

# ON THE INTERPRETATION OF THE SCHWINGER AND LANDAU-ZENER EFFECTS

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## Abstract

The dynamical mechanism of the Schwinger effect of vacuum pair creation (VPC) under action of a strong external electric field is discussed. It is shown that the common interpretation of this process as the tunnelling between states with negative and positive energies is not universal and depends on the choice of electromagnetic field gauge. We discuss the mechanism of VPC based on the growth of virtual vacuum fluctuations (zitterbewegung). Finally, we consider the parallels between the Schwinger effect in QED and the Landau-Zener effect in solid state physics.

1. The original idea [1] about vacuum pair creation (VPC) under action of a stationary spatially homogeneous electric field was based on the model of particle tunnelling between the states with negative and positive energy in the Dirac model of physical vacuum. The authors turn your attention to the obvious picture that the electric field with the 4-potential

$$A^0(\mathbf{x}) = \phi(\mathbf{x}) = -Ex^3, \quad \mathbf{A}(\mathbf{x}, t) = 0 \quad (1)$$

leads to appearance of a potential barrier between the two regions with the dispersion laws (cf. Fig.1)

$$\omega_{pot}^{\pm}(\mathbf{p}, \mathbf{x}) = e\phi(\mathbf{x}) \pm \omega_0(\mathbf{p}), \quad (2)$$

where  $\omega_0 = \sqrt{m^2 + \mathbf{p}^2}$  (below we concentrate on the case of QED as the basic quantum field model),  $e$  is the electron charge with its sign,  $E$  is the electric field strength oriented along of the axes  $x^3$ ). This model results in the exponential factor  $\exp(-E_c/E)$  for the tunneling probability, with the critical field  $E_c = m^2/e$  (here and below we use the natural units  $\hbar = c = 1$ ). This result was confirmed and generalized by J. Schwinger [2] who obtained the well-known exact solution of the corresponding QED problem:

$$w = \frac{1}{4}(eE)^2 \exp \left\{ -\frac{\pi m^2}{|e|E} \right\}. \quad (3)$$

However, a constant electric field of the given geometry  $\mathbf{E}(0, 0, E)$  can be described also in the alternative Hamilton gauge

$$A^\mu = (0, 0, 0, A^3 = A(t) = -Et). \quad (4)$$

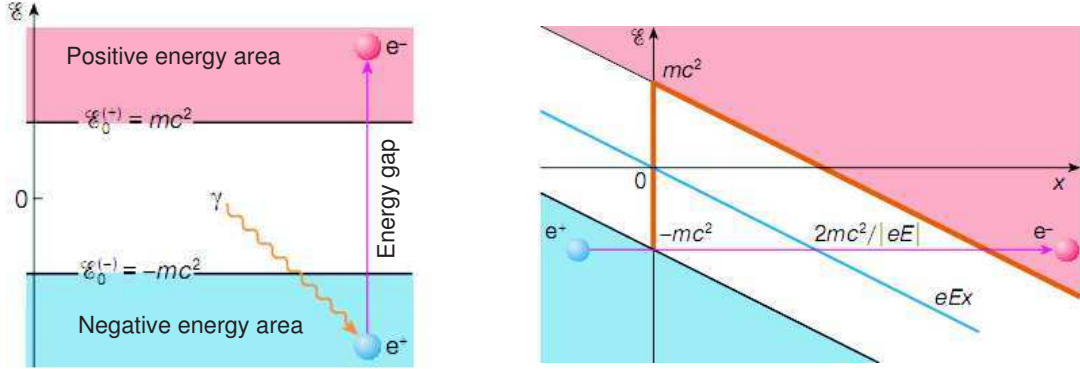


Figure 1: The barrier (tunneling) mechanism of the stationary Schwinger effect for an electron with momentum  $p = 0$ , left without and right with homogeneous stationary electric field applied.

Using the exact solution of the Dirac equation for the potential [3]

$$A_{ex} = E_0 b \left[ \tanh \left( \frac{t}{b} \right) + 1 \right], \quad E_{ex} = E_0 \cosh^{-2} \left( \frac{t}{b} \right), \quad (5)$$

leading to the field strength  $\mathbf{E}(0, 0, E)$ , one can obtain the Euler-Heisenberg-Sauter-Schwinger (EHSS) formula (3) also (at  $b \rightarrow \infty$  [4, 5]). This is an expected consequence of gauge invariance. But the potentials (4) and (5) do not admit a simple tunnelling interpretation of the VPC process. Indeed, the dispersion laws for electrons and positrons have now the following form:

$$\omega^\pm(\mathbf{p}, t) = \pm \sqrt{m^2 + (\mathbf{p} \pm e\mathbf{A}(t))^2} \quad (6)$$

and the energy gap has now no inclination needed for the barrier effect, cf. Fig.2, where we show instantaneous dispersion curves (6) for a fixed  $t \neq 0$ ). Time evolution is accompanied by uniform (or, in general, nonuniform) motion of the dispersion curves (6).

The absence of a tunnelling interpretation of the EHSS effect in the gauge (4) justifies the search for a more general mechanism allowing for universal interpretation of this effect independently of the chosen gauge. This becomes even more obvious if one considers more general electric fields which are still spatially homogeneous, but have an arbitrary time dependence (it is assumed that the applicability of the quasi-classical approximation [6] is not violated). In general, the EHSS formula (3) does not describe such situations (see, however, the work [7]).

This motivates the transition to a more general kinetic approach which is able to describe such strongly non-stationary and non-equilibrium situations, see e.g. [8]. Such a kinetic approach to the dynamical EHSS effect can be extended also to other classes of non-stationary fields (including gravitational and chromo-electromagnetic fields), which are essentially of non-potential nature. However, there are more pronounced examples of the dynamical EHSS effect with a non-potential mechanism,

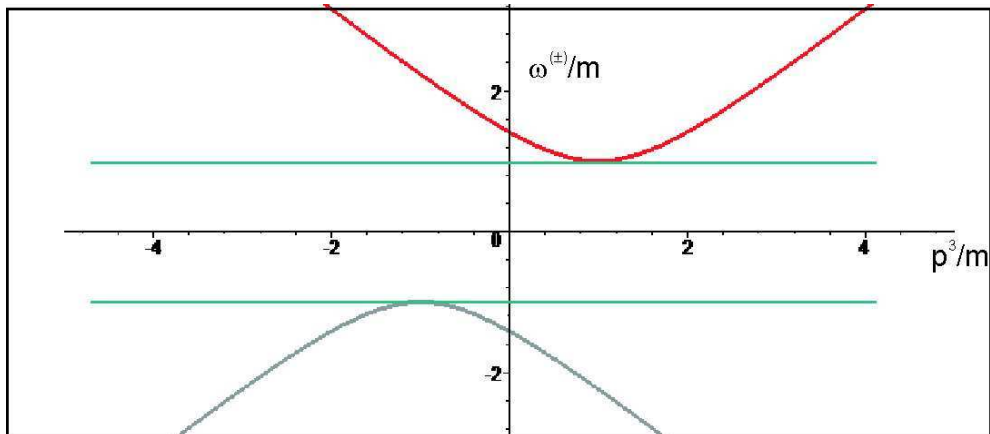


Figure 2: Dispersion laws (6) for  $eA(t) = m$ .

such as the so-called “inertial mechanism” corresponding to changing in time quasi-particle masses [9] and the dynamical Casimir effect [10, 11] stipulated by the time changing boundary conditions.

In this context, a hypothesis can be expressed (apparently, for the first time, this supposition was made in the work [9]), that VPC is the original response of the vacuum on any of its perturbations having quasi-classical or quantum nature. For example, one can expect that also a time-dependent magnetic field can lead to VPC (the influence of a constant strong magnetic field on VPC was considered in the works [5, 8, 16], see also [18]).

2. Apparently, all these manifestations of VPC have some common nature connected with vacuum fluctuations (“zitterbewegung”) under action of some (in general) time-dependent external causes. As a consequence, the life time of virtual vacuum oscillations increases which is also related to virtual particle-antiparticle pairs. Some part of such quasiparticle pairs are found on the instantaneous mass-shell [e.g., Eq. (6) in QED]. During the action of the external force, also pair annihilation processes take place, simultaneously with VPC giving rise to a dynamical quasiparticle plasma [5]. At the same time, a substantial fraction of the particle-antiparticle plasma will result from off-shell contributions. In particular, this leads to different states of in- and out-vacuums [5]. In the considered approximation, any stage of this process is coherent and the particle-antiparticle states are entangled.

The time-development of VPC processes can be investigated in detail. Some steps in this direction were done in the work [12] where, on the basis of numerical investigations, the just quantum processes was selected from the phase of quasi-classical acceleration. This stage can be considered as a multiphoton process with participation of the photon condensate (of the quasiclassical external field).

Thus, VPC processes have in their origin the fluctuation mechanism of virtual vacuum oscillations under the action of different external causes. Some visual confirmations of this hypothesis are demonstrated in Fig. 6 (“dog brush”) of the work [17] and Figs. 3 and 4 of the present paper. These figures show some details of the

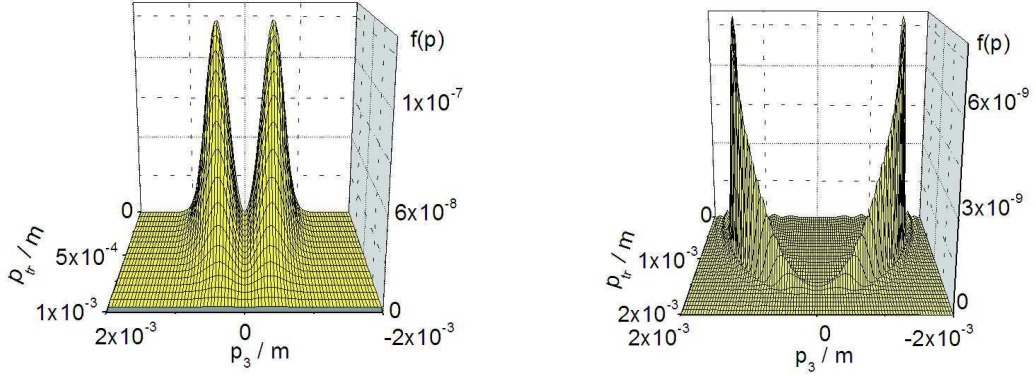


Figure 3: The momentum spectrum of the created particle-antiparticle pairs, left: for a pulsed field at a time long after the pulse, i.e.  $t \gg b = 2\pi/\nu$  and right: for a monochromatic field with  $\lambda = 1000$  nm after 5 field oscillations.

VPC process: initial smooth splashes of the distribution functions, followed by generation of high harmonics and development of instabilities with feasible transition to a chaotic regime. However, all these stages still require more research to gain a deeper understanding.

**3.** The analogy with solid state physics can play an important role for understanding of VPC in QFT. This is due to the simpler experimental observability of solid state phenomena in time-dependent fields. This provides the possibility of detailed comparison of theoretical models with experiments, including verification of different elementary processes. The results appear to be useful also for the understanding of the analogous elementary processes occurring in VPC in quantum field theory. Finally, the Schwinger mechanism of QFT has recently become of primary interest in solid state theory in connection with applications to graphene, e.g. [21].

The two-band model is the simplest and often adequate model for the considered situation in solid state systems such as insulators or semiconductors. Thereby the lower (valence) band corresponds to the states with negative energy in the Dirac picture of the physical vacuum whereas the upper (conduction) band is (at low temperature) vacant, in the absence of an external field, and corresponds to the states with positive energy in the Dirac model. The switch on of an external field is accompanied by perturbation of the electron states in the lower band and creation of electron-hole pairs. This process is connected with an overlap of the electron wave functions of both bands. Here we do not consider the standard optical excitation [13] where the photon energy is in (close to) resonance with the band gap but the analog of the discussed above tunneling process.

The first step in the direction of a nonperturbative kinetic description of such processes in a time dependent electric field was done in the work [14]. In the case of a stationary electric field this process is known as the Landau-Zener effect [15]. For rather fast changing electric fields, one can speak about a dynamical analogy of the Landau-Zener effect (in analogy with the dynamical EHSS effect considered above).

The relevant kinetic equation for the distribution function of electrons  $f(\mathbf{p}, t)$

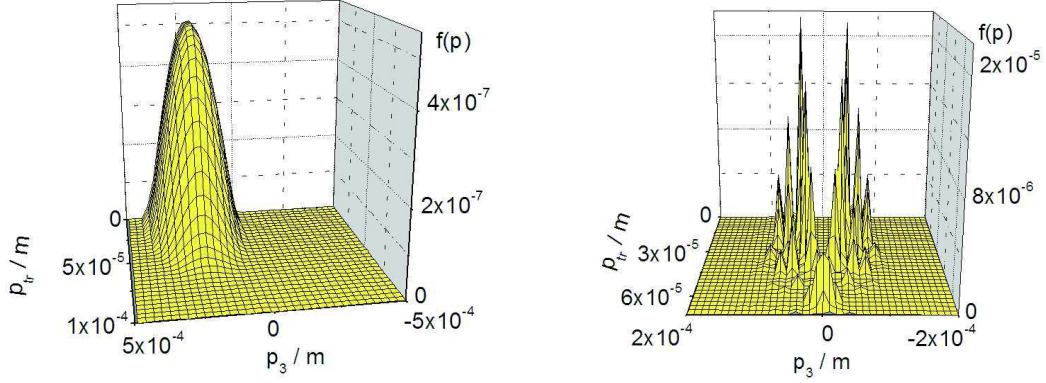


Figure 4: The same as Fig.3 for a  $10^3$  times larger wave length  $\lambda = 0.1$  cm corresponding to a  $10^3$  times larger vector potential.

(it coincides with the distribution function of holes in the case of a non-preexcited system where the initial distributions are zero) is [14]

$$\dot{f}(\mathbf{p}, t) = 2\lambda(\mathbf{p}, t) \int_{-\infty}^t dt' \lambda(\mathbf{p}, t') [1 + 2f(\mathbf{p}, t')] \cos 2\theta(\mathbf{p}, t, t'), \quad (7)$$

where the transition amplitude is  $[\mathbf{P}(t) = \mathbf{p} + e\mathbf{A}(t)]$

$$\lambda(\mathbf{p}, t) = \frac{\dot{\epsilon}(\mathbf{P})}{\Delta + 2\epsilon(\mathbf{P})}, \quad (8)$$

and the dynamical phase is equal to

$$\theta(\mathbf{p}, t, t') = \int_{t'}^t d\tau \epsilon[\mathbf{P}(\tau)]. \quad (9)$$

Here, we use the simplest dispersion law,  $\epsilon(\mathbf{p}) = \mathbf{p}^2/2m$ , where  $m$  is the effective mass and, for simplicity,  $m_e = m_h$  (generalization to arbitrary dispersions are straightforward). The high frequency multiplier in Eq. (7) with the phase (9) corresponds to the “zitterbewegung” parametrized by the external field.

Figs. 3 and 4 show numerical solutions of Eq. (7) for the momentum distributions for two cases: the right figures are for a time-periodic field

$$A_{ex}(t) = -(E_0/\nu) \cos \nu t, \quad E_{ex} = E_0 \sin \nu t, \quad (10)$$

with the frequency  $\nu$  corresponding to the wave length  $\lambda = 1000$  nm (Fig. 3) and  $\lambda = 0.1$  cm (Fig. 4), where  $m = m_e$  and  $E_0 = 1000$  V/cm. The left parts of figures 3 and 4 correspond to a pulsed Sauter potential of the form (5) with  $b = 2\pi/\nu$ , i.e. only one period of (10) is included. Due to the choice of the prefactor, the amplitude of the vector potential in Fig. 4 is 1000 times larger than in Fig. 3. As a consequence, for a pulse, the two peaks of the distribution merge into one. Further, it is easy to see that, in the periodic case, the coherent evolution of the system

is accompanied by an increase of the number of higher harmonics (growth of an instability) which eventually leads to chaotic behavior. This is due to the nonlinear dependence on the field strength.

Similar highly nonlinear behavior of the distribution functions is found on the level of a mean field theory (quantum Vlasov equation) [19] or in electron-ion collisions in strong laser fields (inverse bremsstrahlung) [20]. Thus, we expect that the present results show only a qualitative trend. To obtain a consistent description for the VPC at high excitation densities it will be necessary to include many-particle effects, such as Hartree-Fock and correlation contributions. Furthermore, the KE (7) is based on the assumption that the dispersion of the energy bands is not altered by the action of the external field which, in the case of strong fields, may need a more careful analysis.

## References

- [1] Sauter F., Z. Phys. 1932. V. 73. P. 547; Heisenberg W. and Euler H., Z. Phys. 1936. V. 98. P. 714.
- [2] Schwinger J., Phys. Rev. 1951. V. 82. P. 664.
- [3] Sauter F., Z. Phys. 1931. V. 69. P. 742.
- [4] Nikishov A.I., Trudy FIAN, 1979. V. 111. P. 152.
- [5] Grib A., Mamaev S., and Mostepanenko V., *Vacuum quantum effects in strong external fields*, Friedman Publisher company, St. Peterburg, 1991.
- [6] Landau L. and Lifshitz E., *Quantum Electrodynamics, Volume 4 (Course of Theoretical Physics Series)*, Butterworth-Heinemann; 2 ed., 1982.
- [7] Kluger Y., Eisenberg J.M., Svetitsky B., Cooper F. and Mottola E., Phys. Rev. Lett., 1991. V. 67. P. 2427.
- [8] Pervushin V.N., Skokov V., Reichel A., Smolyansky S., and Prozorkevich A., Int. J. Mod. Phys. A. 2005. V. 20. P.5689.
- [9] Andreev I.V. Phys. Atom. Nucl. 2003. V. 66. P. 1335; hep-ph/0208255.
- [10] Bordag M., Mohideen U., Mostepanenko V.M., Phys. Rept. 2001. V. 353. P.1.
- [11] Smolyansky S.A., Frisen A.V., Prozorkevich A.V., Bravina L.V. and Zabrodin E.E., Proc. of 8-th International Workshop "Relativistic Nuclear Physics: from Hundreds MeV to TeV", May 23 - 28, 2005 JINR, Dubna, P.183.
- [12] Krekora P. , Su Q. and Grobe R., Phys. Rev. Lett. 2004. V. 92. P. 040406.
- [13] Bonitz M. "Quantum Kinetic Theory", B.G. Teubner, Stuttgart/Leipzig (1998)

- [14] Frisen A.V., Prozorkevich A.V., Smolyansky S.A. and Bonitz M., Proceedings of SPIE, 2007. V. 6537. P.653707.
- [15] Zener C. Proc. Roy. Soc. Lond. 1932. V.137. P.696; 1934. V.145. P.523; Landau L.D. Phys. Z. Sowjetunion. 1932. V. 2. P. 46.
- [16] Tarakanov A.V., Reichel A.V., Smolyansky S.A., Vinnik D.V. and Schmidt S.M., In “Progress in Nonequilibrium Green’s Functions II”, M. Bonitz and D. Semkat, Eds., World Scientific Publ., Singapore, 2003. P. 393.
- [17] Vinnik D.V., Mizerny V.A., Prozorkevich A.V., Smolyansky S.A., and Toneev V.D., Phys. Atom. Nucl. 2001. V. 64. P. 775.
- [18] Khalilov V.R., Theor. Math. Phys. 2001. V.126. P.427.
- [19] Bonitz M., Scott D. C., Binder R., Koch S.W., Phys. Rev. B 50 (1994), 15095
- [20] Haberland H., Bonitz M., and Kremp D., Phys. Rev. E 64, 026405 (2001)
- [21] Allor D., Cohen T.D., McGady D.A., preprint at arXiv:0708.1471v2