Comment on "Self-Diffusion in 2D Dusty-Plasma Liquids: Numerical-Simulation Results"

In a recent Letter [1], Hou, Piel, and Shukla (HPS) presented numerical results for the diffusion process in two-dimensional dusty-plasma liquids with Yukawa pair interaction (2DYL), $V(r) = Q^2 \exp(-r/\lambda)/r$, by solving a Langevin equation. The mean-squared displacement

$$u_r(t) = \langle |\vec{r}(t) - \vec{r}(t_0)|^2 \rangle \propto t^{1+\alpha} \quad (1)$$

is used to distinguish normal diffusion ($\alpha = 0$) from subdiffusion ($\alpha < 0$) and superdiffusion ($\alpha > 0$). HPS observed superdiffusion and reported a complicated nonmonotonic dependence of α on the potential stiffness $\kappa = a/\lambda$, where a is the Wigner-Seitz radius. Here we point out that the *behavior* $\alpha(\kappa)$ *is, in fact, regular and systematic*, whereas the observations of Ref. [1] resulted from a comparison of different system states.

As noted in Ref. [1], α depends on κ and the coupling parameter $\Gamma = Q^2/(ak_BT)$, and finding the dependence $\alpha(\kappa)$ requires one to compare states with the same physical coupling. This can be done by fixing, for all κ, the value $\Gamma^{\rm rel} = \Gamma/\Gamma_c$, where $\Gamma_c(\kappa)$ is the crystallization point which is well known for $\kappa \leq 3$ [2]. For larger κ, we obtain $\Gamma_c(\kappa = 3.5) = 2340$ and $\Gamma_c(4) = 4500$.

We have performed detailed investigations of the dependence of α on Γ and κ [3] and observed two different regimes: (i) For $\Gamma^{\rm rel} \lesssim \Gamma_0^{\rm rel} = 0.35$, α is monotonically decreasing with κ, at constant $\Gamma^{\rm rel}$. (ii) For $\Gamma^{\rm rel} \gtrsim \Gamma_0^{\rm rel}$, α increases monotonically with κ, at constant $\Gamma^{\rm rel}$. Around $\Gamma^{\rm rel} = \Gamma_0^{\rm rel}$, α is almost independent of κ. Figure 1 clearly confirms the monotonic κ dependence of α for three fixed values of $\Gamma^{\rm rel}$ corresponding to the parameters shown in Fig. 5 of Ref. [1].

HPS used a different coupling parameter, $\Gamma_{\rm eff}$, which yields an almost constant $\Gamma^{\rm rel}$, for $\kappa \leq 3$. However, for $\kappa > 3$ it corresponds to strongly varying $\Gamma^{\rm rel}$ and thus to different physical situations [4]; cf. the top part of Fig. 1. For example, their value $\Gamma_{\rm eff} = 100$ corresponds to $\Gamma^{\rm rel} = 0.76 > \Gamma_0^{\rm rel}$ for $\kappa = 3$ but to $\Gamma^{\rm rel} = 0.24 < \Gamma_0^{\rm rel}$ for $\kappa = 4$. This explains the nonmonotonicity of $\alpha(\kappa)$ reported by HPS [5].

Thus, we report a *systematic effect of screening* on superdiffusion in 2DYL based on numerical simulations. An increase of κ supports superdiffusion for $\Gamma^{\rm rel} \lesssim 0.6\,\Gamma_0^{\rm rel}$ and results in an increasing diffusion exponent in this range of the coupling. For higher couplings $\Gamma^{\rm rel} \gtrsim 0.6\,\Gamma_0^{\rm rel}$, a stronger screening has the inverse effect and reduces the strength of anomalous diffusion. In conclusion, we have presented numerical evidence for the existence of a monotonic dependence of anomalous diffusion on screening. An

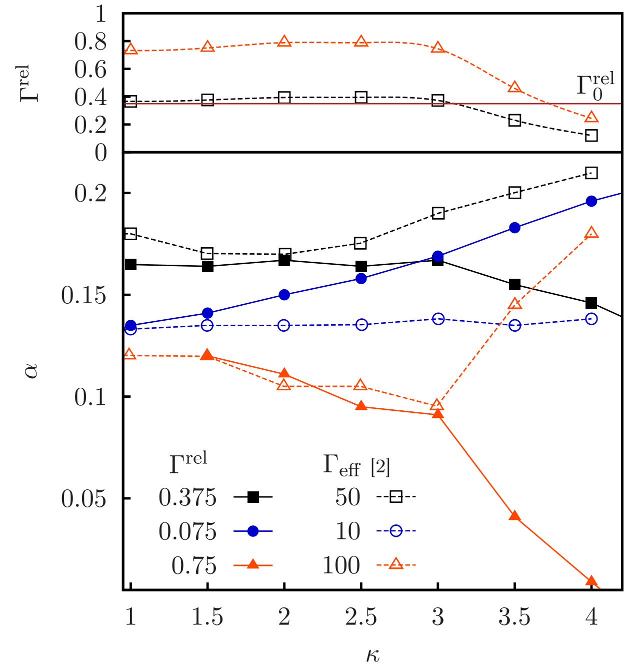

FIG. 1 (color online). Bottom: Exponent α vs κ for three fixed values of $\Gamma^{\rm rel}$ (full lines and symbols) and $\Gamma_{\rm eff}$ (dashed lines, open symbols, data from Ref. [1]). Top: $\Gamma^{\rm rel}(\kappa)$ corresponding to the values $\Gamma_{\rm eff}$ used in Ref. [1].

explanation of this behavior is beyond the present Comment and will be given elsewhere.

T. Ott and M. Bonitz
Institut für Theoretische Physik und Astrophysik
Christian-Albrechts-Universität zu Kiel
Leibnizstraße 15, 24098 Kiel, Germany

P. Hartmann
Research Institute for Solid State Physics and Optics
Hungarian Academy of Sciences
P.O. Box 49, H-1525 Budapest, Hungary

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[2] P. Hartmann *et al.*, Phys. Rev. E **72**, 026409 (2005).
[3] T. Ott, Diploma thesis, University of Kiel, 2008.
[4] $\Gamma_{\rm eff}$ is based on a formula given by G. J. Kalman *et al.*, Phys. Rev. Lett. **92**, 065001 (2004), which was obtained from a numerical fit in the range $\kappa = 0, \ldots, 3$.
[5] The different absolute values of our α, for $\kappa \leq 3$, compared to HPS are most likely due to a different prescription for extracting α from $u_r(t)$. We have always used a constant time interval $\omega_p t \in [100, 320]$ to read off the slope of $u_r(t)$. A friction coefficient $\nu/\omega_p = 0.001$ was used, as in Ref. [1].

