantum simulations
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INTRODUCTION

Systems of charged particles – plasmas – are omnipresent in nature. In astrophysical systems plasmas cover a huge range of temperatures and densities – from low-density plasmas in the interstellar medium to high density plasmas in the core of giant planets and in stars. Furthermore, plasmas occur in many laboratory systems and include the electron gas in metals, the electron-hole plasma in semiconductors or electron ion plasmas produced in gas discharges. More recently new types of unconventional charged particle systems have emerged: strongly correlated ions in traps, highly charged microparticles (dusty or complex plasmas) or ultracold neutral plasmas. Finally, using intense sources of radiation such as lasers or free electron lasers it is now possible to create ultradense plasmas by ionizing solid targets and subsequently compressing the material. As a result, often the electrons may exceed the threshold of quantum degeneracy, when their DeBroglie wavelength becomes larger than the interparticle distance. Even though this may be a short-time transient phenomenon in present laser plasmas, it may have a profound effect on the plasma behavior in many experiments. Therefore a reliable theoretical description requires incorporation of quantum effects.

Due to this interest in laser pulse or ion beam created dense matter, quantum plasmas are attracting increasing attention, for a recent overview see [1, 2]. At the same time these systems are quite complex since one has to simultaneously take into account many effects: electronic and ionic correlations, bound state formation and partial ionization, electronic quantum and spin effects and dynamic effects related to the excitation and relaxation process initiated by strong electromagnetic fields. There exists a variety of theoretical approaches used to study these systems – ranging from semiclassical and first-principle simulations over fluid theory, kinetic theory to quantum field theory – all of which have their strengths and limitations. Progress in this field, therefore, requires parallel development of various methods and careful comparison of results.

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In the present paper I review the method of quantum kinetic equations which has the advantage of being rigorously based upon quantum many-particle physics and thus being well suited to provide a reliable theoretical description of quantum plasmas. Other more simple approaches including hydrodynamic models should be derived from this more fundamental concept by means of controlled approximations, clearly stating the applicability limits. I present two alternative approaches to quantum kinetic theory. The first is based on the reduced density operators and the second on the field-theoretical concept of second quantization. Examples include electron-hole plasmas in semiconductors and laser plasmas. Space limitations do not permit to outline all details of the theory, here the reader is referred to existing text books, e.g. Ref. [3]. Nevertheless is is hoped that the presented overview, together with extensive reference to earlier work, will allow the reader to put different concepts into perspective and apply them successfully in future work.

DERIVATION OF QUANTUM KINETIC EQUATIONS FROM THE BBGKY-HIERARCHY

One possible starting point for deriving quantum kinetic equations for a plasma interacting with an electro-magnetic field are the equations of motion for N-particle quantities such as the wave function or the density operator. Since plasmas always constitute an open system, the wave function is not suitable and we have to resort to the density operator ρ which is an incoherent superposition of all $N-$particle wave functions $|\psi_i\rangle$, $\rho = \sum_i p_i |\psi_i\rangle\langle\psi_i|$, where the p_i are real non-negative probabilities with $\sum_i p_i = 1$.

The equation of motion of the density operator is well known – it is the von Neumann equation [4],

$$i\hbar\frac{\partial}{\partial t}\rho_{1\ldots N} - [H_{1\ldots N}, \rho_{1\ldots N}] = 0, \qquad \mathrm{Tr}_{1\ldots N}\,\rho_{1\ldots N} = 1, \tag{1}$$

where $H_{1\ldots N}$ is the full N-particle hamiltonian. A nonequilibrium many-particle theory can now be constructed by taking the partial trace over $\rho_{1\ldots N}$ giving rise to the reduced s-particle density operators ($s \leq N$, $\mathscr{V}$ is the system volume)[1],

$$F_{1\ldots s} = \mathscr{V}^s\,\mathrm{Tr}_{s+1\ldots N}\,\rho_{1\ldots N}, \quad \frac{1}{\mathscr{V}^s}\mathrm{Tr}_{1\ldots s}F_{1\ldots s} = 1, \tag{2}$$

which obey the quantum version of the BBGKY[2]-hierarchy,

$$i\hbar\frac{\partial}{\partial t}F_{1\ldots s} - [H_{1\ldots s}, F_{1\ldots s}] = n\mathrm{Tr}_{s+1}\sum_{i=1}^{s}[V_{i,s+1}, F_{1\ldots s+1}], \tag{3}$$

where n is the density and $H_{1\ldots s}$ the s-particle hamiltonian. Note that the left hand side has the same form as the von Neumann equation (1), but for the s-particle subset,

[1] For notational simplicity we consider a one-component quantum plasma and do not include the exchange contributions. The proper (anti-)symmetrization of the results will be discussed in a separate section below.
[2] Bogolyubov-Born-Green-Kirkwood-Yvon

whereas the right hand side (r.h.s.) takes into account the coupling to the remaining $N-s$ particles via the pair potential $V_{i,s+1}$. As for classical systems, the hierarchy (3) couples the reduced density operators of all orders. The advantage of the abstract operator notation of the system (3) rests in its compactness. Properties and various representations of the hierarchy, including the coordinate, momentum and Wigner representation, have been discussed in detail in the text book [3].

A quantum kinetic equation follows from the first hierarchy equation upon use of the cluster expansion, $F_{12} = F_1 F_2 + g_{12}$, where g_{12} denotes the pair correlation operator,

$$i\hbar \frac{\partial}{\partial t} F_1 - [\bar{H}_1, F_1] = n \mathrm{Tr}_2 [V_{12}, g_{12}], \tag{4}$$

$$\bar{H}_1 = H_1 + U_1^H, \qquad U_1^H = n \mathrm{Tr}_2 V_{12} F_2. \tag{5}$$

Here $\bar{H}_1$ denotes the effective single-particle hamiltonian that is renormalized by the mean-field (Hartree) potential U_1^H. The collision term [r.h.s. of Eq. (4)] involves the pair correlations which obey their own equation of motion following from the hierarchy (3) which can be found in Ref. [3]. The quality of the resulting kinetic equation is governed by the decoupling approximation for the hierarchy, i.e. by the approximation for g_{12}. The simplest one consists in neglecting the r.h.s. of Eq. (4) entirely. This leads to the mean-field (or Hartree or quantum Vlasov) approximation, e.g. [5, 6]. Including g_{12} in lowest order of perturbation theory, i.e. in second Born approximation, leads to the quantum Landau equation which has been studied in detail in recent years, e.g. [7, 8, 9]. More advanced decoupling schemes include the ladder approximation (ladder sum, T-matrix) giving rise to the quantum Boltzmann equation which is crucial for the description of strong coupling effects and bound state formation, e.g. [10, 11], or the dynamically screened Born approximation (summing the ring diagrams), leading to the quantum Balescu-Lenard equation, which is important for systems with long-range interactions, such as plasmas, e.g. [12, 13, 14], see below. For an overview on different approximations for g_{12} and the corresponding quantum kinetic equations, see [3].

QUANTUM VLASOV EQUATION

The mean-field approximation is the simplest many-body approximation in classical and quantum physics. It entirely neglects correlations and reduces the description of the N-particle system to an effective single-particle problem. It is energy-conserving and time-reversible [3] and, thus, cannot provide thermalization to the correct equilibrium state. Yet, for the description of collective properties of plasmas this approximation is often sufficient, as long as correlations play a minor role. Starting from the first hierarchy equation, using the coordinate representation, changing to center of mass and relative coordinates[3], R and r, and Frourier transforming with respect to r yields the Wigner representation (we perform the obvious generalization to the multi-component case,

[3] All coordinates and momenta are vectors.

denoting the species by subscripts a,b [3])

$$\left\{\frac{\partial}{\partial t}+\frac{p}{m_a}\nabla_R\right\} f_a(R,p,t)-\frac{1}{i\hbar}\int\frac{dr}{(2\pi\hbar)^3}\,d\bar{p}\,\exp\{-i\,(p-\bar{p})\,r/\hbar\}$$
$$\times\left(U_a^{\rm eff}(R+\frac{r}{2},t)-U_a^{\rm eff}(R-\frac{r}{2},t)\right) f_a(R,\bar{p},t)=0, \tag{6}$$

$$\text{with}\quad U_a^{\rm eff}(R,t)=U_a(R,t)+\sum_b\int d\bar{R}\,V_{ab}(R-\bar{R})\,n_b(\bar{R},t), \tag{7}$$

where n_b is the density, $n_b(R,t)=(2s+1)\int dp f_b(R,p,t)/(2\pi\hbar)^3$, and s the particle spin[4].

The simplest solution is obtained in the case of a weak excitation U_a where the equation (6) can be linearized with respect to f_a. The solution for the perturbation f_{a1} is then trivially found together with the result for the polarization and dielectric functions. In fact this result has been obtained nearly 60 years ago by Klimontovich and Silin [15] as well as Bohm and Pines [16]. Note that the solution procedure requires a proper analytic continuation of the dielectric function, e.g. [17, 18, 19]. The mean-field quantum dielectric function (RPA or Lindhard dielectric function) has been studied in great detail. For one-dimensional plasmas, e.g. in semiconductor quantum wires, equilibrium and nonequilibrium plasmons were computed in Refs. [19, 20, 21]. For two-dimensional plasmas the polarization was computed by Stern [22] and frequently used for semiconductor quantum wells and superlattices. The dielectric function and plasmon dispersion for three-dimensional quantum plasmas in the limit of zero temperature are subject of many text books on solid state theory.

While the majority of studies focuses on equilibrium plasmas where the dielectric function is computed for a Fermi distribution experiments in semiconductor transport and optics have also allowed to produce electron-hole populations in nonequilibrium states. This allows to create plasmon instabilities in 1D and 2D systems [23, 19, 20, 21]. In contrast, in isotropic 3D systems a nonequilibrium (non-monotonic) distribution does not lead to an instability, as proven in Ref. [24], but rather to an undamping of acoustic modes [25]. This undamping has a profound effect on carrier thermalization as it opens additional channels for energy and momentum loss of energetic electrons. This could be proven in accurate pump-probe experiments in semiconductors [26, 27].

While the linear theory allows to approximately compute the plasmon dispersion and damping (or growth rate, in case of an instability), it must fail for predicting the long-time behavior of a quantum plasma in case of an instability. To solve this question one has to return to the full nonlinear quantum Vlasov equation (6). This equation was solved numerically in Ref. [5] where the nonlinear stabilization mechanism of a quantum plasma was found. Another interesting nonlinear effect is the creation of wakes of a moving particle. This maybe of relevance for a variety of quantum plasmas and is, therefore, being actively investigated, e.g. [28, 29, 1].

[4] Here we have assumed that the distribution function does not explicitly depend on the spin projection of the particles which is the case if the hamiltonian is spin-independent. The generalization to the spin-dependent case is straightforward.

Finally, we mention that reliable results for the dielectric function and plasmon spectrum of a nonideal quantum plasma which withstand comparison with experiments requires to take into account correlations, i.e. to go beyond the quantum Vlasov approximation. This is a rather complicated problem, as one has to guarantee the conservation laws and sum rules. An early solution within relaxation time approximation has been given by Mermin [30], for a detailed derivation and discussion see [29]. More accurate approaches use e.g. the so-called local-field corrections or selfconsistent solutions of quantum kinetic equations containing a collision term, see [31] and references therein.

SPIN AND EXCHANGE EFFECTS

It has been common in many recent papers on quantum plasmas to derive quantum kinetic equations from their classial counterpart by replacing the classical distribution functions by operators, Poisson brackets by commutators and so on. This procedure is wrong as it violates the symmetry postulate and the indistinguishability of quantum particles. In fact, this also applies to the kinetic equations written above which, therefore, have to be properly (anti-)symmetrized for the case of bosons (fermions). The key is that any wave function or (reduced) density operator used to describe a many-particle system has to be fully (anti-)symmetric with respect to arbitrary particle exchanges.

There are two ways to assure the correct symmetry. The first[5] is to apply "a-posteriori" an (anti-)symmetrization operator to all quantities and equations of motion [32, 3]. For example, the two-particle density operator is replaced by

$$F_{12} \to F_{12}^{S/A} = F_{12}\Lambda^{S/A}, \qquad \Lambda^{S/A} = 1 \pm P_{12}, \tag{8}$$

where we introduced the two-particle (anti-)symmetrization operator $\Lambda^{S/A}$ which involves the pair permutation operator P_{12}. The meaning of this operator becomes clear in the matrix representation using an arbitrary complete orthonormal basis of two-particle states $|12\rangle$:

$$\langle 12|F_{12}^{S/A}|1'2'\rangle = \langle 12|F_{12}|1'2'\rangle \pm \langle 12|F_{12}|2'1'\rangle, \tag{9}$$

where the short notation "$1''$ includes all orbital and spin quantum numbers.

Properly (anti-)symmetrized three-particle and higher order operators are defined analogously [3]. The immediate consequence is that the kinetic equations contain additional exchange contributions and lose the simple commutator structure. For example, the term with the induced potential in the quantum Vlasov equation changes to

$$\begin{aligned} n\mathrm{Tr}_2[V_{12}, F_1F_2] \quad &\to \quad n\mathrm{Tr}_2[V_{12}, F_1F_2\Lambda_{12}^{S/A}] = U_1^{HF}F_1 - F_1U_1^{HF\dagger}, \\ \text{with} \quad U_1^{HF} \quad &= \quad n\mathrm{Tr}_2V_{12}^{S/A}F_2 = n\mathrm{Tr}_2V_{12}F_2\Lambda_{12}^{S/A}; \quad V_{12}^{S/A} = V_{12}\Lambda_{12}^{S/A}, \end{aligned} \tag{10}$$

where U_1^{HF} is now a (non-hermitean) operator and the Hartree-Fock hamiltonian $H_1^{HF} = H_1 + U_1^{HF}$ replaces the mean-field hamiltonian $\bar{H}_1$ which appeared in Eq. (4). Thus, the

[5] The second approach of "a priori (anti-)symmetrization" will be discussed below.

(anti-)symmetrized first hierarchy equation (4) becomes

$$i\hbar\frac{\partial}{\partial t}F_1 - \left\{H_1^{HF}F_1 - F_1H_1^{HF\dagger}\right\} = n\mathrm{Tr}_2\left\{V_{12}^{S/A}g_{12} - g_{12}V_{12}^{S/A\dagger}\right\}. \tag{11}$$

As a result, in addition to the previous mean-field term, in Eq. (4) arises a new "exchange" part (the so-called Fock term) of the hamiltonian. The neglect of the r.h.s. again leads to the mean-field approximation which is the familiar time-dependent Hartree-Fock (TDHF) approximation that is the proper generalization of the classical Vlasov equation to quantum systems. Thus it is clear that even the simple mean-field approximation will exhibit dramatic differences for bosons and fermions, i.e. the different microscopic nature of the two known particle types leads to entirely different macroscopic behaviors, including superconductivity – in the case of fermions – or Bose condensation and superfluidity – in the case of bosons. Some examples can be found in Ref. [33].

Analogously, an exchange contribution appears in the collision integral on the r.h.s. of Eq. (11). All these exchange contributions are, in a quantum plasma in general, of the same order as the direct terms and have to be taken into account. Analogous contributions emerge in all hierarchy equations. Moreover, phase space filling effects arise, assuring e.g. the Pauli principle (in the case of fermions) in all collision integrals, i.e. the probability of a scattering event is not only proportional to the number of particles in the initial state but depends also on the occupation of the final states with factors $1 \pm f_{final}$, see below, [3]. The advantage of the discussed so far "post-(anti-)symmetrization" procedure is that the original [not (anti-)symmetrized] equations have a rather compact form and physical approximations are readily introduced before dealing with the spin. At the same time, the subsequent (anti-)symmetrization procedure may become involved, in particular beyond the two-particle level, see chapter 3 of Ref. [3].

The second (anti-)symmetrization approach, in contrast, uses "a priori" only (anti)symmetric quantities. It is based on the method of "second quantization" where the basic quantities are creation and annihilation operators of a particle in a certain single-particle quantum state $|k\rangle$[6]. Action of operator $a_k^\dagger$ (of operator a_k) on the N-particle state increases (reduces) the number n_k of particles in the k-th orbital by one. These operators have the spin statistics "built-in" via the (anti-)commutation relations for bosons (fermions),

$$[a_k, a_l]_\mp = 0, \qquad [a_k^\dagger, a_l^\dagger]_\mp = 0, \qquad [a_k^\dagger, a_l]_\mp = \delta_{kl}, \tag{12}$$

where the commutator (-, bosons) and anti-commutator (+, fermions) are defined by $[A,B]_\mp = AB \mp BA$. Note that the two operators are not hermitian, but are pairwise adjoint, i.e. the hermitian adjoint of the creator $a_k^\dagger$ is the annihilator a_k. It is easy to show that the relations (12) assure that, for fermions, the number of particles n_k cannot exceed one (thus guaranteeing the Pauli principle) whereas, for bosons, no limitation exists. This property of the creation and annihiliation operators has made the approach of second quantization the major method to describe many-particle quantum systems

[6] A complete orthonormal basis in N-particle Hilbert space $|n_1 n_2 \ldots\rangle$ can be constructed as a direct product of single particle states (orbitals) $|k\rangle$.

in quantum electrodynamics, condensed matter theory, quantum field theory and even quantum chromodynamics (QCD).

Any many-particle operator is readily transformed into second quantization representation. For example, the one-particle reduced density operator is given by a matrix in single-particle Hilbert space that can be written as the expectation value of the associated field operator products[7]. This relation is straightforwardly extended to the two-particle reduced density operator,

$$\langle l|F_1|m\rangle = \langle a_l^\dagger a_m\rangle, \tag{13}$$

$$\langle kl|F_{12}|mn\rangle = \langle a_k^\dagger a_l^\dagger a_m a_n\rangle, \tag{14}$$

and so on. As a special case, the number of particles occupying the state $|k\rangle$ is given by the diagonal element

$$n_k = \langle k|F_1|k\rangle = \langle a_k^\dagger a_k\rangle, \tag{15}$$

whereas off-diagonal elements are related to transitions between different states. Equations of motion for the reduced density operators are then readily derived from the well-known Heisenberg equation of motion for the time-dependent operators a_k and $a_k^\dagger$,

$$i\hbar\frac{\partial}{\partial t}a_k - [H_{1\ldots N}, a_k] = 0. \tag{16}$$

From this we readily recover the correctly (anti-)symmetrized hierarchy equation for the one-particle density operator, Eq. (11), and similarly all higher order equations of the quantum BBGKY hierarchy [3]. For this one simply multiplies Eq. (16) by the necessary operators, evaluates the commutator term and finally performs the ensemble average.

KINETIC EQUATIONS BEYOND THE MEAN-FIELD LIMIT. NON-MARKOVIAN EFFECTS

As mentioned above, the simple mean-field kinetic equation (Vlasov or TDHF) does not provide any relaxation to a stationary state. For a correct kinetic description of transport and thermodynamic properties, therefore, collisions have to be taken into account even if the system is only weakly correlated, giving rise to the collision integrals, arising from the terms on the r.h.s. of Eq. (11). Of special importance for plasmas at moderate coupling is the Born approximation. Within the polarization approximation for the pair correlation operator g_{12} which is modified by quantum effects one obtains the quantum Balescu-Lenard collision integral [3, 12] which we write for the spatially homogeneous

[7] Recall that the symbols $\langle\ldots\rangle$ on the r.h.s. stand for a trace with the full density operator. This is not to be confused with the symbols $|\ldots\rangle$ and $\langle\ldots|$ denoting states in Hilbert space.

case[8],

$$I(\mathbf{p}_1,t) = \frac{2}{\hbar}\int \frac{d\mathbf{p}_2}{(2\pi\hbar)^3}\frac{d\bar{\mathbf{p}}_1}{(2\pi\hbar)^3}\frac{d\bar{\mathbf{p}}_2}{(2\pi\hbar)^3}\left|\frac{V(\mathbf{p}_1-\bar{\mathbf{p}}_1)}{\varepsilon^{RPA}\left[\mathbf{p}_1-\bar{\mathbf{p}}_1,E(\mathbf{p}_1)-E(\bar{\mathbf{p}}_1);t\right]}\right|^2 \times(2\pi\hbar)^3\delta(\mathbf{p}_1+\mathbf{p}_2-\bar{\mathbf{p}}_1-\bar{\mathbf{p}}_2)\cdot 2\pi\delta(E_1+E_2-\bar{E}_1-\bar{E}_2) \times\left\{\bar{f}_1\bar{f}_2(1\pm f_1)(1\pm f_2)-f_1f_2(1\pm\bar{f}_1)(1\pm\bar{f}_2)\right\}\Big|_t. \quad (17)$$

Here, we introduced the quasiparticle (single-particle) energy and momentum (Wigner) distributions, $E_i = E(\mathbf{p}_i)$, $\bar{E}_i = E(\bar{\mathbf{p}}_i)$ and $f_i = f(\mathbf{p}_i,t)$, $\bar{f}_i = f(\bar{\mathbf{p}}_i,t)$. As was mentioned above, quantum exchange gives rise to the Pauli blocking factors $1\pm f(p)$ which reduce the scattering probability into the momentum state $|p\rangle$, in the case of fermions (Pauli principle). For bosons, in contrast, the scattering probability is enhanced (plus sign). As for classical plasmas, the collision integral contains the dynamically screened Coulomb potential, where the dielectric function is now given by the proper quantum expression, in the simplest case, by the Lindhard (RPA) approximation that was discussed above. The collision integral (17) conserves momentum and the mean value of the single-particle energy – which is a result of the corresponding delta functions – and it assures relaxation towards a Fermi or Bose function, respectively for times longer than the relaxation time t_{rel}, in that case $I \to 0$, as is easily verified by direct computation.

The quantum Balescu-Lenard equation was numerically solved for optically excited electron-hole plasmas in semiconductors by Binder et al. [12] who selfconsistently computed the RPA dielectric function with the nonequilibrium distribution functions $f(t)$. To their surprise, they observed that the relaxation of electrons and holes towards a Fermi function can be extremely fast – within times on the order of a few femtoseconds – which they could trace to the appearance of nonequilibrium collective excitations in the plasma [25]. However, this result was in contrast to experiments, where the relaxation time was typically found to be not below 100 fs[9]. The solution of this contradiction was found to be the limited applicability range of the Balescu-Lenard collision integral (17). In fact, in a rapidly evolving plasma, the use of the RPA dielectric function that adiabatically follows the evolution of the distribution functions, i.e. $\varepsilon^{RPA}(p,\omega) \to \varepsilon^{RPA}(p,\omega,t) \approx \varepsilon^{RPA}(p,\omega,[f(t)])$ must fail[10]. Here one has to take into account that binary correlations (and with them the plasmon spectrum) build up on a finite time scale that is of the order of the correlation time τ_{cor} – in a plasma it is of the order of one over the plasma frequency [35]. These observations have stimulated work towards an extension of quantum kinetic theory to finite correlation times leading to generalized non-Markovian collision integrals that take into account a finite collision duration and memory effects (the scattering probability depends on the distribution function at the same time and at its values at earlier moments as well). This has been studied in detail by various groups, e.g. [8, 35, 7], and generalized Balescu integrals have been derived by Haug and Ell [36] and others [13, 3].

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[8] For simplicity, we also leave out the exchange contribution and sum over the different spin projections.
[9] Note that in semiconductors the Coulomb interaction is screened by the lattice. The reduction is given by the square of the background dielectric function ε_b which is typically in the range of $5\ldots20$.
[10] This means the Bogolyubov hypothesis, $\tau_{cor} \ll t_{rel}$, fails, see [3, 8].

A second development to solve this time-scale problem started in the frame of quantum field theory where the basic quantities are two-time correlation functions. Their equations of motion are the Keldysh-Kadanoff-Baym equations (KBE), see below. These equations have been successfully solved for semiconductors and plasmas [7, 37, 38, 31] and led to substantially longer relaxation times, in reasonable agreement with the experiments. Moreover, these non-Markovian quantum kinetic equations have solved two other problems: they correctly take into account correlation effects in the dynamics. Thus instead of the conservation of kinetic energy they conserve the total energy (kinetic plus interaction energy) of the plasma [8] and furthermore, the relaxation terminates not in the equilibrium momentum distribution of an ideal Fermi or Bose gas but in a correlated Wigner distribution function which is consistent with the thermodynamic properties of a nonideal plasma. Finally, we mention that also the gradual build-up of screening could be directly computed [39, 3] and was even directly demonstrated in experiments [40].

RELATIVISTIC FIELD THEORY FOR QUANTUM PLASMAS

As was discussed above, the method of second quantization has a number of advantages for treating quantum many-body systems in nonequilibrium. This concerns not only the selfconsistent inclusion of the spin properties of the microparticles but also the easy incorporation of non-Markovian and memory effects. Even more importantly, using second quantization, it is fairly straightforward to extend the kinetic theory to the relativistic regime and to include the interaction of the plasma with an electromagnetic field of arbitrary strength and temporal structure. Here we briefly sketch the main ideas, outline recent progress and give further references.

The second quantization description of a system of charged particles[11] interacting with an electromagnetic field is rigorously given by the coupled Dirac and Maxwell fields $\hat{\Psi}(1)$, $\hat{A}^{\mu}(1)$, where $\hat{\Psi}$ is a four-spinor[12] of spin $1/2$-particles, the four-vector $\hat{A}^{\mu} = (c\hat{\phi}, \hat{\mathbf{A}})$ comprises the quantized scalar and vector field, and $1 = x_1^{\mu} = (ct, \mathbf{r})$, $\mu = 0,1,2,3$. The Dirac field operator obeys the usual fermionic anti-commutation relations, Eq. (12), where as the photon field satisfies bosonic commutation relations, e.g. [3], guaranteeing the correct spin statistics of the resulting theory. The coupled equations of motion for the two fields are Dirac's and Maxwell's equations and have been discussed already by J. Schwinger [41] in the early 1950s (for a historic overview

[11] To simplify the notation we consider a one-component plasma.

[12] Apart from the spinor structure $\hat{\Psi}(1)$, is fully analogous to the annhilation operator a_k discussed above, the main difference being that the former acts on N-particle states in continuous configuration space and annihilates a particle at space point $\mathbf{r}$. Similarly, the creation operator $a_k^{\dagger}$ has its correspondence in the adjoint spinor $\hat{\bar{\Psi}}$.

see the article of P.C. Martin [42]),

$$\left\{\gamma^\mu\left(\hat{p}_\mu - \frac{e}{c}\hat{A}_\mu\right) - mc\right\}\hat{\Psi} = 0, \tag{18}$$

$$\mathscr{D}^\mu_\nu(1)\hat{A}^\nu(1) = \frac{4\pi}{c}\left\{\hat{j}^\mu(1) + j^{\mu\,ext}(1)\right\}, \tag{19}$$

$$\hat{j}^\mu = ec\hat{\overline{\Psi}}\gamma_\mu\hat{\Psi}, \tag{20}$$

where we also defined the 4-momentum, $\hat{p}^\mu = i\hbar\partial^\mu = i\hbar(\frac{1}{c}\partial_t, -\nabla)$, and the current density operator $\hat{j}^\mu$, which guarantees the charge conservation law, $\partial_\mu \hat{j}^\mu = 0$, and positiveness of the number density $\langle\hat{n}\rangle = \langle\hat{j}^0\rangle/e \geq 0$. Note that the explicit form of the Maxwell operator is gauge-dependent[13]. Furthermore, γ_μ comprises the four Dirac matrices[14] and we use Einstein's convention of summation over repeated matrix indices.

The system (18,19) has the structure of a relativistic single-particle problem in an electromagnetic field, yet the field operator structure generates the full N-particle problem. To compute observables a suitable statistical averaging has to be performed using, as a weight, the density operator $\rho_{1\ldots N}$ of the system, see above. The expectation value of the photon field is then $A(1) \equiv \langle\hat{A}(1)\rangle = \mathrm{Tr}\{\rho\hat{A}(1)\}$ which is related to the classical value of the vector potential (not an operator), whereas the Dirac field has usually a zero mean value, $\langle\hat{\Psi}(1)\rangle = 0$. Thus, the central quantities of the theory are the second moments (fluctuations) which are the nonequilibrium Greens functions of photons, $D^{\mu\nu}$, and fermions G,

$$D^{\mu\nu}(1,1') \sim -i\left\{\langle\hat{A}^\mu(1)\hat{A}^\nu(1')\rangle - A^\mu(1)A^\nu(1')\right\}, \tag{21}$$

$$G(1,1') \sim -i\langle\hat{\Psi}(1)\hat{\overline{\Psi}}(1')\rangle, \tag{22}$$

where we skipped the constant prefactors. Both Greens functions have a four by four matrix structure due to the dimensionality of the 4-vector potential and the 4-spinor structure of Ψ, respectively. The equal-time elements of the Greens functions are directly related to the one-particle density operators, cf. Eq. (13), and thus give access to all relevant observables of the quantum system. Besides, the availability of the time-off-diagonal elements provides important additional information that is related to spectral properties and correlations. Notice that the time arguments of the Greens functions are

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[13] For example in the Feynman gauge, the operator is given by a diagonal matrix, with the standard D'Alambert operator on the diagonal, $\mathscr{D}^\mu_\nu = \partial^\gamma\partial_\gamma\delta^\mu_\nu$, and $\partial_\mu A^\mu = 0$, cf. [3].

[14] Recall the four-vector of the Dirac matrices:

$$\gamma^0 = \begin{pmatrix} \vec{0} & \mathbf{1} \\ \mathbf{1} & \vec{0} \end{pmatrix}, \quad \vec{\gamma} = \begin{pmatrix} \vec{0} & -\vec{\sigma} \\ \vec{\sigma} & \vec{0} \end{pmatrix}, \quad \text{where } \vec{\sigma} = (\sigma_x, \sigma_y, \sigma_z) \text{ are the Pauli matrices}$$

$$\sigma_x = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \quad \sigma_y = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \quad \sigma_z = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \quad \text{and} \quad \mathbf{1} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}.$$

quite complicated, but we skip these details as they are not important for the final kinetic equations[15].

The equations of motion for the two Greens functions are derived in a similar manner as for the reduced density operators, using the Heisenberg equation of motion (16), but now taking into account the two-time dependencies. The result is[16]

$$\left\{\gamma_\mu\left(p_1^\mu - \frac{e}{c}A^\mu(1)\right) - mc\right\}G(1,1') - \int d2\,\Sigma(1,2)G(2,1') = \delta(1-1'), \quad (23)$$

$$\mathscr{D}_\nu^\mu(1)D^{\nu\lambda}(1,1') - \int d2\,\Pi_\nu^\mu(1,2)D^{\nu\lambda}(2,1') = \delta^{\mu\lambda}(1-1'), \quad (24)$$

$$\mathscr{D}_\nu^\mu(1)A^\nu(1) = \frac{4\pi}{c}j^{\mu\,ext}(1), \quad (25)$$

which provides a closed system of coupled equations. The first terms on the left hand sides of Eqs. (23,24) have the structure of the usual Dirac and Maxwell's equations, as equations (18,19) for the field operators, except for the appearance of the ensemble average A^μ. This classical field acts as source for the particle dynamics and obeys its own Maxwell's equations (25) driven by the external currents alone (the ensemble average of the induced current vanishes). Many-particle effects enter the two Greens functions equations via the integral terms which have the form of an additionl energy contribution. In fact, the central quantities entering these expressions are the "selfenergies" of the fermions, Σ, and of the photons, Π, which are defined as follows

$$\Sigma(1,2) = -iec\hbar\gamma_\mu\int d3d4\,G(1,3)\frac{\delta G^{-1}(3,2)}{\delta A^\nu(4)}\frac{4\pi}{c}D^{\nu\mu}(4,1), \quad (26)$$

$$\Pi_\nu^\mu(1,2) = \frac{4\pi}{c}\frac{\delta j^\mu(1)}{\delta A^\nu(2)}, \quad (27)$$

involving the inverse Greens function G^{-1} and the functional derivative of the mean induced current. The evaluation of these quantities is the main task in developing a quantum kinetic theory for fermions coupled to photons and in deriving approximations. Here one takes advantage of the powerful Feynman diagram technique which was generalized to nonequilibrium by L.V. Keldysh and G. Baym and L. Kadanoff[17] [43, 44]. A version of the above equations suitable for further theoretical and computational analysis is given by the Keldysh-Kadanoff-Baym equations which will be discussed below for the non-relativistic limit.

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[15] The times run over the Schwinger-Keldysh time contour, and a contour-ordering of the field operators is implied, see e.g. Ref. [3].

[16] Again the time arguments and integrals run along the Schwinger-Keldysh contour. Also, in the equation for G we omitted a contribution arising from initial field-matter correlations, for details see Ref. [3].

[17] For an overview on the historial developments of the powerful nonequilibrium Greens functions technique in the U.S. and in the Soviet Union in the early 1960s, see the recent reviews by G. Baym [45] and L.V. Keldysh [46].

KELDYSH-KADANOFF-BAYM EQUATIONS FOR FERMIONS, PLASMONS AND PHOTONS

The quantum kinetic equations in the non-relativistic limit are derived from the relativistic theory in standard manner. Then the Dirac equation for the fermion field goes over into the Pauli equation for the field operator $\hat{\psi}_s$ that is a two-spinor and decouples from the antiparticles (s now denotes the particle type) [3]

$$i\hbar\frac{\partial\hat{\psi}_s}{\partial t} = \left[\frac{1}{2m_s}\left(\hat{\mathbf{p}}-\frac{e_s}{c}\hat{\mathbf{A}}\right)^2+e_s\hat{\phi}-\vec{\mu}_s\cdot\nabla\times\hat{\mathbf{A}}\right]\hat{\psi}_s, \tag{28}$$

where $\hat{\mathbf{A}}$ and $\hat{\phi}$ are the field operators of the electromagnetic and scalar component of the electromagnetic field and $\vec{\mu}_s=\frac{e_s\hbar}{2m_sc}\vec{\sigma}$ denotes the magnetic moment due to the spin of the fermions containing the vector of the Pauli matrices. The following procedure is analogous to the relativistic case. One introduces single-particle Greens functions as fluctuations of ensemble averages of two field operators, but now the scalar and transverse part of the electromagnetic field are separated[18] into a scalar "plasmon" Green function D_{00} and a 3×3 matrix "photon" function D_{ik}, where $i,j=1,2,3$,

$$D_{ik}(1,1') \sim -i\left\{\langle\hat{A}_i(1)\hat{A}_k(1')\rangle-A_i(1)A_k(1')\right\}, \tag{29}$$

$$D_{00}(1,1') \sim V(1-1')-i\left\{\langle\hat{\phi}(1)\hat{\phi}(1')\rangle-\phi(1)\phi(1')\right\}, \tag{30}$$

$$G_s(1,1') \sim -i\langle\hat{\psi}_s(1)\hat{\psi}_s^{\dagger}(1')\rangle, \tag{31}$$

where again constant factors have been dropped[19]. Note that the leading term in the plasmon function is the Coulomb potential V. Furthermore, the fermion function has now the structure of a 2×2 spinor matrix.

As in the relativistic case, we obtain coupled equations of motion for the Greens functions, but now of fermion, plasmons and photons which involve the expectation

[18] We use this decomposition to illustrate how the Coulomb interaction among charged particles emerges from the interaction with the photon field. Strictly speaking, this separation is only possible in an isotropic system. In the general case one has to work with a single 4×4 photon Green function.

[19] Again the complex time structure is not explicitly indicated and time ordering on the Schwinger-Keldysh contour is implied.

values of the electromagnetic field (classical fields) $\phi = \langle\hat{\phi}\rangle$ and $\mathbf{A} = \langle\hat{\mathbf{A}}\rangle$,

$$\begin{aligned}
\Delta_1 d^{\gtrless}(11') &= \int_{t_0}^{t_1} d\bar{1}\left[\pi^{>}(1\bar{1}) - \pi^{<}(1\bar{1})\right] d^{\gtrless}(\bar{1}1') \\
&- \int_{t_0}^{t_1'} d\bar{1}\pi^{\gtrless}(1\bar{1})\left[d^{>}(\bar{1}1') - d^{<}(\bar{1}1')\right], \quad (32) \\
\Box_1 d_{ij}^{\gtrless}(11') &= \int_{t_0}^{t_1} d\bar{1}\left[\pi_{ik}^{>}(1\bar{1}) - \pi_{ik}^{<}(1\bar{1})\right] d_{kj}^{\gtrless}(\bar{1}1') \\
&- \int_{t_0}^{t_1'} d\bar{1}\pi_{ik}^{\gtrless}(1\bar{1})\left[d_{kj}^{>}(\bar{1}1') - d_{kj}^{<}(\bar{1}1')\right], \quad (33) \\
\left\{i\hbar\frac{\partial}{\partial t_1} - \frac{1}{2m_a}\left(\mathbf{p}_1 - \frac{e_a}{c}\mathbf{A}_1\right)^2\right. &- \left. e_a\phi_1 - \vec{\mu}_a\nabla\times\mathbf{A}_1\right\} g_a^{\gtrless}(11') = \\
\int_{t_0}^{t_1} d\bar{1}\left[\sigma_{ab}^{>}(1\bar{1}) - \sigma_{ab}^{<}(1\bar{1})\right] g_b^{\gtrless}(\bar{1}1') &- \int_{t_0}^{t_1'} d\bar{1}\sigma_{ab}^{\gtrless}(1\bar{1})\left[g_b^{>}(\bar{1}1') - g_b^{<}(\bar{1}1')\right], \quad (34)
\end{aligned}$$

where contributions from initial correlations have been omitted, e.g. [3] and ϕ and $\mathbf{A}$ obey the ensemble averaged Maxwell's equations (contributions from the induced charge and current densities arising from the fermion field operators vanish since the latter have a zero expectation value),

$$\begin{aligned}
\Delta_1\phi(1) &= -4\pi\rho^{ext}(1), \quad (35) \\
\Box_1\mathbf{A}(1) &= \frac{4\pi}{c}\mathbf{j}^{ext}(1). \quad (36)
\end{aligned}$$

The equations for the Greens functions have to be solved together with the adjoint equations assuring the symmetry with respect to the two arguments of the functions. As in the relativistic case, the equations of motion contain a single-particle contribution (l.h.s. of the equations) and correlation and exchange corrections arising from the integral terms. The latter contain additional (complex) energy contributions expressed by the selfenergies $\pi, \pi_{ik}, \sigma_{ab}$ of plasmons, photons and fermions, respectively. Note that the superscripts $>$ and $<$ of the Green functions and selfenergies distinguish two different correlation functions that arise naturally from the two possible orderings of the field operators in the definitions (30, 29, 31): the function $g^{<} \sim \langle\hat{\psi}^{\dagger}\hat{\psi}\rangle$ is related to the density of particles whereas the function $g^{>} \sim \langle\hat{\psi}\hat{\psi}^{\dagger}\rangle$ relates to the density of hole excitations (and similarly for plasmons and photons)[20].

The above system are the Keldysh-Kadanoff-Baym Equations (KBE) for fermions, plasmons and photons and provide the most general starting point for the development of a selfconsistent non-relativistic quantum kinetic theory for plasmas. These equations can either be used to analytically derive simpler quantum kinetic euqations for the Wigner distribution (or density matrix), see below, or they can be solved numerically directly. This has become possible in recent years for numerous special cases, although a full

[20] The two functions are, in general, independent because the two field operators do not commute, cf. the (anti-)commuation rules (12).

selfconsistent solution of the entire system (32,33,34) has not yet been achieved. The numerical solutions mostly considered the particle dynamics assuming a stationary (relaxed) photon and plasmon field. Among them we mention applications to nuclear matter, e.g. [49, 50], electron-hole plasmas in semiconductors, e.g. [7, 37, 38, 39], dense quantum plasmas [51, 52], the electron gas in metals, e.g. [31] and even the quark-gluon plasma, e.g. [53], and references therein. While most of these applications did concentrate on macroscopic systems that are nearly spatially homogeneous, in recent years new applications to strongly inhomogeneous systems, including electrons in quantum dots [54, 55, 56] and electrons in atoms and molecules have become possible, e.g. [57, 58, 59, 60]. This broad variety of treatable systems underlines the strength and generality of this field-theoretical approach.

QUANTUM KINETIC EQUATIONS

Besides direct solutions of the two-time KBE this theory is perfectly suited for a systematic derivation of more approximate quantum kinetic equations. Most importantly, it allows one to derive equations for the Wigner distribution function that in a selfconsistent manner include correlations, quantum and spin effects, as well as the effect of an arbitrary electromagnetic field. To come to a closed equation for the single-time quantities, such as the Wigner distribution or the single-particle density matrix the two-time correlation functions $g^{\gtrless}$ have to be eliminated.

We illustrate the procedure for the simplest case. We assume that the photons and plasmons have already reached equilibrium, so their distributions are stationary. The consequence is that the Coulomb interaction between the fermions is screened and the electromagnetic field is purley classical, given by the ensemble averaged quantities ϕ and $\mathbf{A}$. We further assume that the plasma is not in a magnetic field, so the term with the magnetic moments in Eq. (34) vanishes. Next, we restrict ourselves to the equation (34) for $g^{<}$ and specify the time arguments to be equal, $t_1 = t_1' = t$. Thus we obtain the equation for the single-particle density matrix, $f_1(\mathbf{r}_1, s_1, \mathbf{r}_1', s_1', t) = \mp i g^{<}(1,1')|_{t_1=t_1'=t}$, the Fourier transform of which (with respect to the relative coordinate $\mathbf{r} = \mathbf{r} - \mathbf{r}'$) yields the Wigner distribution[21] $f(\mathbf{R}, \mathbf{p}, t)$ that, in general, also depends on the macroscopic (center of mass) coordinate, $\mathbf{R} = (\mathbf{r} + \mathbf{r}')/2$.

We illustrate the further derivations for a spatially homogeneous fully ionized electron-ion plasma ($a = e, i$) in a strong time-dependent laser field[22] [61, 14] where we choose the field in the form $\phi^{ext} = 0$ and $\mathbf{A}(t) = -c\int_{-\infty}^{t} d\bar{t}\,\mathbf{E}(\bar{t})$, where $E(t)$ denotes the electric field strength. To avoid ambiguities arising from the gauge freedom, in the resulting kinetic equation, we use an explicitly gauge-invariant form, resulting in a

[21] From now on we will not explicitly write the spin variables. Their inclusion would be important in an external magnetic field, and this generalization presents no problem.

[22] For extension to a weakly inhomogeneous plasma, see Ref. [67].

Greens function depending on the total momentum k which follows according to

$$g_a(\mathbf{k},\omega;t) = \int d\tau d\mathbf{r} \exp\left[i\omega\tau - \frac{i}{\hbar}\mathbf{r}\left(\mathbf{k} + \frac{e_a}{c}\int_{t_1'}^{t_1}\frac{d\bar{t}}{\tau}\mathbf{A}(\bar{t})\right)\right] g_a(\mathbf{r},\tau;t), \tag{37}$$

where $r = r_1 - r_1'$, $\tau = t_1 - t_1'$ and $t = (t_1 + t_1')/2$. The resulting quantum kinetic equations are given by [61]

$$\frac{\partial}{\partial t} f_a(\mathbf{k}_a,t) + e_a\mathbf{E}(t)\cdot\nabla_{\mathbf{k}} f_a(\mathbf{k}_a,t) = -2\mathrm{Re}\int_{t_0}^{t} d\bar{t}\left\{ \sigma_a^{>} g_a^{<} - \sigma_a^{<} g_a^{>}\right\} = I_a(\mathbf{k}_a,t), \tag{38}$$

where the full arguments of the functions on the r.h.s. have a non-trivial time and momentum dependence,

$$\begin{aligned} \sigma_a^{\gtrless} g_a^{\lessgtr} &\equiv \sigma_a^{\gtrless}\left[\mathbf{k}_a + \mathbf{K}_a^A(t,\bar{t});t,\bar{t}\right] g_a^{\lessgtr}\left[\mathbf{k}_a + \mathbf{K}_a^A(t,\bar{t});\bar{t},t\right], \qquad (39)\\ \text{with}\quad \mathbf{K}_a^A(t,t') &\equiv \frac{e_a}{c}\int_{t'}^{t} dt'' \frac{\mathbf{A}(t) - \mathbf{A}(t'')}{t - t'}. \end{aligned}$$

What remains to do is to transform the collision integrals I_a. To this end, the Greens functions and selfenergies which are functionals of the single-particle density matrix, i.e. $g^{\gtrless} = g^{\gtrless}[f_1]$ and $\sigma^{\gtrless} = \sigma^{\gtrless}[g^{\gtrless}]$ should be eliminated, so the collision integrals are expressed entirely in terms of f_a. This is the so-called reconstruction problem which was solved by Baym and Kadanoff for the case of systems close to thermodynamic equilibrium [44] and extended to nonequilibrium by Lipavski and co-workers [62]. Finally, the latter result was extended to a gauge-invariant form suitable for laser plasmas in Refs. [61, 14],

$$\begin{aligned} \pm g_a^{\gtrless}(\mathbf{k};t_1,t_1') &= g_a^R(\mathbf{k};t_1,t_1')\, f_a^{\gtrless}\left[\mathbf{k} - \mathbf{K}_a^A(t',t);t_1'\right]\\ &- f_a^{\gtrless}\left[\mathbf{k} - \mathbf{K}_a^A(t,t');t_1\right] g_a^A(\mathbf{k};t_1,t_1'), \end{aligned} \tag{40}$$

where the upper (lower) sign refers to $g^{>}$ ($g^{<}$), and $f^{<} \equiv f$ and $f^{>} \equiv 1 - f$, and g_a^R is the retarded propagator that can be expressed by the correlation functions according to[23] $g_a^R(t,t') = \Theta(t-t')[g_a^{>}(t,t') - g_a^{>}(t,t')]$, see also Eq. (44).

The results obtained so far are valid for any choice of selfenergy (i.e. any approximation for the pair correlations). Below we will consider two examples: the static second Born approximation (statically screened Boltzmann equation or Landau equation) and the dynamicallly screened Born approximation (nonequilibrium RPA, which is a generalization of the Balescu-Lenard collision integral, Eq. (17), discussed above).

COLLISION INTEGRAL FOR A LASER PLASMA IN RPA

We first consider the collision integral that includes dynamical screening. The Greens functions technique provides an extremely short and elegant way to this approximation.

[23] Note that, for $t_1 \geq t_1'$ only the first term in (40) contributes and for $t_1 < t_1'$ only the second.

In fact, the corresponding selfenergies of particles and plasmons are given by [44]

$$\sigma_a^{\gtrless}(1,1') = i\hbar\, g_a^{\gtrless}(1,1')\, V_{aa}^{s\gtrless}(1,1'), \tag{41}$$

$$\Pi_{ab}^{\gtrless}(1,1') = -i\hbar\, g_a^{\gtrless}(1,1')\, g_b^{\lessgtr}(1,1'). \tag{42}$$

where $V_{aa}^s(1,1')$ denotes the nonequilibrium dynamically screened potential of particles of species "a" (note the dependence on two times), and the correlation function $V_{aa}^{s\gtrless}$ is nothing but the plasmon Greens function $d^{s\gtrless}$ introduced above[24]. Its equation of motion was already formulated above and involves the plasmon selfenergies,

$$\Delta V_{ab}^{s\gtrless}(1,1') = \sum_c \int_{-\infty}^{\infty} d\bar{1}\, \left[\Pi_{ac}^R(1,\bar{1}) V_{cb}^{s\gtrless}(\bar{1},1') - \Pi_{ac}^{\gtrless}(1,\bar{1})\, V_{cb}^{sA}(\bar{1},1')\right], \tag{43}$$

where we introduced the short notation for advanced ("A") and retarded ("R") propagators,

$$F^{R/A}(1,1') = \pm\Theta[\pm(t_1 - t_1')]\left\{F^{>}(1,1') - F^{<}(1,1')\right\}. \tag{44}$$

Despite its simplicity, the system (38–43) [14] describes remarkably complex physical processes: the evolution in space and time of charged carriers interacting via the full dynamic Coulomb potential which in turn evolves selfconsistently (screening build up) and may have nonequilibrium modes (including instabilities and nonlinear phenomena). Furthermore, the dynamics is influenced by the transverse electromagnetic field $\mathbf{A}$ which contains external fields (e.g. a laser field) and induced contributions and obeys Maxwell's equations.

We can now apply the reconstruction ansatz (40) for the Greens functions and obtain for the plasmon selfenergies (longitudinal polarization functions)

$$\begin{aligned}\Pi_{aa}^R(\mathbf{q};t,t') &= -\frac{i}{\hbar}\Theta(t-t')\, e^{\frac{i}{\hbar}\mathbf{q}\mathbf{R}_a(t,t')} \int \frac{d^3k}{(2\pi\hbar)^3}\, e^{-\frac{i}{\hbar}\left(\varepsilon_{\mathbf{k}+\mathbf{q}}^a - \varepsilon_{\mathbf{k}}^a\right)(t-t')} \\ &\times \left\{f_a\left[\mathbf{k}+\mathbf{Q}_a(t,t');t'\right] - f_a\left[\mathbf{k}+\mathbf{q}+\mathbf{Q}_a(t,t');t'\right]\right\},\end{aligned} \tag{45}$$

where we used for $g^{R/A}$ in Eq. (40) the propagators of a free particle with energy ε^a in an electromagnetic field. Further, $\mathbf{Q}_a$ and $\mathbf{R}_a$ are, respectively, the momentum gain and displacement of a free particle in a field $E(t)$ during the time interval $[t',t]$ given by

$$\mathbf{Q}_a(t,t') = -e_a \int_{t'}^{t} dt''\, \mathbf{E}(t''), \quad \mathbf{R}_a(t,t') = \frac{e_a}{m_a}\int_{t'}^{t} d\tilde{t} \int_{t'}^{\tilde{t}} d\bar{t}\, \mathbf{E}(\bar{t}). \tag{46}$$

In the absence of an external field, $E = 0$, also Q and R vanish, and the function Π^R yields just the Lindhard polarization function[25]. Obviously, here we have obtained a

[24] The notation V^s is more conventional in plasma physics.

[25] To verify this one just has to perform a Fourier transform with respect to $t - t'$ that yields a frequency dependent function where the theta function takes care of causality.

far-reaching generalization. In the presence of an external field, the plasmon spectrum is modified in two ways: first, by the field-dependent prefactor of the standard RPA-polarization and, second, by the field-dependent momentum arguments of the distribution functions. From this it is clear that the dielectric function will be influenced by the field in a similar way. Indeed, since [3] $\varepsilon^R(q,t,t') = \delta(t-t') - \sum_a V_{aa}(q)\Pi_a^R(q,t.t')$, the field dependence of Π^R leads directly to a modification of the plasmon spectrum of a plasma in a strong laser field $E(t)$, in particular to the appearance of plasmon side bands due to plasmon-photon coupling [14].

The collision integrals of the electrons and ions involving the two-time dynamically screened potential V^s is rather complicated and will not be written here. For details and further discussion we refer to [14]. So far, no numerical solutions of this equations were possible. In the limiting case of zero laser field the evolution of laser generated electrons in a semiconductor under the influence of dynamical screening build up was computed by Banyai et al. [39].

COLLISION INTEGRAL FOR A LASER PLASMA IN BORN APPROXIMATION

We now neglect the plasmon dynamics and assume that a stationary spectum had time to emerge. Furthermore, we neglect dynamics effects and consider the static limit, $\omega \to 0$ giving rise to a statically screened potential (Debye potential) between the electrons and ions, $V_{ab}^{s,R/A}(q,t_1,t_1') \to V_{ab}^{st}(q,t_1)\delta(t_1-t_1')$ where we allowed for a time dependence of the screening length (via the nonequilibrium distribution functions). As a result, the collision integral in the quantum kinetic equation (38) becomes [61]

$$\begin{aligned} I_a(\mathbf{k}_a,t) &= 2\sum_b \int \frac{d\mathbf{k}_b d\bar{\mathbf{k}}_a d\bar{\mathbf{k}}_b}{(2\pi\hbar)^9} |V_{ab}^{st}(\mathbf{k}_a-\bar{\mathbf{k}}_a)|^2 (2\pi\hbar)^3 \delta(\mathbf{k}_a+\mathbf{k}_b-\bar{\mathbf{k}}_a-\bar{\mathbf{k}}_b) \\ &\times \int_{t_0}^{t} d\bar{t}\, \cos\left\{\frac{1}{\hbar}\left[(\varepsilon_{ab}-\bar{\varepsilon}_{ab})(t-\bar{t}) - (\mathbf{k}_a-\bar{\mathbf{k}}_a)\mathbf{R}_{ab}(t,\bar{t})\right]\right\} \\ &\times \left\{\bar{f}_a\bar{f}_b\left[1-f_a\right]\left[1-f_b\right] - f_a f_b\left[1-\bar{f}_a\right]\left[1-\bar{f}_b\right]\right\}\Big|_{\bar{t}}, \end{aligned} \tag{47}$$

where we denoted $\varepsilon_{ab} \equiv \varepsilon_a + \varepsilon_b$, $\varepsilon_a \equiv p_a^2/2m_a$, $f_a \equiv f(K_a,\bar{t})$, $\bar{f}_a \equiv f(\bar{K}_a,\bar{t})$, $\mathbf{K}_a \equiv \mathbf{k}_a + \mathbf{Q}_a$ and $\mathbf{R}_{ab} \equiv \mathbf{R}_a - \mathbf{R}_b$. Although plasmon and screening dynamics are no longer included, Eq. (38) with the collision term (47) still contains important physics reflecting the influence of the electromagnetic field: (I) field-induced change of the arguments of the distribution functions[26]; (II) modification of the energy balance in electron-ion scattering[27] [argument of the cosine in Eq. (47)]; (III) nonlinear (exponential) dependence of the collision integral on the field strength – this leads to the generation of higher field harmonics in the scattering processes. Furthermore, it gives rise to scattering processes which involve emission (absorption) of photons, i.e. (inverse) bremsstrahlung. Indeed, it

[26] i.e. time-dependent generalization of the so-called intra-collisional field effect

[27] For two electrons (or two ions) the field terms cancel, $R_{aa} = 0$, because both particles oscillate in phase.

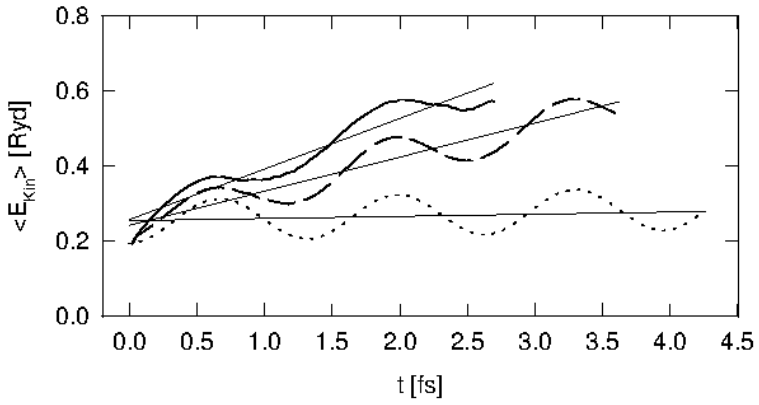

FIGURE 1. Electron heating of a fully ionized hydrogen plasma due to electron-ion collisions in a strong laser field (inverse bremsstrahlung heating) for different densities: $n_e = 10^{22} cm^{-3}$ (full line), $n_e = 10^{23} cm^{-3}$ (dashes) and $n_e = 10^{24} cm^{-3}$ (dots). Laser amplitude is $E_0 = 10^8$V/cm. Computed by solving the quantum kinetic equation (38) with the collision integral (47). (The straight lines are a guide for the eye.) From Ref. [63].

is straightforward to show [61] that, for a monochromatic electric field, $E(t) = E_0 \cos\Omega t$, transport quantities computed from the integral (47) will contain contributions proportional to $J_n^2(z)\delta[\varepsilon_{ab} - \bar{\varepsilon}_{ab} + \mathbf{q}\mathbf{w}_{ab}(t) - n\hbar\Omega]$, $(-\infty < n < \infty)$, where the amplitude of an n-photon process is given by the Bessel functions J_n. The argument z of J_n is determined by the field strength and frequency, $z = \mathbf{q}[\mathbf{v}_a^0 - \mathbf{v}_b^0]/\hbar\Omega$, where $\mathbf{q} = \mathbf{k}_a - \bar{\mathbf{k}}_a$, $v_a^0 = e_a E_0/m_a\Omega$ and $\mathbf{w}_{ab} = [\mathbf{v}_a^0 - \mathbf{v}_b^0]\sin\Omega t$.

The quantum kinetic equation (38) with the collision integral (47) has been solved numerically for a many parameters, e.g. [63, 64]. An example is shown in Fig. 1. Here a fully ionized hydrogen plasma is exposed to a strong optical laser pulse. If collisions are neglected electrons and ions would oscillate in the field with the external frequency gaining and losing energy periodically and no net heating would occur. Only by including electron-ion collisions via the collision integral (47) correctly captures the collisional heating mechanism via inverse bremsstrahlung (photon absorption). The heating effect is strongly density dependent, and an optimum is observed near resonance, i.e. when the laser frequency is close to the electron plasma frequency. Interestingly, the distribution function is strongly modified by the laser field and the collisions with the ions, see Fig. 2. In particular, inverse bremsstrahlung in e-i collisions is accompanied by photon absorption. This gives rise to equally spaced side peaks of the electron distribution that can be detected experimentally in a plasma of small geometrical size. In a bulk plasma, electron-electron scattering will tend to thermalize the system and to wash out the side peaks and the dominant effect is a broadening of the distribution [64], associated with the strong heating, as shown in Fig. 1. Details of the numerics and further results are presented in Ref. [64]. Besides direct numerical solution the presented quantum kinetic equation (38) is well suited for analytical investigation for special cases, see e.g. Refs. [65, 66, 63].

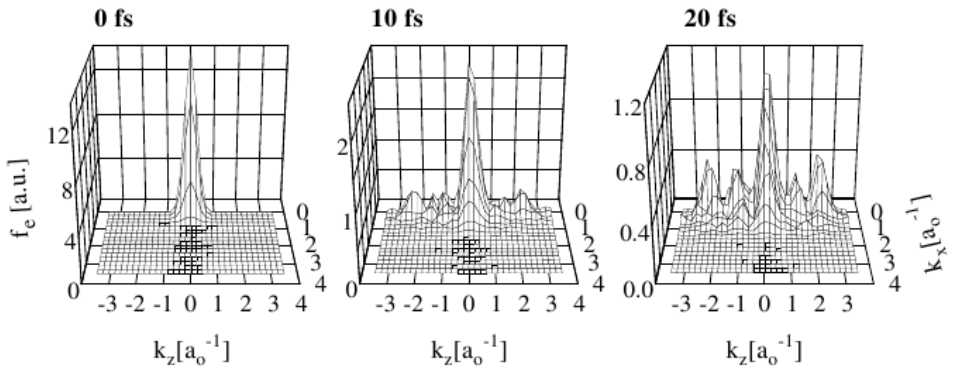

FIGURE 2. Time evolution of the electron momentum distribution in a strong monochromatic laser field (in z-direction) under the combined action of the field and electron-ion collisions after 0, 6 and 12 laser cycles, respectively. Notice the side peaks in the distribution which are due to photon absorption (inverse bremsstrahlung). From Ref. [63].

SUMMARY

In this paper, we have presented a brief overview on a kinetic theory approach to quantum plasmas. A kinetic description is crucial in many cases where nonequilibrium distributions are present and Coulomb collisions in the presence of a strong electromagnetic field are relevant. This is clearly the case in laser plasmas or in ion beam produced plasmas where collisions set the relevant time scales for thermalization and determine the heating rate of the plasma. These effects are of high current interest in experiments and theory-experiment comparisons are becoming possible, e.g. via Thomson scattering using free electron lasers, e.g. [68]. The increasing accuracy of these experiments will be a driving force for theory developments in the near future. Quantum kinetic theory has proven successful in this respect but further improved time resolution in the experiments and novel intensity and photon energy regimes are a constant challenge requiring new developments.

The goal of the present review was to outline two approaches suitable to develop a quantum kinetic theory. The first is based on the theory of reduced density operators (nonequilibrium BBGKY hierarchy) and the second on quantum field theory leading to the powerful concept of nonequilibrium Greens functions. Both approaches are practically equivalent and may serve as the starting point for future developments in quantum plasma theory. For further reading on the theory and computational treatment of quantum plasmas, see the two review chapters in the recent text book [34].

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