RESEARCH ARTICLE | JANUARY 01 2000

Quantum kinetic equations for nonideal plasmas: Bound states and ionization kinetics

D. Kremp; Th. Bornath; M. Bonitz; W. D. Kraeft; M. Schlanges

Phys. Plasmas 7, 59–67 (2000)
https://doi.org/10.1063/1.873781

Quantum kinetic equations for nonideal plasmas: Bound states and ionization kinetics

D. Kremp, Th. Bornath, and M. Bonitz
Department of Physics, Universität Rostock, Universitätsplatz 3, D-18051 Rostock, Germany

W. D. Kraeft[a] and M. Schlanges
Institute of Physics, Ernst-Mortiz-Arndt Universität Greifswald, Domstr. 10a, 17487 Greifswald, Germany

(Received 25 May 1999; accepted 1 July 1999)

In this paper, nonequilibrium properties of strongly coupled plasmas are considered. Usually, such problems are dealt with using Boltzmann– or Lenard–Balescu-type equations. However, for the application to strongly coupled plasmas, these equations exhibit several shortcomings. So, it is not possible (i), to describe the short time kinetics, (ii), to recover the correct (energy) conservation laws and thermodynamics, and, (iii), to account for the formation or destruction of bound states. Therefore, the kinetics of strongly coupled plasmas is considered starting from the Kadanoff–Baym equations, which are known to overcome the above limitations. This is demonstrated by a numerical solution of the two-time Kadanoff–Baym equations in second Born approximation. To be able to discuss approximations which are physically more interesting, it is advantageous to proceed to the time diagonal Kadanoff–Baym equations. In first order gradient expansion, generalizations of the Boltzmann and of the Lenard–Balescu kinetic equations are derived accounting for the bound state problem, too. Thus, the shortcomings (i)–(iii) mentioned above are overcome. Finally, the kinetic equations are applied to the problem of ionization kinetics. © *2000 American Institute of Physics.* [S1070-664X(00)01801-2]

I. INTRODUCTION

In this paper, we discuss the nonequilibrium properties of strongly coupled plasmas, which are essentially influenced by degeneracy and correlation effects such as dynamical screening, self-energy, phase space occupation, bound states, and lowering of the ionization energy. We will show which generalizations of conventional kinetic theories have to be performed in order to properly include these effects.

Usually, nonequilibrium properties of many-particle systems are described by kinetic equations of the Boltzmann-type,

$$\left\{\frac{\partial}{\partial t}+\frac{\partial E}{\partial \mathbf{p}}\frac{\partial}{\partial \mathbf{R}}-\frac{\partial E}{\partial \mathbf{R}}\frac{\partial}{\partial \mathbf{p}}\right\} f(\mathbf{p},\mathbf{R}t)=I(\mathbf{p},\mathbf{R}t). \tag{1}$$

Equation (1) is an equation of motion for the Wigner distribution function f which is the quantum mechanical generalization of the classical one-particle distribution function. Of special interest here is the collision integral I, because it accounts for the change of the Wigner function as a result of correlations. Its form is determined by the type of the interaction between the particles.

In the case of short-range interactions, it is appropriate to apply the binary collision approximation, which leads to the Boltzmann collision integral

$$I(\mathbf{p}_1,t)=\frac{1}{V}\int\frac{d\mathbf{p}_2 d\overline{\mathbf{p}}_1 d\overline{\mathbf{p}}_2}{(2\pi)^9}2\pi\delta(E_{12}-\overline{E}_{12})\times|\langle\mathbf{p}_1\mathbf{p}_2|T(E_{12}+i\epsilon)|\overline{\mathbf{p}}_1\overline{\mathbf{p}}_2\rangle|^2\times\{\overline{f}_1\overline{f}_2(1\pm f_1)(1\pm f_2)-f_1f_2(1\pm\overline{f}_1)(1\pm\overline{f}_2)\}, \tag{2}$$

with $f_1=f(\mathbf{p}_1,t),\overline{f}_1=f(\overline{\mathbf{p}}_1,t),E_{12}=E_1+E_2$, etc. This collision integral shows the typical transition probability for a two-particle process in terms of the *on-shell T*-matrix. Furthermore, we have the usual combination of distribution functions. Due to the phase space occupation factors $(1\pm f)$, the integral (2) is applicable to fully degenerate systems, too, where the upper (lower) sign refers to bosons (fermions). For systems with Coulomb interaction, it is appropriate to apply the Lenard–Balescu collision integral which describes dynamical screening effects within the random phase approximation (RPA),

$$I(\mathbf{p}_1,t)=\int\frac{d\mathbf{p}_2 d\overline{\mathbf{p}}_1 d\overline{\mathbf{p}}_2}{(2\pi)^9}\left|\frac{V(\mathbf{p}_1-\overline{\mathbf{p}}_1)}{\epsilon^R(\mathbf{p}_1-\overline{\mathbf{p}}_1,E_1-\overline{E}_1,t)}\right|^2(2\pi)^3\times\delta(\mathbf{p}_1+\mathbf{p}_2-\overline{\mathbf{p}}_1-\overline{\mathbf{p}}_2)2\pi\delta(E_{12}-\overline{E}_{12})\times\{\overline{f}_1\overline{f}_2(1\pm f_1)(1\pm f_2)-f_1f_2(1\pm\overline{f}_1)(1\pm\overline{f}_2)\}. \tag{3}$$

In Eq. (3), the transition probability is expressed in terms of

[a]Author to whom correspondence should be addressed. Electronic mail: wolf.kraeft@physik.uni-greifswald.de

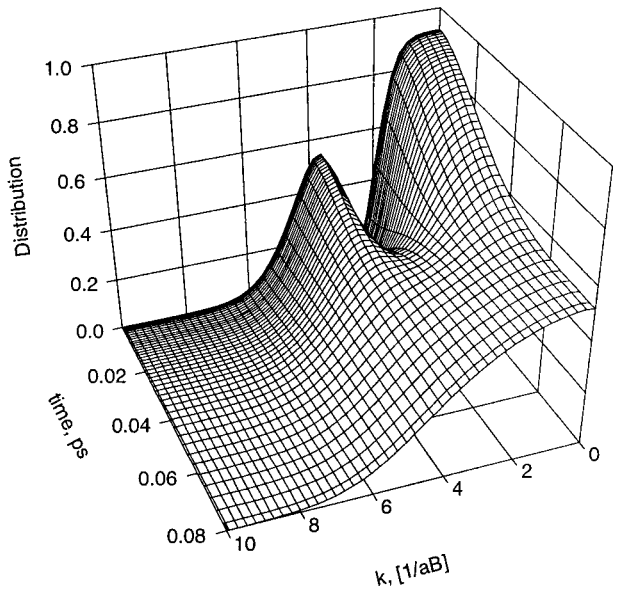

FIG. 1. Numerical demonstration of the irreversible time evolution of the Wigner distribution function from the solution of the Boltzmann equation with the collision integral (3) using the RPA dielectric function in the static limit. Parameters are chosen for electrons in a bulk semiconductor (GaAs). The initial state is taken to be a special nonequilibrium distribution. The final state is the Fermi distribution function.

the *dynamically screened* potential. In both collision integrals, (2) and (3), there is a delta distribution providing the conservation of the *kinetic* energy (*only*) before and after the collision.

Kinetic equations of Boltzmann-type (1) are of fundamental importance because they describe the irreversible relaxation towards the equilibrium state starting from arbitrary initial conditions, and they are the basis of transport theory. Figure 1 gives a numerical demonstration for a spatially homogeneous nonequilibrium plasma showing the time evolution of the Wigner distribution based on a numerical solution of a Boltzmann-type equation with the collision integral (3) using the RPA dielectric function in the static limit. The final state is the Fermi distribution function, and during the relaxation, kinetic energy (only) is conserved.

Though Boltzmann-type kinetic equations are essential tools in physics, there exist many problems and substantial shortcomings. We give some examples. (i) The short-time behavior ($t<t_{\mathrm{corr}}$) cannot be described correctly. (ii) The kinetic equation conserves the kinetic energy $\langle T\rangle$ only instead of the total energy $\langle T\rangle+\langle V\rangle$. This is unphysical, especially for strongly correlated many-particle systems. (iii) No bound states (atoms) are accounted for. (iv) The equilibrium distribution function describes ideal particles only. Therefore, generalizations have to be formulated. Rather general kinetic equations were given already by Prigogine,[1] Resibois,[2] Balescu,[3] Zwanzig, Zubarev, and others. These authors considered evolution equations for classical distribution functions having the shape

$$\frac{\partial f(t)}{\partial t}=F^{+}(t)+\int_0^t dt_1 G_0^{+}(t-t_1)f(t_1). \tag{4}$$

Here, the first right-hand side (RHS) term accounts for the influence of initial correlations, and the integral describes memory effects by integration over the past. For such type of equations, Prigogine introduced the term *non-Markovian* evolution equations.

In this paper, we show how to systematically set up a *quantum* kinetic theory for nonideal plasmas which accounts for the features just mentioned and which overcomes the shortcomings of Boltzmann-type equations.

II. GENERALIZED KADANOFF–BAYM EQUATIONS

The starting point for our further considerations are the generalized Kadanoff–Baym equations[4] (KBE), which are equations of motion for the two-time single-particle correlation functions $g_1^{\gtrless}(11')$,

$$\left(i\frac{\partial}{\partial t_1}+\frac{\nabla_1^2}{2m}\right)g_1^{\gtrless}(11')-\int d\bar{r}_1\Sigma_{\mathrm{HF}}(r_1\bar{r}_1t_1)g_1^{\gtrless}(\bar{r}_1t_1t_1')$$
$$=\int_{t_0}^{t_1}d\bar{1}[\Sigma^{>}(1\bar{1})-\Sigma^{<}(1\bar{1})]g_1^{\gtrless}(\bar{1}1')$$
$$+\int_{t_0}^{t_1'}d\bar{1}[\Sigma^{\gtrless}(1\bar{1})+\Sigma^{in}(1\bar{1})][g_1^{>}(\bar{1}1')-g_1^{<}(\bar{1}1')];$$
$$1=(\mathbf{r}_1,t_1). \tag{5}$$

The KBE are essentially determined by the self energy functions $\Sigma^{\gtrless}(1\bar{1})$ which are defined by

$$\int_C d\bar{1}\Sigma(1\bar{1})g(\bar{1}1')=\pm i\int d2V(12)g_{12}(121'2^{+}), \tag{6}$$

where C means integration along the Keldysh time contour. In generalization of the usual KBE, Eq. (5) contains a term $\Sigma^{in}(1\bar{1})$ which describes the influence of initial correlations on the evolution.[5,6] This term is defined by the two-particle correlation function at the initial time, $g_{12}^{C}(t_0)$, and is given by

$$\Sigma^{in}(11')=\pm i\int d2V(1-2)$$
$$\times\int d\bar{r}_1d\bar{r}_2d\bar{\bar{r}}_1d\bar{\bar{r}}_2\{g_{12}^{R}(1,2;\bar{r}_1t_0,\bar{r}_2t_0)$$
$$\times g_{12}^{C}(\bar{r}_1\bar{r}_2r_1'\bar{\bar{r}}_2;t_0)g_1^{A}(\bar{\bar{r}}_2t_0,2^{+})\}\delta(t_1'-t_0). \tag{7}$$

The correlation functions $g_1^{\gtrless}$ occurring in the KBE can be understood as a far-going generalization of the distribution function. The evolution of $g_1^{<}$ along the diagonal $t_1=t_1'$ in the t_1-t_1'-plane yields the time-dependent Wigner function, $f(\mathbf{p},t)=\pm ig^{<}(\mathbf{p},tt)$. However, due to the two-time dependence, $g_1^{\gtrless}$ contain more information. Their behavior perpendicular to the diagonal reflects the correlations in the plasma (single-particle spectrum). This is most clearly seen in the spectral function $a(\mathbf{p}tt')=i[g^{>}(\mathbf{p},tt')-g^{<}(\mathbf{p},tt')]$, for an illustration, see Figs. 2 and 3. Furthermore, the correlation functions determine the thermodynamic and transport quantities. For example, the average potential energy is given by

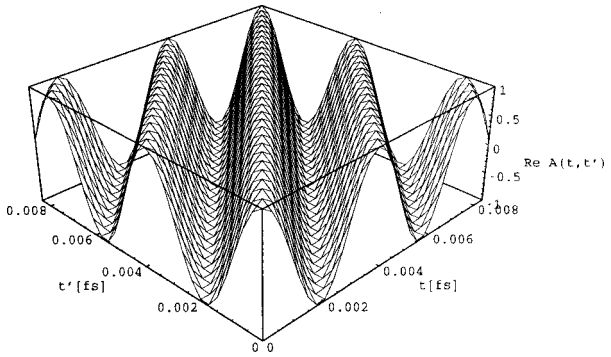

FIG. 2. In order to demonstrate specific features, the Kadanoff–Baym equations were solved for ideal particles. For fixed momentum, the results for the spectral function of electrons are plotted in the t',t-plane. Perpendicular to the diagonal $t=t'$, the spectral function oscillates with the one-particle energy, $\omega=E(p)/\hbar$. Parameter, electron degeneracy $n\lambda^3/2=0.39$.

$$\langle V\rangle = \pm\frac{i}{4}\int d\mathbf{r}\left\{ i\frac{\partial}{\partial t_1} - i\frac{\partial}{\partial t_1'} - \frac{\nabla_1^2}{2m} - \frac{\nabla_{1'}^2}{2m}\right\} \times g^{<}(\mathbf{r}_1 t_1, \mathbf{r}_1' t_1')_{\mathbf{r}_1=\mathbf{r}_1', t_1=t_1'}. \tag{8}$$

Thus, the correlation functions $g_1^{\gtrless}(11')$ determine all one-particle and two-particle properties of plasmas both in equilibrium and nonequilibrium.

The self-energy functions Σ occurring in (5) have to be approximated conveniently. If only the Hartree–Fock self-energy $\Sigma_{\mathrm{HF}}(r_1\overline{r}_1 t_1)$ is retained and the correlation contributions $\Sigma^{\gtrless}(1\overline{1})$ are neglected, we arrive at Vlasov-type equations. In general, the self-energy is, according to Eq. (6), defined by the two-particle correlation function. An important advantage of the nonequilibrium Green's function approach is that it provides highly efficient quantum field-theoretical methods for systematically deriving consistent approximations to the self-energy.

The Kadanoff–Baym equations (5) have the following properties: (i) They include all quantum effects (arbitrary degeneracy). (ii) The equations are valid without any restriction with respect to the time and allow to include arbitrary binary correlations at the initial time t_0. (iii) From these equations, the correct conservation laws of an interacting

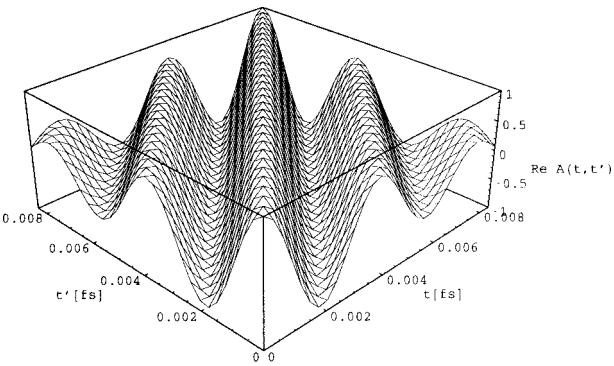

FIG. 3. If the interaction according to (9) is included, we see that the oscillations of the single particle excitations are now modified by damping. The parameters are, electron degeneracy $n\lambda^3/2=0.39$, nonideality $\Gamma=|\langle V\rangle|/\langle T\rangle=0.0098$.

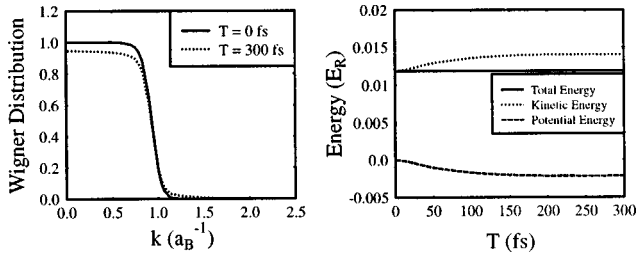

FIG. 4. The left figure shows the relaxation of an electron gas from a Fermi function at $T=3$ K (full line) towards a correlated equilibrium distribution (dotted line). The parameters are the same as in Fig. 1. The right figure shows the buildup of correlation energy and the corresponding increase of kinetic energy, at constant total energy.

many-particle system are recovered. (iv) The structure of approximations is completely determined by the approximation for a single function, the self-energy.

Therefore, essential progress in the nonequilibrium theory of strongly correlated plasmas can be achieved by solving the Kadanoff–Baym equations. Due to the complicated structure of the equations, only numerical evaluations are possible. Such calculations were performed, up to now, for simple approximations for the self-energy, for example for the 2nd Born approximation for Σ,

$$\Sigma^{\gtrless}(\mathbf{p},t,t') = \int\frac{d\mathbf{p}'}{(2\pi)^3}\int\frac{d\mathbf{q}}{(2\pi)^3}|V(\mathbf{q})|^2 g_1^{\gtrless}(\mathbf{p}+\mathbf{q},t,t') \times g_1^{\gtrless}(\mathbf{p}'-\mathbf{q},t,t') g_1^{\gtrless}(\mathbf{p}',t',t). \tag{9}$$

For plasmas, $V(\mathbf{q})$ is taken to be the statically screened potential

$$V(\mathbf{q}) = \frac{4\pi e^2}{q^2+\kappa^2}, \tag{10}$$

where $\kappa^{-1}=\kappa^{-1}(t)$ is the nonequilibrium generalization of the Debye– or Thomas–Fermi-radius, respectively. To demonstrate specific features, first solutions of the Kadanoff–Baym equations for ideal particles are shown. For fixed momentum, the results for the spectral function are plotted in the $t'-t$-plane, cf. Fig. 2. We see that the spectral function is constant along the diagonal $t'=t$ whereas, in perpendicular direction, an oscillatory behavior is observed, where the frequency is determined by the energy of the undamped single particle excitations. If the interaction according to (9) is included, we see, in Fig. 3, that the oscillations of the single particle excitations are now modified by damping.

Details of the relaxation of the distribution function, starting from a Gauss-type initial distribution, are shown for two time steps in the left part of Fig. 4. The final result is a correlated Wigner distribution function. At very low temperatures ($T\to 0$), there may be strong deviations from the Fermi $T=0$-distribution as a result of correlations. In the right part of Fig. 4, we show the relaxation of kinetic, potential and total energy. If initially the plasma is uncorrelated, $E_{\mathrm{pot}}(t_0)=0$. As time proceeds, interparticle correlations build up leading to an increase of the absolute value of correlation energy (in a stable system, the total correlation energy is negative). As a consequence of total energy conser-

vation, this leads to an increase of kinetic energy. The numerical results confirm that indeed the problem of conservation of the total energy is solved if one uses the Kadanoff–Baym equations.

Thus, we demonstrated with this relatively simple example that many of the shortcomings of the Boltzmann equation are overcome. Further numerical results have been given in Refs. 7–10.

III. TIME DIAGONAL KADANOFF–BAYM EQUATIONS

The static Born approximation discussed in the last section is rather simple and does not describe a number of important physical phenomena such as bound states and dynamical screening. To take these effects into account, more complex approximations are necessary, for which it is advantageous to consider the time-diagonal limit of the Kadanoff–Baym equation which yields essentially an equation for the Wigner function. Information related to nondiagonal time points, i.e., for $t' \neq t$, is lost. The diagonal equation reads

$$\left(\frac{\partial}{\partial t}+\frac{\mathbf{p}}{m}\cdot\nabla_{\mathbf{R}}\right)f(\mathbf{p},\mathbf{R}t)$$
$$=F^{+}(t)+\int_{t_0}^{t}d\bar{t}_1\{\Sigma^{>}(\mathbf{p},t\bar{t}_1)g^{<}(\mathbf{p},\bar{t}_1t)$$
$$-\Sigma^{<}(\mathbf{p},t\bar{t}_1)g^{>}(\mathbf{p},\bar{t}_1t)+g^{<}(\mathbf{p},t\bar{t}_1)\Sigma^{>}(\mathbf{p},\bar{t}_1t)$$
$$-g^{>}(\mathbf{p},t\bar{t}_1)\Sigma^{<}(\mathbf{p},\bar{t}_1t)\}, \tag{11}$$

and is the quantum mechanical form of the general kinetic equation (4). However, like in the case of the Kadanoff–Baym equations, we retained still (i) the influence of initial correlations, (ii) the effect of retardation, and (iii) the validity of conservation laws. The RHS of Eq. (11) represents a fargoing generalization of usual collision integrals.

For explicit expressions for the collision integral, we have to solve two problems. First, we have to find appropriate approximations for the self-energy. Second, we have to express the correlation functions as a functional of the Wigner function in order to find a closed equation for the latter.

For the determination of the self-energy $\Sigma^{\gtrless}$ we adopt, as a physically relevant possibility, the binary collision approximation, see, e.g., Refs. 11–13, which is the simplest approximation to account for bound states. In this approximation

$$\Sigma_1^{\gtrless}(t_1t_1')=\pm iT_{12}^{\gtrless}(t_1t_1')g_2^{\lessgtr}(t_1't_1). \tag{12}$$

It is convenient to apply the optical theorem to express the T-matrices $T_{12}^{\gtrless}$ by retarded or advanced ones,

$$T_{12}^{\gtrless}(t_1t_1')=i\int d\bar{t}_1d\bar{t}_2T_{12}^{R}(t_1\bar{t}_1)\mathcal{G}_{12}^{\gtrless}(\bar{t}_1\bar{t}_2)T_{12}^{A}(\bar{t}_2t_1'), \tag{13}$$

where we introduced the special two-particle correlation function,

$$\mathcal{G}_{12}^{\gtrless}(t_1t_1')=g_1^{\gtrless}(t_1t_1')g_2^{\gtrless}(t_1t_1'), \tag{14}$$

and the retarded and advanced T-matrices have to be determined from a generalized Lippmann–Schwinger equation,

$$T_{12}^{R/A}(tt')=V_{12}\delta(t-t')+i\int d\bar{t}V_{12}\mathcal{G}_{12}^{R/A}(t\bar{t})T_{12}^{R/A}(\bar{t}t'), \tag{15}$$

where

$$\mathcal{G}_{12}^{R/A}(tt')=\pm\Theta(\pm(t-t'))\{\mathcal{G}_{12}^{>}-\mathcal{G}_{12}^{<}\}.$$

Another physically relevant approximation is the random phase approximation (RPA). The RPA describes especially the dynamical screening effects. Then we have for the self-energy

$$\Sigma^{\gtrless}(t_1t_1')=iV_s^{\gtrless}(t_1t_1')g^{\gtrless}(t_1t_1'). \tag{16}$$

Again we have an optical theorem which introduces retarded and advanced dynamically screened potentials

$$V_s^{\gtrless}(t_1t_2)=\int d\bar{t}_1d\bar{t}_2V^{R}(t_1\bar{t}_1)\Pi^{\gtrless}(\bar{t}_1\bar{t}_2)V^{A}(\bar{t}_2t_2), \tag{17}$$

which obey a Dyson-type equation which is physically intuitive

$$V^{R/A}(t_1t_2)=V\delta(t_1-t_2)+\int d\bar{t}_1V\Pi^{R/A}(t_1\bar{t}_1)V^{R/A}(\bar{t}_1t_2). \tag{18}$$

Here, Π is the nonequilibrium polarization function which is, in our approximation, taken in RPA for Fermi particles,

$$\Pi^{\gtrless}(t_1t_2)=-ig^{\gtrless}(t_1t_2)g^{\gtrless}(t_2t_1),$$
$$\Pi^{R/A}(t_1t_2)=\pm\Theta(\pm(t_1-t_2))\{\Pi^{>}(t_1t_2)-\Pi^{<}(t_1t_2)\}.$$

The second problem which has to be solved is the *reconstruction problem*, i.e., to express the correlation functions $g^{\gtrless}$ as functionals of Wigner functions. This is necessary as the equation for the determination of the latter, Eq. (11), still contains the correlation functions $g^{\gtrless}$. The first possibility to do this is the Kadanoff–Baym-ansatz (KBA).[4] It accounts for quasiparticles, however it does not describe retardation effects as $f^{\gtrless}$ is taken only at the time diagonal,

$$\pm g^{\gtrless}(t_1t_1')=f^{\gtrless}\left(\frac{t_1+t_1'}{2}\right)\{g^{R}(t_1-t_1')-g^{A}(t_1-t_1')\},$$
$$\pm ig^{\gtrless}(\mathbf{p}\omega,t)=2\pi\delta(\omega-E(\mathbf{p}))f^{\gtrless}(\mathbf{p},t), \tag{19}$$

where we have denoted $f^{<}=f, f^{>}=1\pm f, t=(t_1+t_1')/2$; ω arises from the Fourier transformation with respect to t_1-t_1'. An improved further possibility is the generalized Kadanoff–Baym-ansatz (GKBA) which is due to Lipavský *et al.*[14] and includes the quasiparticle approximation and full retardation,

$$\pm g^{\gtrless}(t_1t_1')=\{g^{R}(t_1t_1')f^{\gtrless}(t_1')-f^{\gtrless}(t_1)g^{A}(t_1t_1')\}. \tag{20}$$

Finally, the third possibility goes beyond the quasiparticle approximation, however it is 1st order with respect to retardation. This approximation includes the Kadanoff–Baym ansatz, but moreover it includes correlations between the quasiparticles,[12]

$$\pm ig^{<}(\omega,t)=2\pi\delta(\omega-E)f(t)-\frac{\mathcal{P}'}{\omega-E}\frac{\partial}{\partial t}(\pm f(t))$$
$$-\frac{\mathcal{P}'}{\omega-E}(\pm i)\Sigma^{<}(\omega,t)+2\pi\delta(\omega-E)$$
$$\times\int\frac{d\overline{\omega}}{2\pi}\frac{\mathcal{P}'}{\overline{\omega}-E}(\pm i)\Sigma^{<}(\overline{\omega},t), \tag{21}$$

where $\mathcal{P}'$ denotes the derivative of the principal value $\mathcal{P}$.

IV. FIRST-ORDER GRADIENT EXPANSION, BOUND STATES, CONSERVATION LAWS

Let us come back to the time-diagonal Kadanoff–Baym equation (11), in which, on the RHS, still occur two-time correlation functions. Whereas in the equilibrium case these quantities depend on the differences of the two times only, now the situation is of course more complicated. Writing $g^{\gtrless}(t_1,t_1')$ and $\Sigma^{\gtrless}(t_1,t_1')$ as functions of difference and sum variables t_1-t_1' and $(t_1+t_1')/2$ (microscopic and macroscopic times), respectively, the two time scales are coupled in (11) due to the non-Markovian property of the collision integral. A decoupling can be achieved within the so-called gradient expansion.[4] Following the ideas outlined in Ref. 12, we consider the Kadanoff–Baym equations in the so-called first-order gradient expansion which is here a first-order expansion with respect to the retardation

$$\frac{\partial f(\mathbf{p},t)}{\partial t}=I^0(\mathbf{p},t)+I^1(\mathbf{P},t), \tag{22}$$

with

$$I^0(\mathbf{p}_1t)=\int_{-\infty}^{+\infty}\frac{d\omega}{2\pi}[(\pm i)\Sigma^{<}(\omega t)ig^{>}(\omega t)$$
$$-i\Sigma^{>}(\omega t)(\pm i)g^{<}(\omega t)], \tag{23}$$

and

$$I^1(\mathbf{p}_1t)=\frac{\partial}{\partial t}\int_{-\infty}^{+\infty}\frac{d\omega_1d\omega_2}{(2\pi)^2}\frac{\mathcal{P}'}{\omega_1-\omega_2}[i\Sigma^{>}(\omega_1t)(\pm i)$$
$$\times g^{<}(\omega_2t)-(\pm i)\Sigma^{<}(\omega_1t)ig^{>}(\omega_2t)]. \tag{24}$$

Equation (22) represents a kinetic equation for the Wigner distribution function. On the RHS, the collision integrals are given in terms of the one particle correlation functions $g^{\gtrless}$ and the self energy functions $\Sigma^{\gtrless}$. The latter are, again, functionals of the $g^{\gtrless}$, for instance one could use the binary collision approximation, or the RPA, respectively. Therefore, using the reconstruction formula (21) in I^0 and I^1, one gets a closed kinetic equation for the Wigner distribution function f.

In T-matrix approximation, the collision integral I^1 can be written as

$$I^1=\frac{\partial}{\partial t}\int\frac{dp_2}{(2\pi)^3}$$
$$\times\left\{\left(\frac{\partial}{\partial\omega}\mathrm{Re}\langle\mathbf{p}_1\mathbf{p}_2|T^R(\omega)|\mathbf{p}_2\mathbf{p}_1\rangle\right)_{|\omega=\epsilon(p_1)+\epsilon(p_2)}f_1f_2\right.$$
$$-\int\frac{d\omega}{2\pi}\langle\mathbf{p}_1\mathbf{p}_2|T^{<}(\omega+\epsilon(p_2))|\mathbf{p}_2\mathbf{p}_1\rangle$$
$$\left.\times\frac{\mathcal{P}'}{\omega-\epsilon(p_1)}(1-f_1-f_2)\right\}. \tag{25}$$

In the second contribution at the RHS of Eq. (25), the T-matrix has to be taken off-shell. That means that the whole two-particle spectrum, scattering as well as binding energies, becomes accessible. Inserting a bilinear expansion for the T-matrix, one is able to separate bound state and scattering state parts. This expansion reads

$$iT_{12}^{\gtrless}(\omega,t)=\sum_K V|\Psi^K\rangle\langle\Psi^K|VN_K^{\gtrless}2\pi\delta(\omega-E_K), \tag{26}$$

with wave functions $|\Psi^K\rangle$ and energy eigenvalues E_K following from the effective Schrödinger equation:

$$(E_K-\epsilon(\mathbf{p}_1,t)-\epsilon(\mathbf{p}_2,t))\langle\mathbf{p}_1\mathbf{p}_2|\Psi^K\rangle$$
$$-(1-f(\mathbf{p}_1,t)-f(\mathbf{p}_2,t))$$
$$\times\int\frac{d\overline{\mathbf{p}}_1d\overline{\mathbf{p}}_2}{(2\pi)^6}\langle p_1p_2|V|\overline{p}_2\overline{p}_1\rangle\langle\overline{p}_1\overline{p}_2|\Psi^K\rangle=0. \tag{27}$$

For bound states we have $N_K^{<}(t)=F_j(\mathbf{P},t)$, where $F_j(\mathbf{P},t)$ is the distribution function of bound particles in the state $|j\rangle$; for scattering states, it holds $N_K^{<}(t)=f(\mathbf{p}_1t)f(\mathbf{p}_2t)$.

A bound state contribution in I^1 follows only from the off-shell T-matrix in (25), and we get

$$I^1=I^1_{\mathrm{scatt}}+\frac{\partial}{\partial t}\int\frac{d\mathbf{p}_2}{(2\pi)^3}\sum_{jP}\langle\mathbf{p}_1\mathbf{p}_2|\Psi^{jP}\rangle\langle\tilde{\Psi}^{jP}|\mathbf{p}_2\mathbf{p}_1\rangle F_j(\mathbf{P}), \tag{28}$$

with j, $\mathbf{P}$ being the set of quantum numbers of bound states. If we consider the bound states to be a new species, we have now a three component system consisting of free electrons, ions and atoms. It is obvious to introduce the distribution function of *free* electrons by

$$f^F(\mathbf{p}_1t)=f(\mathbf{p}_1t)-\int\frac{d\mathbf{p}_2}{(2\pi)^3}\sum_{jP}\langle\mathbf{p}_1\mathbf{p}_2|\Psi^{jP}\rangle$$
$$\times\langle\tilde{\Psi}^{jP}|\mathbf{p}_2\mathbf{p}_1\rangle F_j(\mathbf{P};t). \tag{29}$$

We arrive at a generalization of the Boltzmann equation which has the compact structure,[12]

$$\frac{d}{dt}f^F(\mathbf{p}t)=\left(1+\frac{1}{2}\epsilon\frac{d}{dt}\frac{d}{d\epsilon}\right)I_B(\mathbf{p},\epsilon)=I_B(\mathbf{p})+I^R(\mathbf{p}), \tag{30}$$

with $\epsilon\to0$, and where $I^R(\mathbf{p})$ includes all 1st order retardation terms which arise from I^1 as well as from I^0 [from the use of the optical theorem (12), and from (21)]. The term $I_B(\mathbf{p})$

$=\lim_{\epsilon\to 0} I_B(\mathbf{p},\epsilon)$ is the local contribution, i.e., without retardation, cf. Eq. (2). The collision integral $I_B(\mathbf{p},\epsilon)$ is given by

$$I_B(\mathbf{p}_1,\epsilon)=\int \frac{d\mathbf{p}_2 d\overline{\mathbf{p}}_1 d\overline{\mathbf{p}}_2}{(2\pi)^3}\int\frac{d\omega}{2\pi} \times\left\{\frac{2\epsilon}{(\omega-E_{12})^2+\epsilon^2}\frac{2\epsilon}{(\omega-\overline{E}_{12})^2+\epsilon^2} \times|T^R(\omega+i\epsilon)|^2[\overline{N}^<_{12}N^>_{12}-\overline{N}^>_{12}N^<_{12}]\right\}, \quad (31)$$

where we used the abbreviations $\overline{N}^<_{12}=\overline{f}_1\overline{f}_2, N^>_{12}=(1\pm f_1)\times(1\pm f_2)$.

Similar considerations can be done in the case of a generalized Balescu–Lenard equation. Here, one gets from a retardation expansion a kinetic equation similar to Eq. (30), but now one has instead of $I_B(\epsilon)$ a generalized Balescu–Lenard term,[15]

$$I_{\mathrm{BL}}(\mathbf{p}_1,\epsilon)=\int \frac{d\mathbf{p}_2 d\overline{\mathbf{p}}_1 d\overline{\mathbf{p}}_2}{(2\pi)^9}\int\frac{d\omega}{2\pi} \times\frac{2\epsilon}{(\omega-E_1+\overline{E}_1)^2+\epsilon^2}\frac{2\epsilon}{(\omega+E_2-\overline{E}_2)^2+\epsilon^2} \times\left|\frac{V(\mathbf{p}_1-\overline{\mathbf{p}}_1)}{\epsilon^R(\mathbf{p}_1-\overline{\mathbf{p}}_1,\omega)}\right|^2[\overline{N}^<_{12}N^>_{12}-\overline{N}^>_{12}N^<_{12}]. \quad (32)$$

On behalf of Eqs. (28), (29), the kinetic properties of the plasma are influenced by bound states only, if the temporal change of the occupation number of bound states is different from zero, i.e., if there occurs a formation or a decay of bound states in three-particle collisions. Furthermore, for nonequilibrium systems, the distribution functions f^F and F_j are independent. Therefore, an extension of the binary collision approximation to three-particle collisions is necessary. Moreover, we still have to consider an equation for the bound states. The result is a set of coupled equations

$$\frac{df^F(\mathbf{p},t)}{dt}=I_B(\mathbf{p},t)+I^R(\mathbf{p},t)+I^0_3(\mathbf{p},t),$$
$$\frac{dF_j(\mathbf{P},t)}{dt}=I^{\mathrm{scatt}}_j(\mathbf{P},t)+I^{\mathrm{react}}_j(\mathbf{P},t). \quad (33)$$

Here, in the equation for the free particles there was included a three-particle collision term describing all possible three-particle collision processes (including bound state formation and decay). The reaction term I^{react} provides for the possibility of changes of the composition by three-particle collisions in the kinetic equation for the bound state distribution function and will be discussed in Sec. V. In the way sketched above, we have solved a further problem, the introduction of bound states into the kinetic theory.

In the framework of the kinetic theory in gradient expansion presented above, we could show that the energy conservation holds,[12] for an approach basing on density operator techniques see Ref. 13,

$$\frac{d}{dt}\{\langle T\rangle+\langle V\rangle\}=0. \quad (34)$$

In the case of the binary collision approximation, we get for the mean value of the potential energy

$$\langle V\rangle^{\mathrm{bin.coll.}}=\langle V\rangle^{\mathrm{HF}}+\frac{1}{2}Tr_{12} \times\left[|T^R(\overline{E})|^2\frac{\mathcal{P}}{E-\overline{E}}\{N^<_{12}\overline{N}^>_{12}-\overline{N}^<_{12}N^>_{12}\}\right], \quad (35)$$

where Tr denotes the trace. Here, the bound states come into the play via the off-shell T-matrix.

In RPA, the mean value of the potential energy is expressed in terms of the dynamically screened potential

$$\langle V\rangle^{\mathrm{RPA}}=\langle V\rangle^{\mathrm{HF}}+\frac{1}{2}Tr_{12} \times\left[|V^R_s(\overline{E})|^2\frac{\mathcal{P}}{E-\overline{E}}\{N^<_{12}\overline{N}^>_{12}-\overline{N}^<_{12}N^>_{12}\}\right]. \quad (36)$$

With these expressions for the mean value of the potential energy, the equation of state (EOS) may be derived from the well-known charging formula,[11]

$$p-p_0=-\frac{1}{\Omega}\int_0^1\frac{d\lambda}{\lambda}\langle\lambda V\rangle_\lambda. \quad (37)$$

Inserting (35) or (36) into (37), one gets the thermodynamics in the approximation of the second virial coefficient, or of the quantum ring sum, respectively.

V. DYNAMIC SCREENING EFFECTS IN IONIZATION AND RECOMBINATION RATES

In the previous section, the system of kinetic equations (33) was given to describe the nonequilibrium properties of plasmas which are influenced by the existence of bound states. In addition to the free particle kinetics, the kinetics of bound states in different states $|j\rangle$ is included taking into account elastic, inelastic, and reactive three-particle processes. In this way, the kinetic equations (33) form the basic equations to describe the ionization and population dynamics in dense nonequilibrium plasmas. In this section, we will focus on the population dynamics of atomic and ionic bound states in spatially homogeneous hydrogenlike plasmas consisting of free electrons and ions with densities n_e and n_i, and of two-particle bound states with density n_j.

We will show how the many-particle effects relevant for strongly coupled plasmas influence the reaction rates. To do this we start from a quantum kinetic equation for the distribution functions of bound states including dynamic screening in the collision terms of elastic, inelastic and reactive processes. In Ref. 16, such a kinetic equation was derived starting from the Bethe–Salpeter equation for the two-particle correlation functions. The effective interaction kernel was determined in the frame of the polarization approximation and could be expressed in terms of the dynamically

screened Coulomb potential. Then, the following Lenard–Balescu-type kinetic equation for the distribution function of bound particles in the state $|j\rangle$ could be obtained,

$$\frac{\partial}{\partial t}F_j(\mathbf{P},t)=I_j^{\mathrm{scatt}}(\mathbf{P},t)+I_j^{\mathrm{react}}(\mathbf{P},t). \tag{38}$$

Here, the first collision integral describes the elastic and inelastic scattering processes

$$(a+b)+c\leftrightarrow(\overline{a}+\overline{b})+\overline{c}.$$

The explicit expression is

$$\begin{aligned}I_j^{\mathrm{scatt}}(\mathbf{P},t)=&\frac{(2\pi)^4}{V}\sum_{j,c}\int\frac{d\overline{\mathbf{P}}}{(2\pi)^3}\frac{d\mathbf{p}}{(2\pi)^3}\frac{d\mathbf{q}}{(2\pi)^3}z_c^2|\mathcal{P}_{j\mathbf{P},\overline{j}\,\overline{\mathbf{P}}}(\mathbf{q})|^2\\&\times|V^R(\mathbf{q},E_{\overline{j}}(\overline{\mathbf{P}})-E_j(\mathbf{P})|^2\delta(E_j(\mathbf{P})+\epsilon_c(\mathbf{p})\\&-E_{\overline{j}}(\overline{\mathbf{P}})-\epsilon_c(\mathbf{p}+\mathbf{q}))\times\{[1-f_c][1+F_j]\overline{f}_c\overline{F}_{\overline{j}}\\&-f_cF_j[1-\overline{f}_c][1+\overline{F}_{\overline{j}}]\}.\end{aligned} \tag{39}$$

The second contribution describes the formation and the decay of bound states in three-particle collisions

$$(a+b)+c\leftrightarrow\overline{a}+\overline{b}+\overline{c}$$

and is given by

$$\begin{aligned}&I_j^{\mathrm{react}}(\mathbf{P},t)\\&=\frac{(2\pi)^4}{V}\sum_{j,c}\int\frac{d\overline{\mathbf{p}}_a}{(2\pi)^3}\frac{d\overline{\mathbf{p}}_b}{(2\pi)^3}\frac{d\mathbf{p}}{(2\pi)^3}\frac{d\mathbf{q}}{(2\pi)^3}\\&\times z_c^2|\mathcal{P}_{j\mathbf{P},\overline{\mathbf{p}}_a\overline{\mathbf{p}}_b}(\mathbf{q})|^2|V^R(\mathbf{q},E_j(\mathbf{P})-\overline{\epsilon}_a-\overline{\epsilon}_b)|^2\\&\times\delta(E_j(\mathbf{P})+\epsilon_c(\mathbf{p})-\epsilon_a(\overline{\mathbf{p}}_a)-\epsilon_b(\overline{\mathbf{p}}_b)-\epsilon_c(\mathbf{p}+\mathbf{q}))\\&\times\{\overline{f}_a\overline{f}_b\overline{f}_c[1+F_j][1-f_c]-[1-\overline{f}_a]\\&\times[1-\overline{f}_b][1-\overline{f}_c]F_jf_c\}.\end{aligned} \tag{40}$$

In (39) and (40), c labels the free plasma species. Like above, we introduced the abbreviations $F_j=F_j(\mathbf{P}),\overline{F}_j=F_{\overline{j}}(\overline{\mathbf{P}}),f_c=f_c(\mathbf{p}),\overline{f}_c=f_c(\mathbf{p}+\mathbf{q})$, etc., the atomic form factors $\mathcal{P}_{KK'}$ are given below, (43).

Let us discuss the expression for the collision integrals. The energies ϵ_a are quasiparticle energies which we consider in the so-called *rigid shift approximation*,[11] i.e., $\epsilon_a(p)=\epsilon_a^0(p)+\Delta_a$ with Δ_a being a thermally averaged self-energy shift. The bound state energies $E_j(P)=E_j^0(P)+\Delta_j$ are determined by an effective Schrödinger equation.[11,16] The transition rates in (39) and (40) are given in terms of the dynamically screened interaction potential $V^s(\mathbf{q},\omega)$, by the atomic form factor $\mathcal{P}_{K,K'}$, and by the set of distribution functions including phase space occupation effects. In general, the screened potential is determined by the screening equation (18). In local approximation and after Fourier transformation with respect to the difference variables, the retarded screened potential can be written as

$$V_s^R(\mathbf{q},\omega)=\frac{V(q)}{\epsilon^R(\mathbf{q},\omega)} \tag{41}$$

with $V(q)=4\pi e^2/q^2$ being the Coulomb potential, and $\epsilon(\mathbf{q},\omega)$ being the retarded plasma dielectric function. The macroscopic variables were dropped for simplicity. The dielectric function is considered in RPA,

$$\epsilon(\mathbf{q},\omega)=1-\sum_a V_{aa}(q)\Pi_{aa}^{\mathrm{RPA}}(\mathbf{q},\omega), \tag{42}$$

which accounts for the contribution of free particles to the screening determined by the free particle polarization function Π_{aa}^{RPA}. Of course, a consistent treatment of screening in partially ionized plasmas requires the inclusion of the contributions of two-particle bound and scattering states, i.e., one has to go beyond the RPA. This was considered in the work of Klimontovich,[17] of Röpke and Der,[18] and recently, for nonequilibrium systems, based on real-time Green's function technique, in Ref. 16.

The atomic form factor describing the bound–bound, and bound-free transitions is given by

$$\begin{aligned}\mathcal{P}_{K,K'}(\mathbf{q})=&\int d\mathbf{r}_a d\mathbf{r}_b\Psi_K^*(\mathbf{r}_a\mathbf{r}_b)(z_ae^{-i\mathbf{q}\cdot\mathbf{r}_a}\\&+z_be^{-i\mathbf{q}\cdot\mathbf{r}_b})\Psi_{K'}(\mathbf{r}_a\mathbf{r}_b),\end{aligned} \tag{43}$$

where K and K' denote the set of quantum numbers, i.e., $K=\mathbf{P},j$ for bound states and $K=\mathbf{p}_a,\mathbf{p}_b$ for scattering states, and z_a is the charge number. The wave functions Ψ_K are determined by an effective Schrödinger equation as given by (27) which accounts for the influence of many-particle effects on the two-particle states.

Now, the kinetic equation (38) can be used as the starting point for the derivation of rate equations, which are obtained by integration with respect to the momentum. Only the collision terms of inelastic and reaction processes contribute to the rate equation,

$$\frac{\partial n_j}{\partial t}=\sum_{\overline{j}}(n_en_{\overline{j}}K_{\overline{j}j}-n_en_jK_{j\overline{j}})+(n_in_e^2\beta_j-n_en_j\alpha_j). \tag{44}$$

$K_{jj}^{\overline{\ }}$ are the coefficients of collisional excitation (deexciation), and α_j,β_j are the impact ionization and three-body recombination coefficients, respectively. Let us consider the coefficient of electron impact ionization. From the reaction term of the kinetic equation (38), we get in adiabatic approximation $(m_e\ll m_i)$,[19,20]

$$\begin{aligned}\alpha_j=&\frac{(2\pi)^3}{n_e}\int\frac{d\overline{\mathbf{p}}}{(2\pi)^3}\frac{d\mathbf{p}}{(2\pi)^3}\frac{d\mathbf{q}}{(2\pi)^3}|\mathcal{P}_{j,\overline{\mathbf{p}}}(\mathbf{q})|^2\\&\times\left|\frac{V(q)}{\epsilon(\mathbf{q},I_j^{\mathrm{eff}}+\epsilon_{\overline{p}}^0)}\right|^2 2\pi\delta(I_j^{\mathrm{eff}}+\epsilon_{\mathbf{p}}^0-\epsilon_{\mathbf{p}}^0+\epsilon_{\mathbf{p}-\mathbf{q}}^0)\\&\times[1-f_e(\mathbf{p}-\mathbf{q})][1-f_e(\overline{\mathbf{p}})]f_e(\mathbf{p}),\end{aligned} \tag{45}$$

where we introduced the effective ionization energy

$$I_j^{\mathrm{eff}}=|E_j^0|+\Delta_e+\Delta_i-\Delta_j. \tag{46}$$

In the following, the electron distribution function is assumed to be a local equilibrium one. Furthermore, we consider the nondegenerate case which allows to write α_j in terms of a cross section, i.e.,

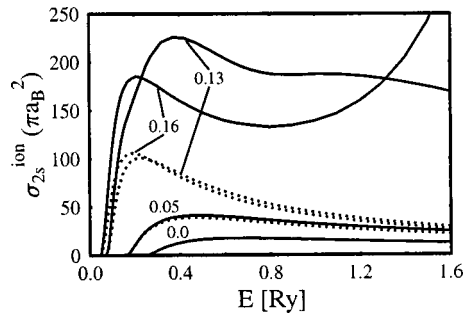

FIG. 5. Total cross section for ionization from the $2s$-level of hydrogen vs impact energy for different screening parameters κa_B (κ, inverse Debye screening length). The hydrogen plasma is considered at temperature $T=30\,000$ K. Two approximations are compared; dynamic screening included in the scattering potential (solid), and static screening (dashed).

$$\alpha_j=\frac{8\pi m_e}{(2\pi m_e k_B T)^{3/2}}\int_{I_j^{\rm eff}}^{\infty} d\epsilon\,\epsilon\sigma_j^{\rm ion}(\epsilon)\exp\left(-\frac{\epsilon}{k_B T}\right) \tag{47}$$

with the total cross section for impact ionization from the atomic state j,

$$\sigma_j^{\rm ion}=\frac{8\pi\hbar^2}{p_e^2 a_B^2}\int\frac{d\Omega_{p_e}}{4\pi}\int_0^{\bar{p}_{\max}} d\bar{p}\bar{p}^2 d\Omega_{\bar{p}}\int_{q_{\min}}^{q_{\max}} dq q \times\left|\frac{V_{ee}(q)}{\epsilon(q,\epsilon_p^0+I_j^{\rm eff})}\mathcal{P}_{j\bar{p}}(q)\right|^2. \tag{48}$$

In comparison to the ionization cross section of an isolated electron–atom problem, there are the following differences: (i) the threshold is determined by the effective ionization energy $I_j^{\rm eff}$, (ii) the wave functions in the atomic form factor have to be calculated from an effective Schrödinger equation, (iii) dynamic screening is accounted for in the electron–atom scattering potential.

In Fig. 5, numerical results for the cross section are shown to demonstrate the influence of the many-particle effects. To discuss dynamic screening effects, two different approximations are considered; (i) static screening, and (ii), the scattering potential is taken with the dynamic RPA dielectric function, whereas the effective ionization energies and the form factor are calculated assuming static screening like in the model used in Refs. 21 and 22. We observe small deviations if the screening parameter κa_B is small. However, at higher plasma densities, the cross section including dynamic screening shows an irregular behavior for high energies, i.e., it increases with increasing energy. This behavior is connected with collective excitations in dense plamas, and it indicates that the picture of a single electron scattering on an atom breaks down. Indeed, one can show that, with a lowered ionization energy, the energy argument in the dielectric function can take values near the plasma frequency. Therefore, ionization and recombination are governed by collective effects, i.e., by absorption and emission of plasmons. This makes the usual cross section definition unsuitable. Although it is possible to calculate the ionization coefficient according to (47), it is more appropriate to rewrite the basic statistical expression of the rate coefficients. For the ionization coefficient, it follows[19,20]

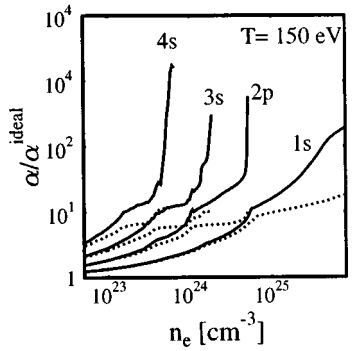

FIG. 6. Ionization coefficients of hydrogenlike carbon ions vs free electron density at a temperature $T=150$ eV. Two approximations are compared; dynamic screening included in the scattering potential (solid), and static screening (dashed). There are shown the ratios of the rate coefficients to their ideal values.

$$\alpha_j=\frac{1}{n_e}\int\frac{d^3\bar{p}}{(2\pi)^3}d^3q|\mathcal{P}_{j\bar{\mathbf{p}}}(\mathbf{q})|^2 V(q)2 \times{\rm Im}\,\epsilon^{-1}(\mathbf{q},\epsilon_{\bar{\mathbf{p}}}+I_j^{\rm eff})n_B(\epsilon_{\bar{\mathbf{p}}}+I_j^{\rm eff}), \tag{49}$$

with $n_B(\omega)=[\exp(\beta\omega)+1]^{-1}$ being the Bose function.

The expressions derived above make it possible to calculate ionization and recombination coefficients for nonideal plasmas. Effects of dynamic screening on ionization rates were discussed, e.g., by Murillo and Weisheit[23] who used a modified version of the expression given by (49), and by Schlanges, Bornath *et al.*[19,20] on the basis of quantum kinetic theory.

Now, we will present some results for the rate coefficients. Figures 6 and 7 show the coefficients of ionization and recombination for different states of hydrogenlike carbon ions in a dense carbon plasma as a function of free electron density. The recombination coefficients were calculated using the relation

$$\beta_j=\Lambda_e^3\alpha_j\exp(I_j^{\rm eff}/k_B T), \tag{50}$$

with $\Lambda_e=(2\pi\hbar^2/m_e k_B T)^{1/2}$ being the thermal wavelength. We observe a strong density dependence of the rates with increasing density. This is mainly an effect of the lowering of the ionization energy. But there is a considerable contribution determined by the scattering continuum. Especially,

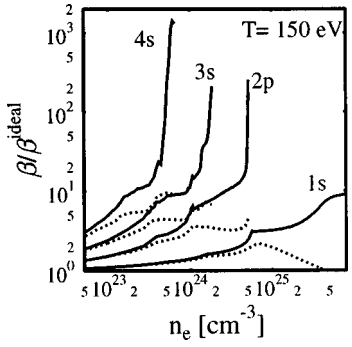

FIG. 7. Recombination coefficients of hydrogenlike carbon ions vs free electron density. Same parameters as in Fig. 6.

higher lying bound states merge into the continuum (Mott effect) and contribute there as resonances enhancing the cross section,[21,22] see also Ref. 24.

The inclusion of RPA dynamic screening in the scattering potential leads to higher rates compared to pure static screening. At this point, it should be mentioned that dynamic screening is included here in a simple approximation. Of course, further improvements are necessary to calculate rate coefficients for high density plasmas. A self-consistent treatment of dynamic screening in the electron–atom potential as well as in the atomic form factor is required. Screening has to be considered beyond the simple RPA scheme. Furthermore, damping effects should be considered in the spectral functions of the one- and two-particle correlation functions for plasmas at high densities.

ACKNOWLEDGMENTS

We would like to thank R. Prenzel and D. Semkat for useful discussions and preparation of figures. This work was supported by the Deutsche Forschungsgemeinschaft, Sonderforschungsbereich 198: Kinetik Partiell Ionisierter Plasmen.

[1] I. Prigogine, *Nonequilibrium Statistical Mechanics* (Wiley–Interscience, New York 1963).
[2] P. Resibois, Physica (Amsterdam) **31**, 645 (1965).
[3] R. Balescu, Phys. Fluids **3**, 52 (1960).
[4] L. P. Kadanoff and G. Baym, *Quantum Statistical Mechanics* (Benjamin, New York, 1962).
[5] P. Danielewicz, Ann. Phys. (N.Y.) **152**, 239 (1984).
[6] D. Semkat, D. Kremp, and M. Bonitz, Phys. Rev. E **59**, 1557 (1999).
[7] M. Bonitz, D. Kremp, D. C. Scott, R. Binder, W. D. Kraeft, and H. S. Köhler, J. Phys.: Condens. Matter **8**, 6057 (1996).
[8] M. Bonitz, D. Semkat, N. H. Kwong, and D. Kremp, in *Proceedings of the Conference on Strongly Coupled Coulomb Systems*, Boston, MA, August 1997, edited by G. Kalman, M. Rommel, and K. Blagoev (Plenum, New York, 1998), p. 515.
[9] M. Bonitz, D. Semkat, and D. Kremp, Phys. Rev. E **56**, 1246 (1997).
[10] M. Bonitz, *Quantum Kinetic Theory* (Teubner-Verlag, Stuttgart, Leipzig, 1998).
[11] W. D. Kraeft, D. Kremp, W. Ebeling, and G. Röpke, *Quantum Statistics of Charged Particle Systems* (Akademieverlag, Berlin, and Plenum, New York, 1986).
[12] Th. Bornath, D. Kremp, W. D. Kraeft, and M. Schlanges, Phys. Rev. E **54**, 3274 (1996).
[13] D. Kremp, M. Bonitz, W. D. Kraeft, and M. Schlanges, Ann. Phys. (N.Y.) **258**, 320 (1997).
[14] P. Lipavský, V. Špička, and B. Velický, Phys. Rev. B **34**, 6933 (1986).
[15] D. Kremp, Th. Bornath, M. Bonitz, and M. Schlanges, Physica B **228**, 72 (1996).
[16] M. Schlanges and Th. Bornath, Contrib. Plasma Phys. **37**, 239 (1997).
[17] Yu. L. Klimontovich, *Kinetic Theory of Nonideal Gases and Plasmas* (Nauka, Moscow, 1975; Academic, New York, 1982); Yu. L. Klimontovich, *Kinetic Theory of Electrodynamic Processes* (Nauka, Moscow, 1980; Springer-Verlag, Berlin, Heidelberg, New York, 1982).
[18] G. Röpke and R. Der, Phys. Status Solidi B **92**, 501 (1979).
[19] M. Schlanges, Th. Bornath, R. Prenzel, and D. Kremp, in Proceedings of the Tenth APS Conference, San Francisco, 1996, edited by A. L. Osterheld and W. H. Goldstein, AIP Conf. Proc. No. **381** (American Institute of Physics, New York 1996), p. 215.
[20] Th. Bornath, M. Schlanges, F. Morales, and R. Prenzel, J. Quant. Spectrosc. Radiat. Transf. **58**, 501 (1997).
[21] M. Schlanges and Th. Bornath, Physica A **192**, 262 (1993).
[22] Th. Bornath and M. Schlanges, Physica A **196**, 427 (1993).
[23] M. S. Murillo and J. C. Weisheit, in Proceedings of the International Conference on Strongly Coupled Plasma Physics, edited by H. M. Van Horn and S. Ichimaru (University of Rochester Press, Rochester, 1993), p. 233; Phys. Rep. **302**, 1 (1998)
[24] C. A. Iglesias and R. W. Lee, J. Quant. Spectrosc. Radiat. Transf. **58**, 637 (1997).