My experimental airplane. 200hp 200mph

Some applications of Kadanoff-Baym eqs.

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Topics of this talk

1. Looking back:
   Correlations and equilibration
2. Initial correlations
3. Imaginary time-stepping
4. Temperature?
5. Levinson.
6. Separable Interaction
7. Coupled eqs Nucleons,Pions, Deltas.

Kadanoff-Baym Equations

$$\left[i\hbar \frac{\partial}{\partial t_1} - \frac{\hbar^2 p^2}{2m} \right] G^{\gtrless}(p, t_1, t_2) = I_1^{\gtrless}(p, t_1, t_2)$$

with a similar equation for $\dfrac{\partial}{\partial t_2}$

$$I_1^{\gtrless}(p, t_1, t_2) = \int_{t_0}^{t_1} \frac{d\bar{t}}{\hbar} \left[\sum^{>}(p, t_1, \bar{t}) - \sum^{<}(p, t_1, \bar{t} \right] G^{\gtrless}(p, \bar{t}, t_2)$$

$$- \int_{t_0}^{t_2} \frac{d\bar{t}}{\hbar} \sum (p, t_1, \bar{t}) \left[G(p, \bar{t}, t_2) - G(p, \bar{t}, t_2) \right]$$

and similar for I_2.

The total energy is given by

$$E(t) = -\frac{i}{4}\int \frac{d^3p}{(2\pi)^3}\left(\frac{p^2}{2m} + i\left(\frac{\partial}{\partial t} - \frac{\partial}{\partial t'}\right)\right) G^<(p,t,t')\big|_{t=t'}$$

The "correlated" kinetic energy is:

$$K_\rho(t) = \int \frac{d^3p}{(2\pi)^3}\frac{p^2}{2m}\rho(p,t)$$

$$\rho(p,t) = -iG^<(p,t,t)$$

$\rho(p,t)$ is the "correlated" fermi-distribution

while the correlation energy is

$$E_{corr}(t) = E(t) - K_\rho(t)$$

The uncorrelated kinetic energy is:

$$K_f(t) = \int \frac{d^3p}{(2\pi)^3} \frac{p^2}{2m} f(p,t)$$

$f(p,t)$ Is the **uncorrelated** fermi-distribution

only defined at equilibrium; need temperature to define

Th

For a local interaction $V(r)$ the selfenergies are :

$$\sum\nolimits^{\gtrless}(\mathrm{p},\mathrm{t}_1,t_2)=\int\frac{d^3p_1}{(2\pi)^3}G^{\gtrless}(p_1,t_1,t_2)[V(p-p_1)]^2$$

$$\times\int\frac{d^3p_2}{(2\pi)^3}G^{\gtrless}(p-p_1+p_2,t_1,t_2)G^{\lessgtr}(p_2,t_2,t_1)$$

In the classical limit $G^{<}$ is the distribution function f.

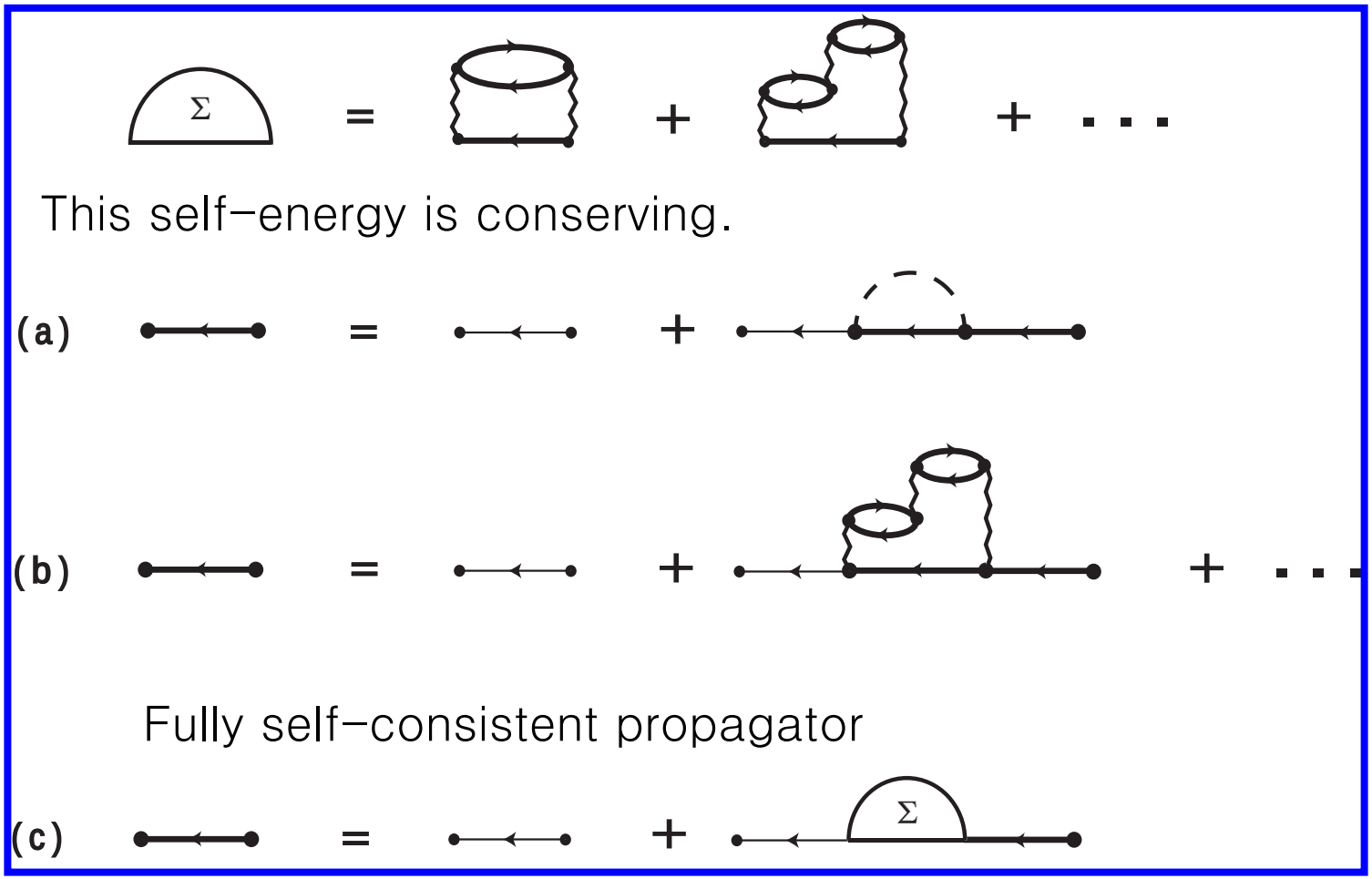
$\mathrm{G}^{>}$ is $1-f$.

Computer program in

HSK, Kwong, Yousif

Comp Phys Communications 123(1999)123

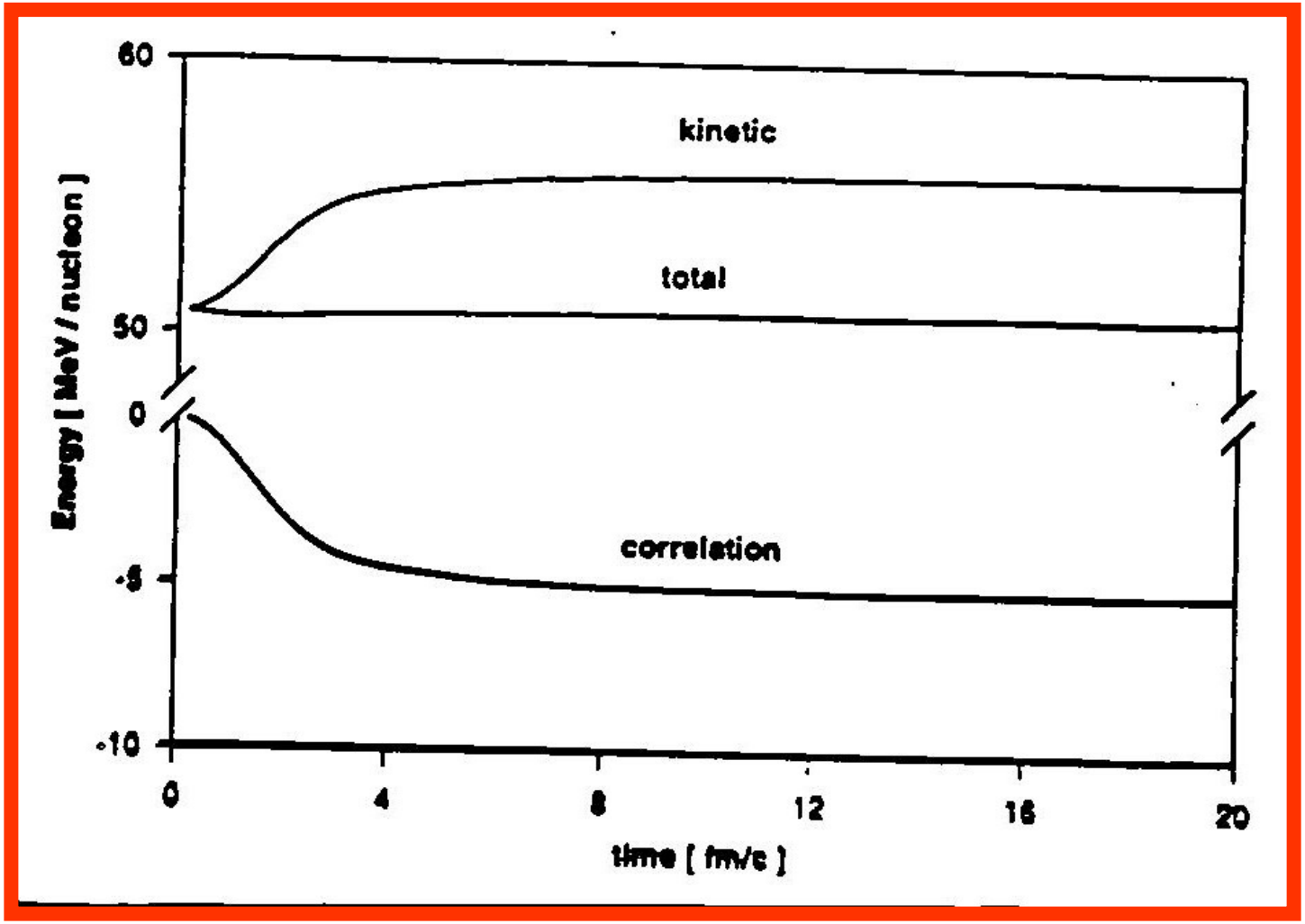
SIGMA is Self-energy.
a,b and c some def. of propagator.
(c) is fully self-consistent.



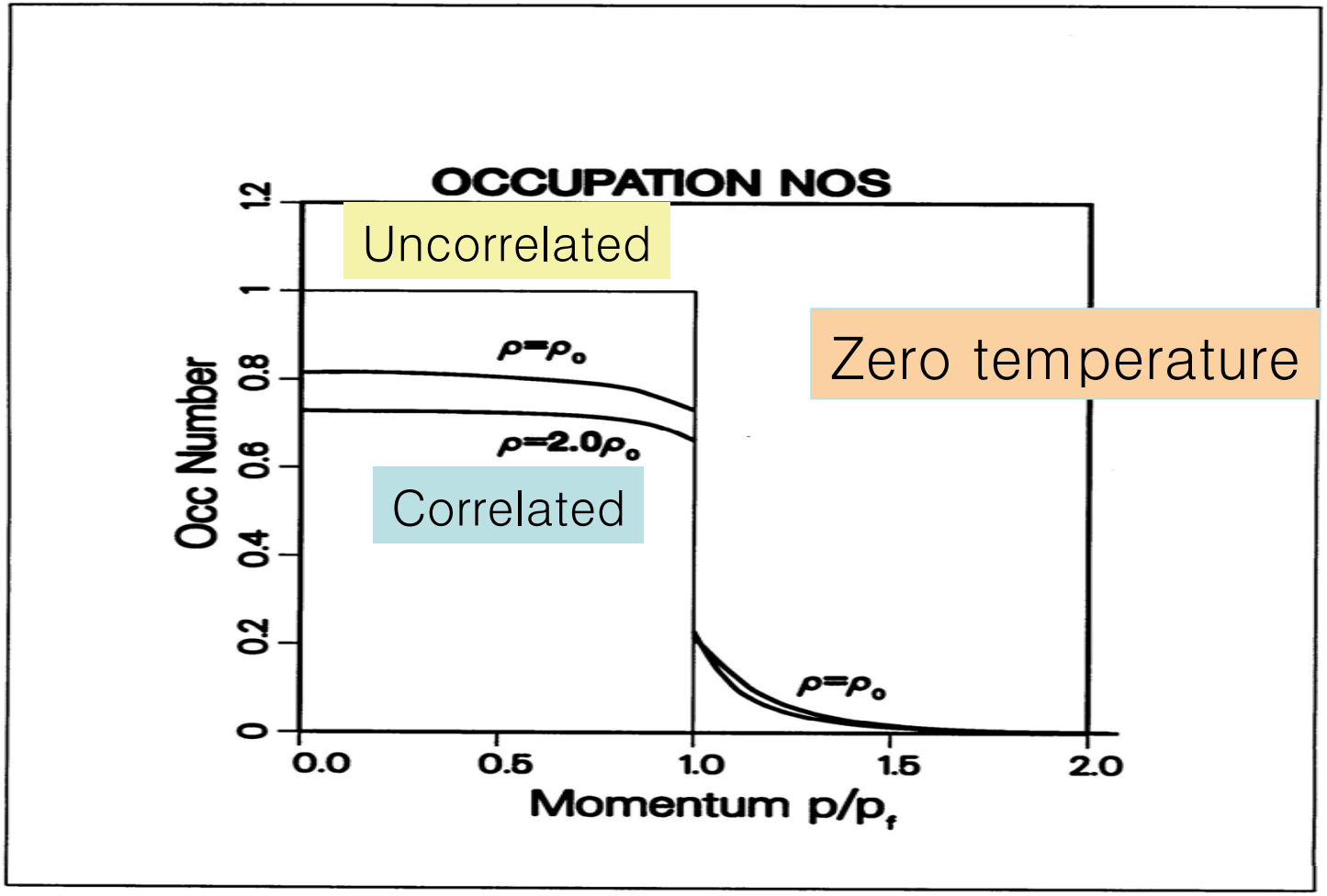
- The diagrams included by the complex self-energies are not included in perturbative quasi-particle approximations like Brueckner. They are included in Green's function calculations by the width of the spectral functions.

TOTAL ENERGY IS CONSERVED
kinetic energy converges.

Temperature increases



Effect of correlations on Distribution-function



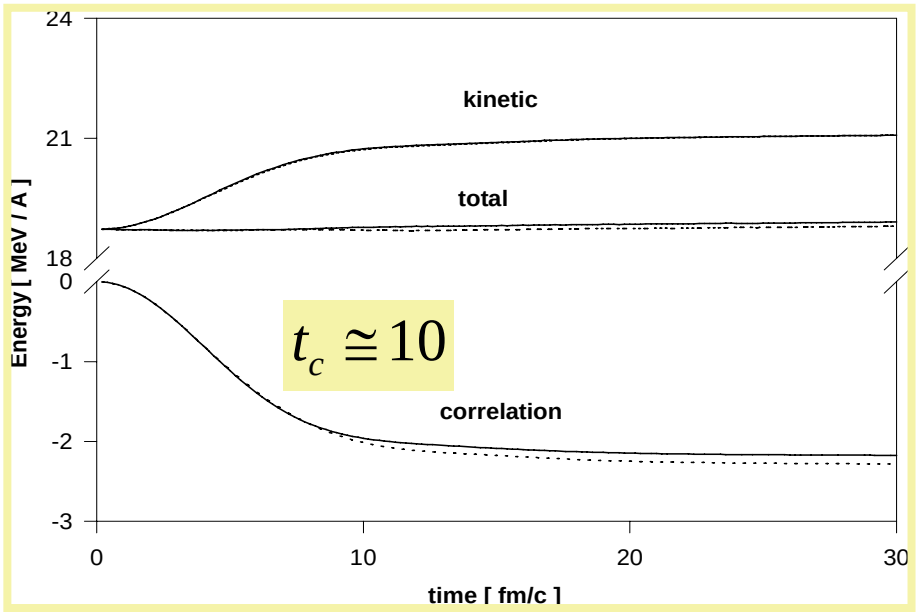
t_c and t_{rel}

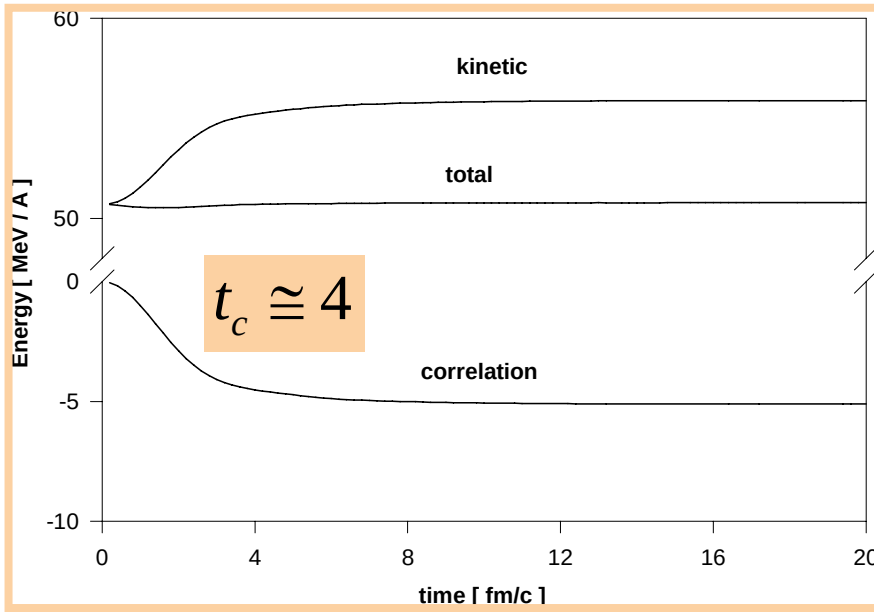
t_c is correlation time, i.e. time at which the correlation - energy is constant assuming an initial uncorrelated system.

t_{rel} is relaxation time, i.e. time for the system to equilibrate or thermalise from an initial non - equilibrium state.

I am here going to discuss (show) some results regarding t_c

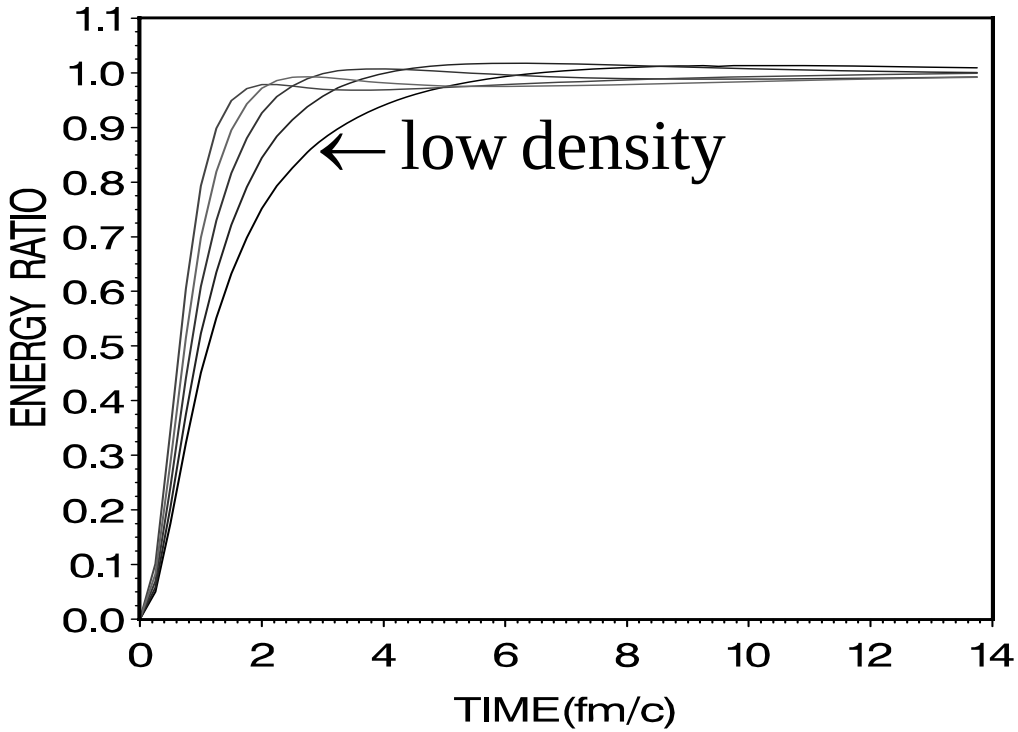
Correlation times



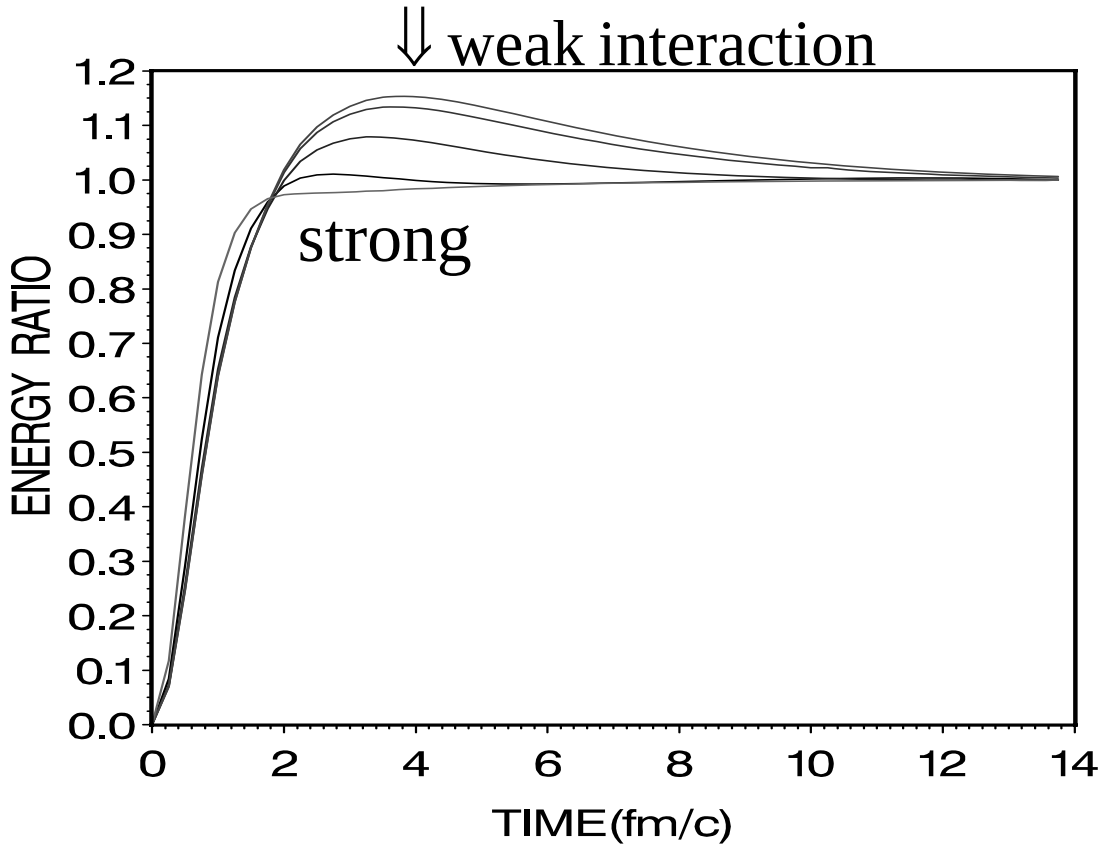


Low density

High density



Correlation energy at densities from left to right
2,1,0.5,.25, and 0.125 times normal.



Correlation energy at strengths 2.,1.,.5,.25, and 0.125 normal

Correlation time

$$t_C = \frac{1}{2E_f}$$

Independent of interaction strength

Kohler, Morawetz PRC 64 (2001) 024613
Morawetz, Kohler Eur Phys J A4 (1999) 291

Temperature

* In the QP-calculations (Boltzmann) the temperature of the initial state is preserved after the time-iterations are completed.

* This is in general not the case in the KB-calculations.

* The system is heated when correlations build up.

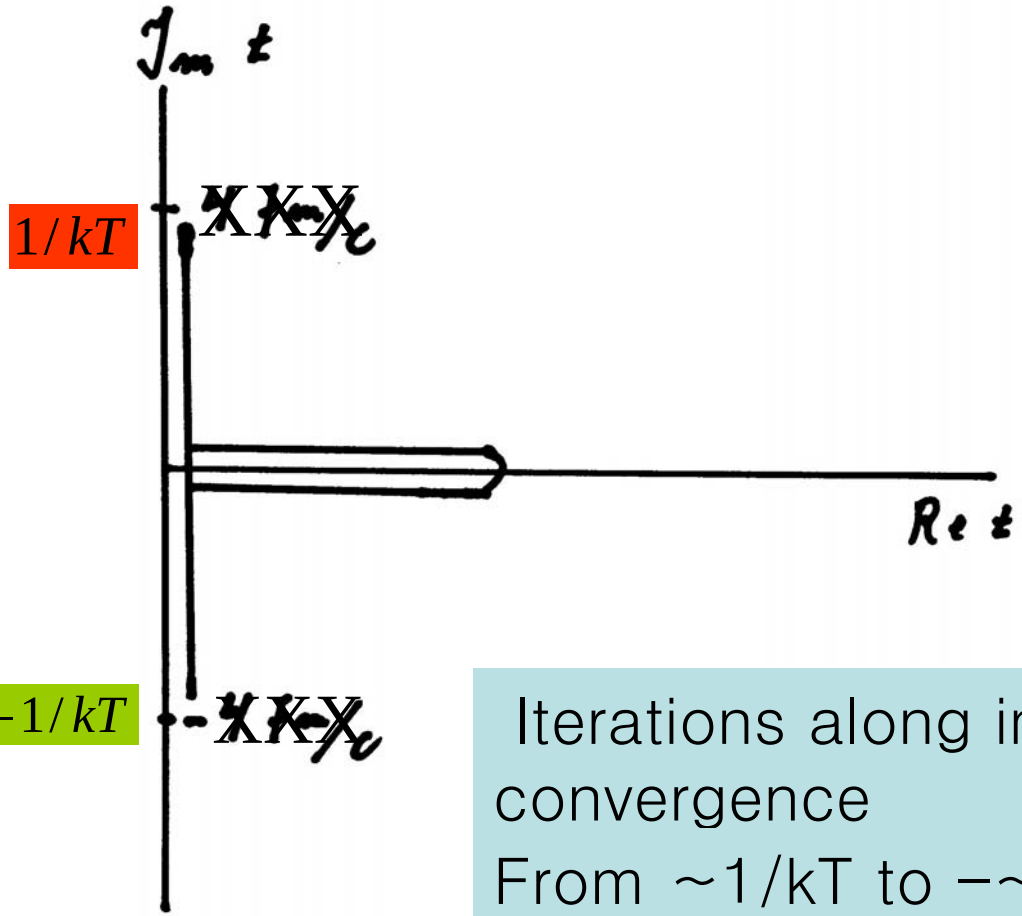
* The final equilibrium temperature T and chemical potential μ are in this case obtained from:

$$\sum{}^{<}(p,\omega) = -e^{\beta(\omega-\mu)}\sum{}^{>}(p,\omega)$$

Note that this implies that $\Sigma^{>}/\Sigma^{<}$ is independent of p which has to be satisfied computationally.

Note also that this requires a Fourier - transformation from the computed $\Sigma(p,t,t') \Rightarrow \Sigma(p,\omega)$ that needs a sufficiently large $t-t'$ range to be accurate. This formalism has been succesfully applied in some unpublished work.

Preparing correlated system at temperature T



Iterations along imaginary time-axis until convergence

From ~1/kT to -~1/kT

Following work by Pawel Danielewicz a computer program has been built applying the imaginary time-stepping technique to specifically realize low-temperature systems.

After choosing some upper and lower imaginary times the temperature of the so obtained correlated system is calculated by the method described above. I do not know of any other.

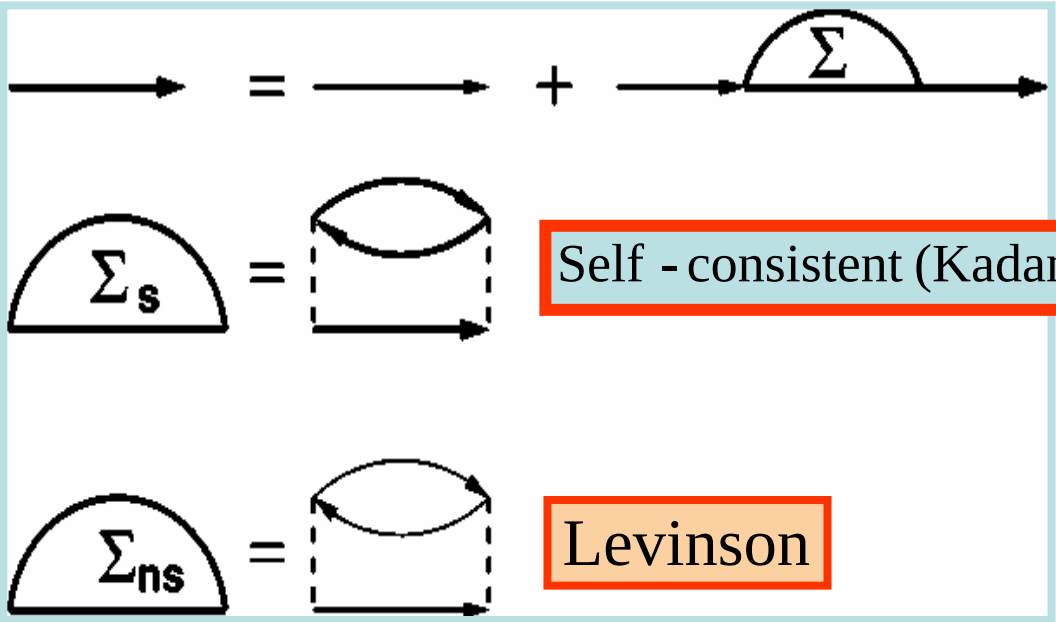
- The imaginary time-stepping technique was used by Pawel Danielewicz in early work. In some unpublished work with Nai Kwong it was used together with the Temperature –routine for some RPA-calculations.

Approximations of KB eqs

- There are several methods of trying to improve upon Boltzmann Markovian eq.

- Of interest is the Levinson eq

Levinson equation



Self - consistent (Kadanoff - Baym)

Levinson

Kremp, Bonitz, Kraeft and Schlanges, Ann. Phys. (NY) 258 (1997)320

Morawetz and HSK, Eur. Phys J. A4,291 (1999)

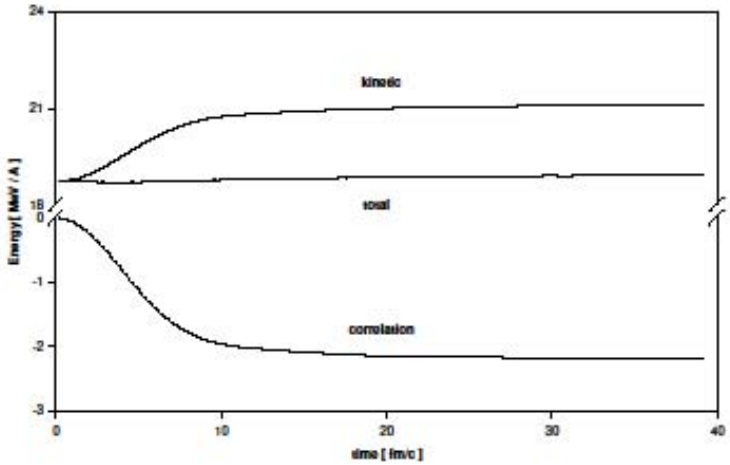
HSK and Morawetz, Phys. Rev. C64, 024613 (2001)

Non-Markovian kinetic equation

Time diagonal part of Kadanoff/Baym equation Levinson equation [$\Delta_E = \frac{k^2}{2m_a} + \frac{p^2}{2m_b} - \frac{(k-q)^2}{2m_a} - \frac{(p+q)^2}{2m_b}$]

$$\frac{\partial}{\partial t}\rho_a(t) \;=\; \frac{2}{\hbar^2}\sum_{bpq} V_{\rm D}^2(q)\int_0^t d\bar{t}\cos\left\{\frac{1}{\hbar}(t-\bar{t})\Delta_E\right\}\;\left(\rho'_a\rho'_b(1-\rho_a)(1-\rho_b) - \rho_a\rho_b(1-\rho'_a)(1-\rho'_b)\right)_{\bar{t}}$$

- particle-number conservation $\frac{\partial}{\partial t}n = 0$
- momentum conservation $\frac{\partial}{\partial t}\langle p_1\rangle = 0$
- Time variation of kinetic energy $\frac{\partial}{\partial t}(\langle\frac{p_1^2}{2m}\rangle + \langle V_{12}\rangle) \equiv 0$ Phys. Lett. A 199 (1995) 241

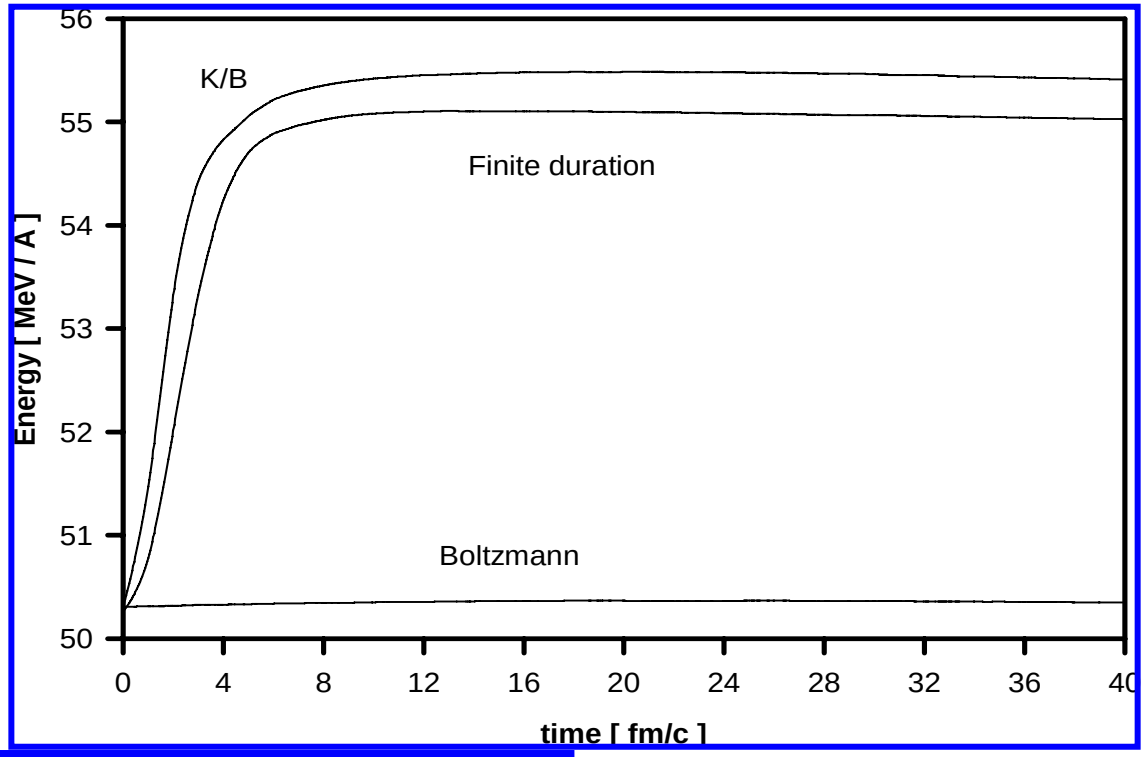


Time evolution of correlation and kinetic energy for nuclear matter with density and temperature $n_1 = n_o/60$ $T_1 = 0.5$ MeV and $n_2 = n_o/20$ $T_2 = 0.1$ MeV moving with relative velocity of $1\hbar/fm$ which corresponds to a collision energy of 21 MeV/n.

Europ. J. Phys. A (1999) 291-305
Phys. Rev. C 64 (2001) 024613

The inclusion of memory effects produces the full energy conservation. But, hides several effects: Off-shell tails in ρ, Renormalization of scattering rates and wave function, quasiparticle energies collision delays Ann. Phys. (NY) 294 (01) 135

F tion



$$E_{corr(t)} = \frac{1}{4}\left\langle \frac{\cos\frac{\Delta E(t-t_0)}{\hbar}-1}{\Delta E} F^{(0)}(t) \right\rangle$$

$F^{(0)}$ means zero order gradient expansion of collision integral.

- There are all kind of problems with the Levinson eq. It is just one example of the “danger” of introducing approximations into the self-consistent KBEs, that are in themselves very stable. Another example is the Boltzmann (transport) eq.
- One initially puzzling problem with Levinson was the following:

From above:

$$E_{corr}(t) = E(t) - K_{\rho}(t)$$

It has been shown* that in the long-time limit Levinson gives:

$$E_{corr} = -1/2 \int d\mathrm{p} \mathrm{V}^2 \rho_1 \rho_2 \rho'_3 \rho'_4 / (\omega_1 + \omega_2 - \omega_3 - \omega_4)$$

with

$$\rho' = 1 - \rho$$

* Eur. Phys. J. A4 (291) (1999)

This looks much like the second order Born, which is no surprise but on closer inspection there is a discrepancy in the factor!!! (It should be ¼.)

Actually we have:

$$E = K_\rho + E_{corr} \quad \text{(Levinson)}$$

while

$$E = K_f + \frac{1}{2} E_{corr} \quad \text{(Born)}$$

and

$$K_\rho \neq K_f$$

suggesting that

$$K_\rho = K_f - \frac{1}{2} E_{corr}$$

EQP-approximation

This gives me a chance to bring in the EQP (Extended Quasi Particle) approximation which like Levinson is a weak interaction limit.

$$S_{EQP} = 2\pi\delta(\omega-\omega_0)Z(p) - P\frac{2\,\mathrm{Im}\,\Sigma^{+}(p,\omega)}{(\omega-\omega_0)^2}$$

$$Z(p) = 1 + \left(\frac{\partial\,\mathrm{Re}\,\Sigma^{+}(p,\omega)}{\partial\omega}\right)$$

which implies

$$G^{<}(p,\omega) = i\ f(p)2\pi Z(p)\delta(\omega-\omega_0) + \Sigma^{<}(p,\omega)\frac{P}{(\omega-\omega_0)^2}$$

This leads to a relation between the correlated and uncorrelated distributions

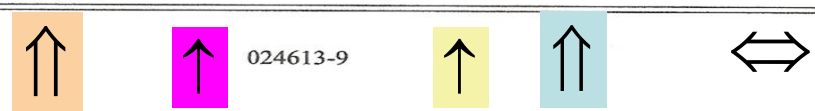
$$\rho(p) = f(p) - i\int \frac{d\omega}{2\pi} \frac{P}{(\omega-\omega_0)^2} \left[\Sigma^{<}(p,\omega)(1-f(p)) + \Sigma^{>}(p,\omega) f(p)\right]$$

from which the relation between the two kinetic energies suggested above is obtained.

agreement between all methods for this case. If the situation very well with the Levinson energies at all densities and

TABLE I. Correlation energies as a function of the density of nuclear matter. At normal density the temperature dependence is also shown. All results are here with $V_0 = 453.0$ MeV. The energies E^{eq}_{corr} are (the negative of) the equilibrium correlation energies. The Born energy E^{eq}_{corr} (Born) is calculated with three different distribution functions as discussed in the text.

ρ (fm^{-3})	T (MeV)	E^{eq}_{corr} (KB) (MeV)	E^{eq}_{corr} (Lev) (MeV)	E^{eq}_{corr} (Born) (MeV) KB	Lev.	Init.	t_c (KB) (fm/c)	$\frac{\hbar}{2E_F}$ (fm/c)
0.380	0	53.63				65.16	2.0	1.5
0.183	0	35.95	49.67		49.69	43.97	2.4	2.4
0.181	10	36.03	48.60	50.52	48.54	49.16		
0.182	20	35.94	46.74		46.65	52.84		
0.182	40	34.31	42.14		42.09	50.52		
0.182	60	31.59	37.22	38.00	37.20	44.65		
0.095	0	23.55	31.33		31.76	28.83	3.4	3.8
0.047	0	14.40	17.84	18.24	18.26	17.47	5.2	6.2
0.023	0	8.42	9.80	9.96	9.96	9.96	8.5	9.7



KB Lev Born

Numerical example

- Some other points.

Separable interaction

Separable interaction

$$< k \,|V\,| \,k' >= v(k)v(k')$$

One technical advantage with local potentials
Is that the FFT routines can be applied throughout.
In nuclear (and other) problems one mostly deals
with non-local interactions. The nucleon-
nucleon interaction (in particular the
singlet-S) is well represented by a separable (non-local)
potential. The FFT can then not be used for
all convolutions. A computer-program was
developed for this case. The run-time was (only) about a
factor of 2 longer. One advantage over FFT is that the
'tails' of the functions can be shorter.

The modern low-momentum renormalized nuclear (N-N) interactions (e.g. Vlowk) are designed so that second order Born would be adequate. The separable interaction would then be useful.

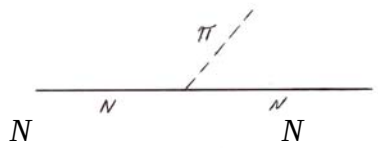
Another application would be for the unitary limit with scattering phase-shifts $\pi/2$. From inverse scattering one finds an analytic expression

$$v^2(k) = \frac{4\pi}{\sqrt{\Lambda^2 - k^2}}$$

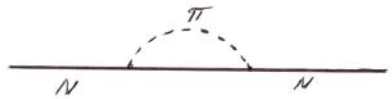
Coupled equations

One problem that has intrigued me is the following: Take nucleons, pions and include also deltas and let them interact via some coupling constants and define self-energies by:

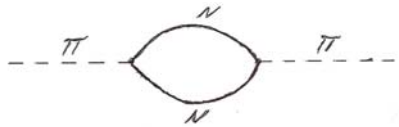
Vertex and coupled propagators



πN vertex



N - propagator



π - propagator

Deltas also included!!

SUMMARY and CONCLUSIONS

- The two-time Greens functions with conserving self-energies provide a valuable tool for studying the physics of many-body systems.
- It has several advantages over the energy-dependent formalism.
- With modern low-momentum separable nuclear interactions it would suffice to use second (or low order) self-energies at low density.
- Future developments should however include going beyond second order Born.
- Coupled KB-equations are an important application for several studies.

END