Electron-phonon interaction. Can dispersionless phonons provide relaxation?

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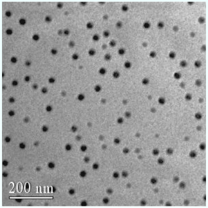
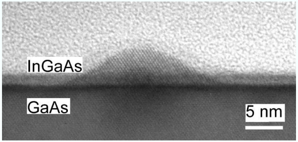
Kiel, 2010

Introduction

Self-assembled quantum dots





S. Anders et al., Phys. Rev. B **66**, 125309 (2002)

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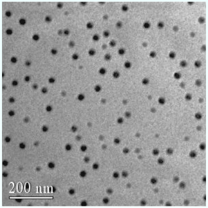

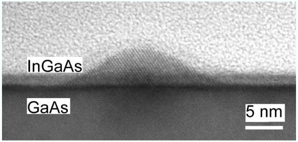

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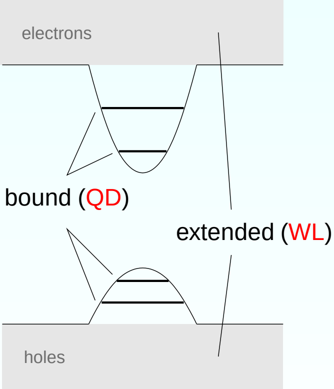
Energy spectrum



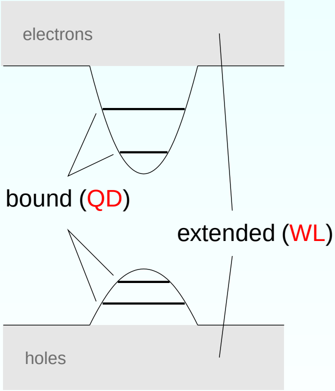
Introduction

Discrete spectrum

- important for optoelectronics
- controlled by quantum dot geometry

Carrier kinetics

Energy spectrum



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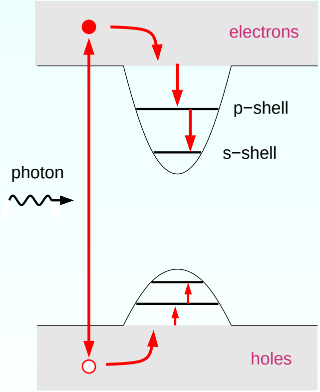
Capture and relaxation



Introduction

Low carrier densities

- carrier-carrier scattering negligible
- carrier-LO-phonon interaction dominant

Perturbation theory:

- capture and relaxation prevented

  phonon bottleneck

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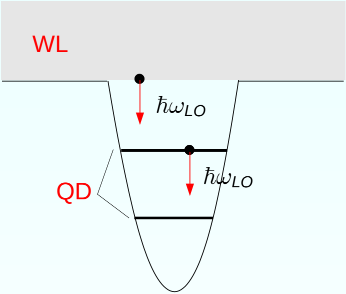


U.Bockelmann, G. Bastard, PRB **42**, 8947 (1990)

H. Benisty *et al.*, PRB **44**, 10945 (1991)

Introduction

Experiment:

fast (ps) capture and relaxation

Introduction

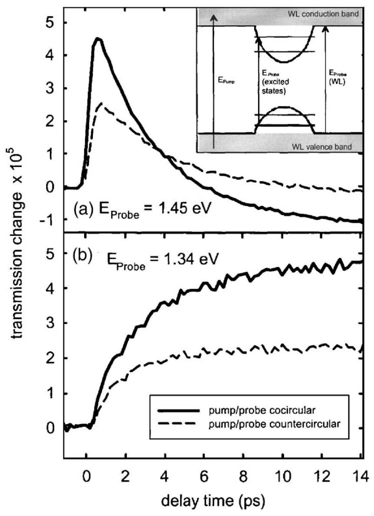
Experiment:

fast (ps) capture and relaxation

Example:

S. Trumm *et al.* APL **87**, 153113 (2005)

- rapid relaxation inside wetting layer
- rapid transfer to bound states
- characteristic times (ps) independent of excitation power



Outline

- Boltzmann kinetics
  - relaxation properties, in general and with dispersionless LO-phonons
- Carrier-phonon system in equilibrium
  - carrier renormalization $\Longrightarrow$ the polaron
- Quantum kinetics
  - 2-time quantum kinetics
  - 1-time approximation
- Results (QWs and QDs)
- Numerical difficulties

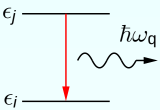
Boltzmann kinetics

Carrier-phonon Hamiltonian:

$$H = \sum_i \epsilon_i a_i^\dagger a_i + \sum_{\boldsymbol{q}} \hbar\omega_q b_{\boldsymbol{q}}^\dagger b_{\boldsymbol{q}} + \sum_{i,j,\boldsymbol{q}} M_{i,j}(\boldsymbol{q}) a_i^\dagger a_j (b_{\boldsymbol{q}} + b_{-\boldsymbol{q}}^\dagger)$$

$$\omega_q = \omega_{LO}\ , \qquad M_{i,j}(\boldsymbol{q}) = g_q \langle \varphi_i | e^{i\boldsymbol{qr}} | \varphi_j \rangle$$

$$g_q^2 \propto \alpha \frac{1}{q^2}$$ Fröhlich coupling



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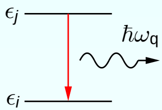

Time evolution of population $f_i = \langle a_i^\dagger a_i \rangle$:

$$\frac{\partial f_i}{\partial t} = \sum_j \{ W_{i,j}(1 - f_i) f_j - W_{j,i}(1 - f_j) f_i \}$$

Boltzmann kinetics

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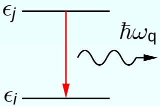

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$W_{i,j}$ = Golden Rule transition rate $\quad j \longrightarrow i \quad$:

$$W_{i,j} = \frac{2\pi}{\hbar} \sum_q |M_{i,j}(q)|^2 \{ N_q \delta(\epsilon_i - \epsilon_j - \hbar\omega_q) + (N_q + 1) \delta(\epsilon_i - \epsilon_j + \hbar\omega_q) \}$$

Boltzmann kinetics

Relaxation properties (analytic):

- conserves positivity:

$$f_i(0) \geqslant 0 \quad \Longrightarrow f_i(t) \geqslant 0$$

- stationary solution:

$$f_i(t) = 1/[e^{\beta(\epsilon_i-\mu)} + 1] = F(\epsilon_i)$$

- for any $f_i(0) \geqslant 0$

$$f_i(t) \longrightarrow F(\epsilon_i) \quad \text{as} \quad t \longrightarrow \infty$$

provided all states can be connected using paths with nonzero transition rates.

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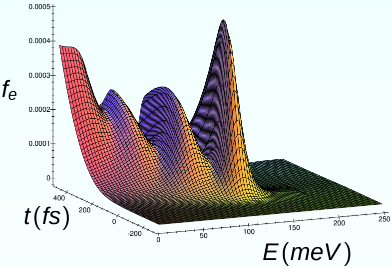
LO-phonons + continuous spectrum (QW)



phonon cascade

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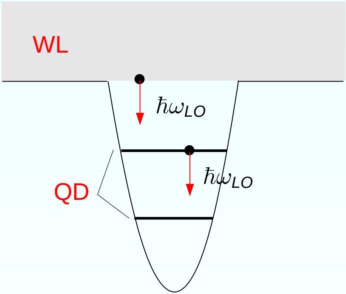
LO-phonons in quantum dots



phonon bottleneck

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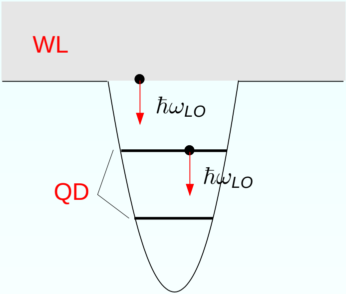
Proposed solutions:

- LO+LA: T.Inoshita, H.Sakaki, PRB.**46**, 7260 (1992)
- LO+LO: A. Knorr group (Berlin) in press

LO-phonons in quantum dots



phonon bottleneck

Boltzmann vs. quantum kinetics

Experiment: fast capture and relaxation (ps)

Boltzmann kinetics:

- weak coupling
- slow evolution
- long time
- no memory (Markovian)

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Quantum kinetics:

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- fast evolution
- both early and later times
- memory effects included

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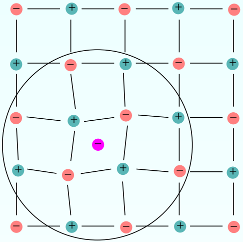
Relaxation properties (numeric)

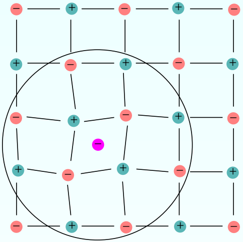
- is a steady state reached ?
- steady state = thermal equilibrium ?

Polarons



electron polarizes crystal

Polarons



electron polarizes crystal

free carrier retarded GF

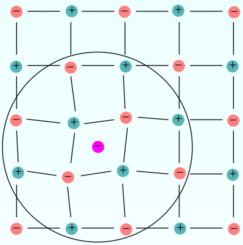
$$G^{\mathrm{R}}_{\alpha}(t) = \frac{\theta(t)}{i\hbar} e^{-\frac{i}{\hbar}\epsilon_\alpha t}$$

$$G^{\mathrm{R}}_{\alpha}(\omega) = \frac{1}{\hbar\omega - \epsilon_\alpha + i\delta}$$

spectral function

$$\begin{aligned} \widehat{G}_{\alpha}(\omega) &= -2\mathrm{Im}\, G^{\mathrm{R}}_{\alpha}(\omega) \\ &= 2\pi\,\delta(\hbar\omega - \epsilon_\alpha) \end{aligned}$$

Polarons



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$$G^{\mathrm{R}}_{\alpha}(\omega) = \frac{1}{\hbar\omega - \epsilon_\alpha + i\delta}$$

polaron retarded GF

$$G^{\mathrm{R}}_{\alpha}(\omega) = \frac{1}{\hbar\omega - \epsilon_\alpha + \Sigma^{\mathrm{R}}_{\alpha}(\omega)}$$

$\Sigma^{\mathrm{R}}_{\alpha}(\omega)$ self-energy for carrier-phonon interaction

spectral function

$$\begin{aligned}\widehat{G}_{\alpha}(\omega) &= -2\mathrm{Im}\, G^{\mathrm{R}}_{\alpha}(\omega) \\ &= 2\pi\,\delta(\hbar\omega - \epsilon_\alpha)\end{aligned}$$

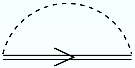
$$\widehat{G}_{\alpha}(\omega) = \quad ?$$

Quantum well polaron

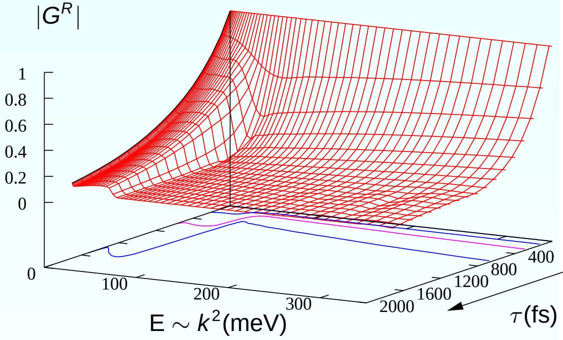
Self-consistent Random Phase Approximation

$$\left\{ i\hbar \frac{\partial}{\partial \tau} - \epsilon_k \right\} G_k^R(\tau) =$$

$$= \int_0^{\tau} d\tau' \Sigma_k^R(\tau') G_k^R(\tau - \tau')$$

$\Sigma =$ 

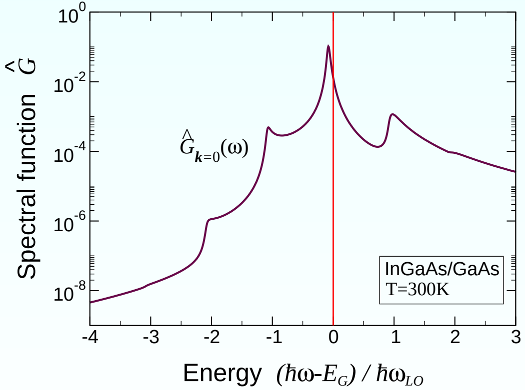
- arbitrary order in α
- multi-phonon processes



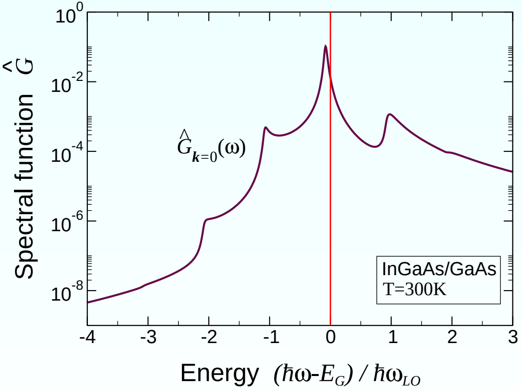
GaAs at T= 300K

$\alpha = 0.06, \quad \hbar\omega_{LO} = 36$ meV

Quantum well polaron



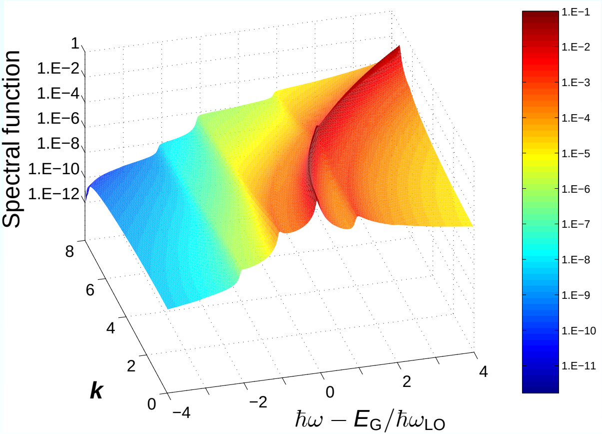
Quantum well polaron

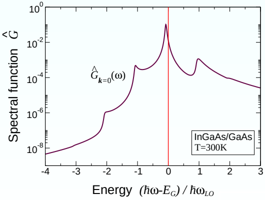


- polaron shift
- energy broadening
- phonon satellites

J. Seebeck, T.R. Nielsen, P. Gartner, F. Jahnke, Phys. Rev. B **71**, 125327 (2005)

Quantum well polaron





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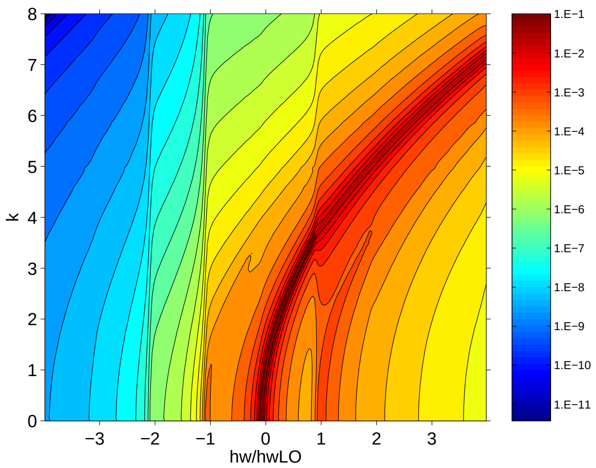
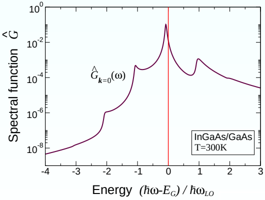




Quantum dot polaron

LO-phonons + quantum dot states only

- Self-consistent Random Phase Approximation
  - T.Inoshita, H.Sakaki, PRB **56**, 2061 (1998)
  - K.Král, Z.Khás, PRB **57**, 4355 (1998)
- finite number of phonons $\Longrightarrow$ diagonalization
  - O.Verzelen, R.Fereira, G.Bastard, PRB **62**, 4809 (2000)
  - T.Stauber, R.Zimmermann, H. Castella, PRB **62**, 7336 (2000)

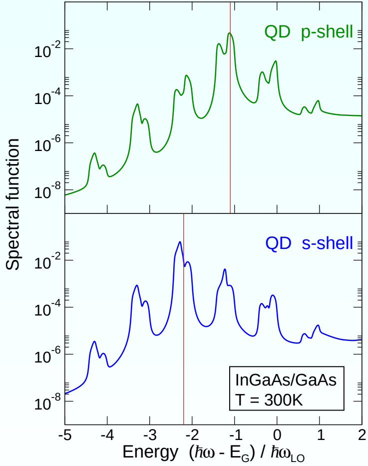
BUT wetting layer states important for both kinetics and spectrum

Quantum dot polaron

LO-phonons + quantum dot + wetting layer states

Quantum dot polaron

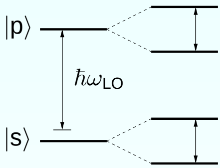
LO-phonons + quantum dot + wetting layer states

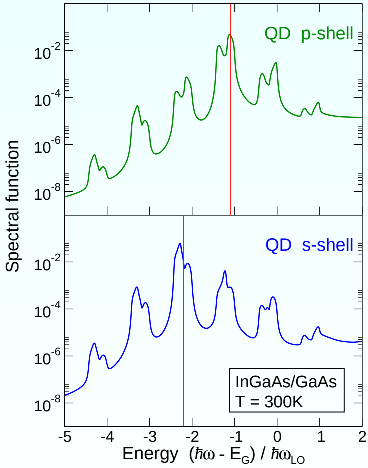


$$\Delta E = 1.1\hbar\omega_{\text{LO}}$$

Quantum dot polaron

LO-phonons + quantum dot + wetting layer states





Hybridization

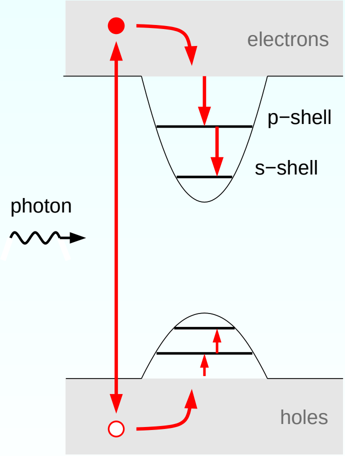
$$\Delta E = 1.1\hbar\omega_{\text{LO}}$$

Quantum kinetics

Quantity of interest:

$$\rho_\alpha(t) = \begin{pmatrix} f^e_\alpha(t) & \psi_\alpha(t) \\ \psi^*_\alpha(t) & 1 - f^h_\alpha(t) \end{pmatrix} = -i\hbar G^<_\alpha(t,t)$$

2x2 matrix in the band index, $\alpha = \boldsymbol{k},\ s,\ p$



Methods:

- Equation of Motion ✘
  - hierarchy problem
- Nonequilibrium Green's Functions ✔
  - polaronic effects
  - 2-time quantities

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Nonequilibrium Green's Functions

Closed Equations for

$$G(t_1, t_2) = \frac{1}{i\hbar} \left\langle T \left[\psi(t_1) \psi^\dagger(t_2) \right] \right\rangle$$

Procedure:

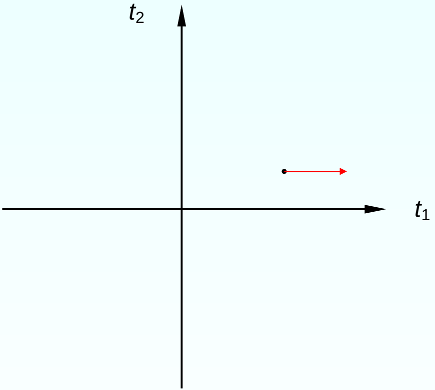
- solve for $G_\alpha(t_1, t_2)$
- recover physical data from the time-diagonal $t_1 = t_2$

Quantum kinetics 2-time

Kadanoff-Baym equations:

$$\left\{ i\hbar \frac{\partial}{\partial t_1} - H_0(1) \right\} G_\alpha(t_1, t_2) =$$

$$= \{\Sigma \cdot G\}_\alpha (t_1, t_2)$$



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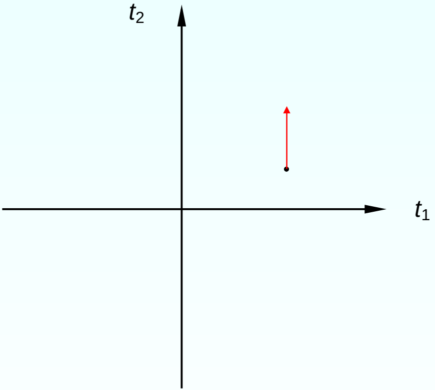
$$= \{\Sigma \cdot G\}_\alpha (t_1, t_2)$$

$$\left\{ i\hbar \frac{\partial}{\partial t_2} + H_0(2) \right\} G_\alpha(t_1, t_2) =$$

$$= -\{G \cdot \Sigma\}_\alpha (t_1, t_2)$$



Quantum kinetics 2-time

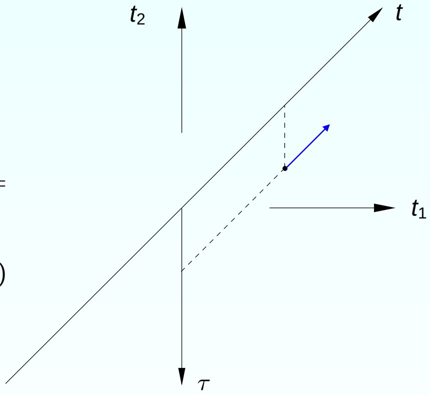
Reparametrize:

$$(t_1, t_2) = (t, t - \tau)$$

t - main time τ - relative time

$$\left\{ i\hbar \frac{\partial}{\partial t} - H_0(1) + H_0(2) \right\} G_\alpha(t, t - \tau) =$$

$$= \{\Sigma \cdot G\}_\alpha (t, t - \tau) - \{G \cdot \Sigma\}_\alpha (t, t - \tau)$$

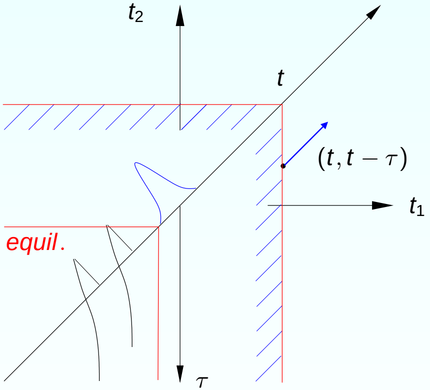


Quantum kinetics 2-time

Polaronic Green's Function:

$G^R_\alpha(t, t - \tau) = G^R_\alpha(\tau)$

- "initial" condition
- natural memory cutoff



Quantum kinetics 2-time

$$
\begin{aligned}
i\hbar \frac{\partial}{\partial t} G^{R,<}_{\boldsymbol{k}}(t,t-\tau) &= \Sigma^{\delta}_{\boldsymbol{k}}(t) G^{R,<}_{\boldsymbol{k}}(t,t-\tau) - G^{R,<}_{\boldsymbol{k}}(t,t-\tau)\Sigma^{\delta}_{\boldsymbol{k}}(t-\tau) + \\
&+ i\hbar \frac{\partial}{\partial t} G^{R,<}_{\boldsymbol{k}}(t,t-\tau)\Big|_{coll} \quad ; \quad \tau \geqslant 0
\end{aligned}
$$

Instantaneous self-energy:

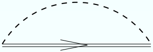
$$
\Sigma^{\delta}_{\boldsymbol{k}}(t) = \begin{pmatrix} \epsilon^{c}_{\boldsymbol{k}} & -d_{cv} \cdot E(t) \\ -d_{vc} \cdot E(t) & \epsilon^{v}_{\boldsymbol{k}} \end{pmatrix}
$$

The optical excitation connects the two bands.

Quantum kinetics 2-time

$$i\hbar \frac{\partial}{\partial t} G_{\boldsymbol{k}}^{R,<}(t,t-\tau) = \Sigma_{\boldsymbol{k}}^{\delta}(t) G_{\boldsymbol{k}}^{R,<}(t,t-\tau) - G_{\boldsymbol{k}}^{R,<}(t,t-\tau)\Sigma_{\boldsymbol{k}}^{\delta}(t-\tau) + \\ + i\hbar \frac{\partial}{\partial t} G_{\boldsymbol{k}}^{R,<}(t,t-\tau)\Big|_{coll} \quad ; \quad \tau \geqslant 0$$

RPA self-energies:



$$\Sigma_{\boldsymbol{k}}^{R}(t,t') = i\hbar \sum_{\boldsymbol{q}} g_q^2 \left[D^{>}(t-t') G_{\boldsymbol{k}-\boldsymbol{q}}^{R}(t,t') + D^{R}(t-t') G_{\boldsymbol{k}-\boldsymbol{q}}^{<}(t,t') \right]$$

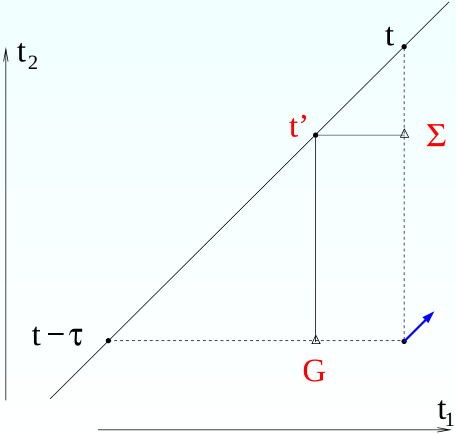
$$\Sigma_{\boldsymbol{k}}^{<}(t,t') = i\hbar \sum_{\boldsymbol{q}} g_{\boldsymbol{q}}^2 \, D^{<}(t-t') G_{\boldsymbol{k}-\boldsymbol{q}}^{<}(t,t')$$

$$i\hbar D^{>}(\tau) = (N_{LO}+1)\, e^{-i\hbar\omega_{LO}\tau} + N_{LO}\, e^{i\hbar\omega_{LO}\tau} = i\hbar D^{<}(-\tau)$$

Quantum kinetics 2-time

Collision terms for G^R :

$$i\hbar\frac{\partial}{\partial t}G^R_{\boldsymbol{k}}(t,t-\tau)\Big|_{coll} = \int_{t-\tau}^{t} dt' \left[\Sigma^R_{\boldsymbol{k}}(t,t')G^R_{\boldsymbol{k}}(t',t-\tau) - G^R_{\boldsymbol{k}}(t,t')\Sigma^R_{\boldsymbol{k}}(t',t-\tau)\right]$$



Quantum kinetics 2-time

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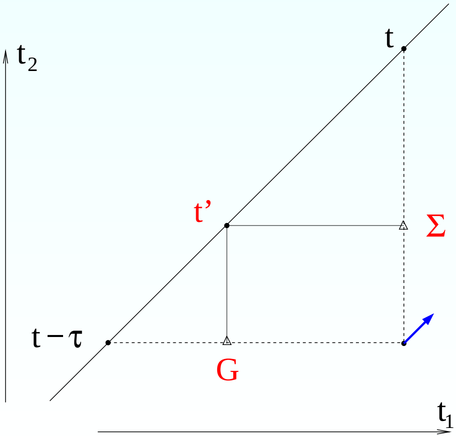
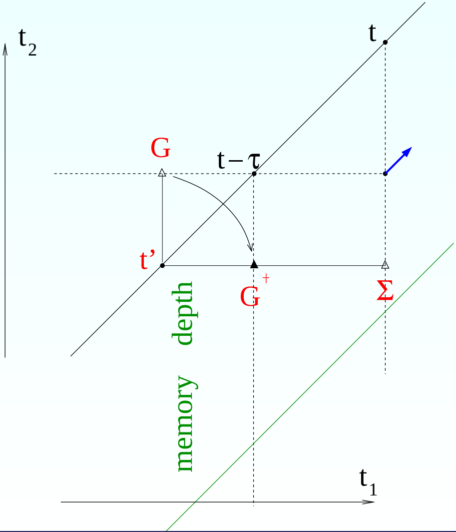


Quantum kinetics 2-time

Collision terms for $G^<$:



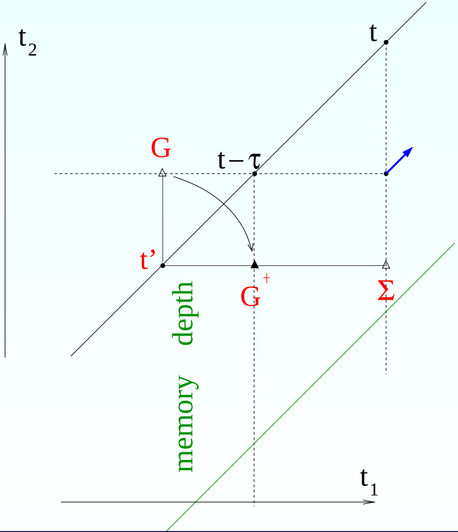
$$i\hbar \frac{\partial}{\partial t} G_{\boldsymbol{k}}^<(t, t-\tau)\Big|_{coll} =$$

$$\int_{-\infty}^{t} dt' \Big[\Sigma_{\boldsymbol{k}}^R(t,t') G_{\boldsymbol{k}}^<(t', t-\tau)$$

$$+ \Sigma^< G^A - G^R \Sigma^< - G^<(t,t')\Sigma^A \Big]$$

Quantum kinetics 2-time

Collision terms for $G^<$:



$$i\hbar \frac{\partial}{\partial t} G^<_{\boldsymbol{k}}(t, t-\tau)\Big|_{coll} =$$

$$\int_{-\infty}^{t} dt' \left[\Sigma^R_{\boldsymbol{k}}(t,t') G^<_{\boldsymbol{k}}(t', t-\tau) \right.$$

$$\left. + \Sigma^< G^A - G^R \Sigma^< - G^<(t,t') \Sigma^A \right]$$

$$G^<(t_1, t_2) = -\left[G^<(t_2, t_1) \right]^\dagger$$

$$G^A(t_2, t_1) = \left[G^R(t_1, t_2) \right]^\dagger$$

Quantum kinetics 2-time

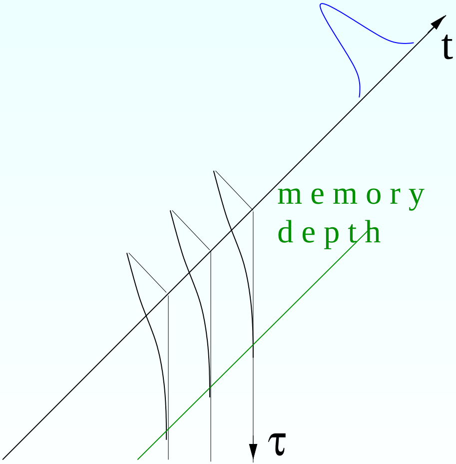
Initial conditions - the polaron

Before the pulse $\Longrightarrow$ equilibrium

$$G^R(t,t-\tau) = \begin{pmatrix} g_c^R(\tau) & 0 \\ 0 & g_v^R(\tau) \end{pmatrix}$$

$$G^<(t,t-\tau) = \begin{pmatrix} 0 & 0 \\ 0 & -g_v^R(\tau) \end{pmatrix}$$

$$\begin{aligned} g_c^<(\tau) &= 0 \\ g_v^>(\tau) &= 0 \\ g_v^R(\tau) &= g_v^>(\tau) - g_v^<(\tau) \end{aligned}$$



Quantum kinetics 2-time

No t-evolution before excitation:

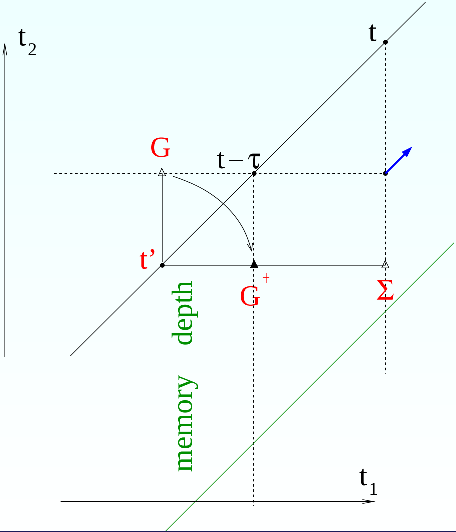
$$i\hbar\frac{\partial}{\partial t}G^{R,<}_{\boldsymbol{k}}(t,t-\tau) = \Sigma^{\delta}_{\boldsymbol{k}}(t)G^{R,<}_{\boldsymbol{k}}(t,t-\tau) - G^{R,<}_{\boldsymbol{k}}(t,t-\tau)\Sigma^{\delta}_{\boldsymbol{k}}(t-\tau) + \\ + i\hbar\frac{\partial}{\partial t}G^{R,<}_{\boldsymbol{k}}(t,t-\tau)\Big|_{coll} \quad ; \quad \tau \geqslant 0$$

$$\Sigma^{\delta}_{\boldsymbol{k}}(t) = \begin{pmatrix} \epsilon^{c}_{\boldsymbol{k}} & 0 \\ 0 & \epsilon^{v}_{\boldsymbol{k}} \end{pmatrix}$$

$$\Sigma^{\delta}_{\boldsymbol{k}}G^{R,<}_{\boldsymbol{k}}(t,t-\tau) - G^{R,<}_{\boldsymbol{k}}(t,t-\tau)\Sigma^{\delta}_{\boldsymbol{k}} = 0$$

Quantum kinetics 2-time

No t-evolution before excitation:

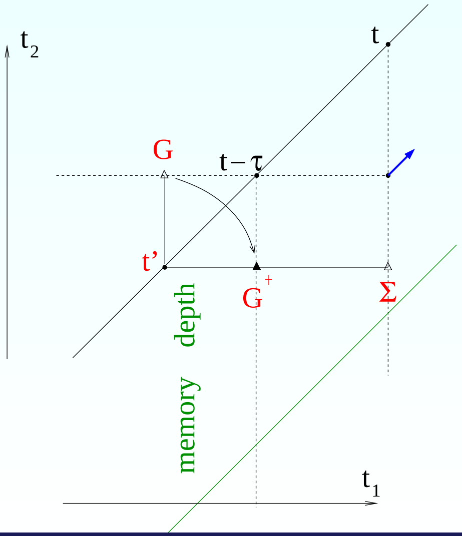


$$i\hbar \frac{\partial}{\partial t} G^<_{\boldsymbol{k}}(t, t-\tau)\Big|_{coll} =$$

$$\int_{-\infty}^{t} dt' \left[\Sigma^R_{\boldsymbol{k}}(t,t') G^<_{\boldsymbol{k}}(t', t-\tau) + \Sigma^< G^A - G^R \Sigma^< - G^<(t,t')\Sigma^A \right]$$

Quantum kinetics 2-time

No t-evolution before excitation:



$$i\hbar \frac{\partial}{\partial t} G^<_{\boldsymbol{k}}(t, t-\tau)\Big|_{coll} =$$

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$$\left. + \Sigma^< G^A - G^R \Sigma^< - G^<(t,t')\Sigma^A \right]$$

$$G^R(t, t-\tau) = \begin{pmatrix} g^R_c(\tau) & 0 \\ 0 & g^R_v(\tau) \end{pmatrix}$$

$$G^<(t, t-\tau) = \begin{pmatrix} 0 & 0 \\ 0 & -g^R_v(\tau) \end{pmatrix}$$

Quantum kinetics 1-time

- Generalized Kadanoff-Baym ansatz (GKBA)

$$G^{\lessgtr}_{\alpha}(t,t') \approx i\hbar G^{R}_{\alpha}(t-t')\, G^{\lessgtr}_{\alpha}(t',t') \qquad t > t'$$

$$\frac{d}{dt} f_\alpha(t) = 2\mathrm{Re} \int_{-\infty}^{t} dt' \sum_{\beta\, q} |M_{\alpha,\beta}(q)|^2 \; G^{\mathrm{R}}_{\beta}(t-t') \left[G^{\mathrm{R}}_{\alpha}(t-t')\right]^* \times \Big\{ -f_\alpha(t') \; \left[1 - f_\beta(t')\right] D^{>}_{q}(t-t') + \left[1 - f_\alpha(t')\right] \; f_\beta(t') D^{<}_{q}(t-t') \Big\}$$

$$D^{\lessgtr}_{q}(t) = N_{LO}\, e^{\mp i\omega_{\mathrm{LO}} t} + (1 + N_{LO})\, e^{\pm i\omega_{\mathrm{LO}} t}$$

- renormalized quasiparticles (polarons)
- beyond Markov approximation (memory effects)

Quantum kinetics 1-time

- Generalized Kadanoff-Baym ansatz (GKBA)

$$G^{\lessgtr}_{\alpha}(t,t') \;\approx\; i\hbar G^{R}_{\alpha}(t-t')\, G^{\lessgtr}_{\alpha}(t',t') \qquad t > t'$$

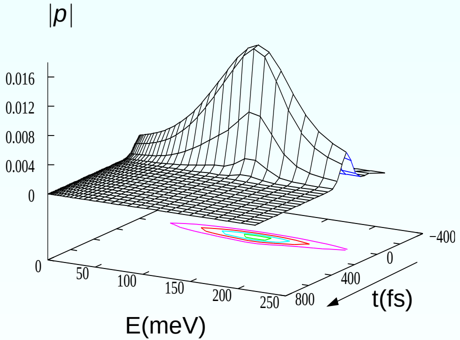
$$\frac{d}{dt} f_\alpha(t) = 2\mathrm{Re} \int_{-\infty}^{t} dt' \sum_{\beta\, q} |M_{\alpha,\beta}(q)|^2 \; e^{-i\epsilon_\beta (t-t')} \left[e^{-i\epsilon_\alpha (t-t')}\right]^* \times \Big\{ -f_\alpha(t)\; \left[1 - f_\beta(t)\right] D^{>}_{q}(t-t') + \left[1 - f_\alpha(t)\right]\; f_\beta(t) D^{<}_{q}(t-t') \Big\}$$

$$D^{\lessgtr}_{q}(t) = N_{LO}\, e^{\mp i\omega_{\mathrm{LO}} t} + (1+N_{LO})\, e^{\pm i\omega_{\mathrm{LO}} t}$$

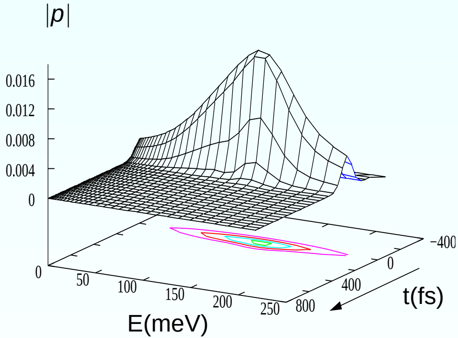
- bare particles
- Markov approximation (no memory effects)

Quantum well results: polarization

CdTe $\alpha = 0.31$ T = 300K



2-time

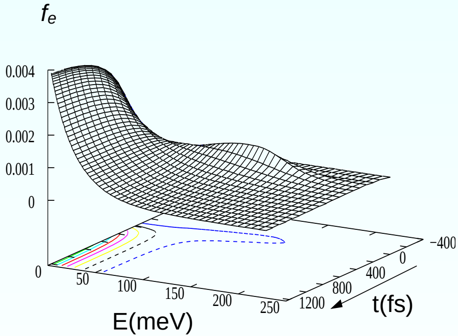


1-time

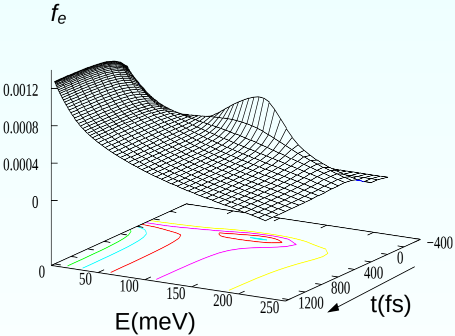
Strong dephasing

Quantum well results: electron population

CdTe $\alpha = 0.31$ T = 300K



2-time



1-time

Steady state reached
Not the same!

Kubo-Martin-Schwinger condition

- Exact relation in thermal equilibrium

KMS condition

$$f_\alpha = \int \frac{d(\hbar\omega)}{2\pi} F(\omega)\, \widehat{G}_\alpha(\omega)$$

Kubo-Martin-Schwinger condition

- Exact relation in thermal equilibrium

KMS condition

$$f_\alpha = \int \frac{d(\hbar\omega)}{2\pi} F(\omega)\, \widehat{G}_\alpha(\omega)$$

- Non-interacting system

→ $\widehat{G}_\alpha(\omega) = 2\pi\, \delta(\hbar\omega - \epsilon_\alpha)$

→ $f_\alpha = F(\epsilon_\alpha)$

Kubo-Martin-Schwinger condition

- Exact relation in thermal equilibrium

KMS condition

$$f_\alpha = \int \frac{d(\hbar\omega)}{2\pi} F(\omega)\, \widehat{G}_\alpha(\omega)$$

- Non-interacting system

  → $\widehat{G}_\alpha(\omega) = 2\pi\, \delta(\hbar\omega - \epsilon_\alpha)$

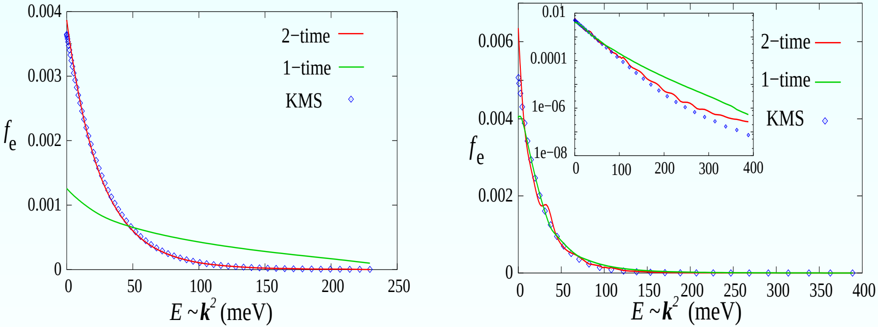
  → $f_\alpha = F(\epsilon_\alpha)$

- Interacting system

  → Generalization of Fermi-Dirac distribution

Kubo-Martin-Schwinger test in quantum wells

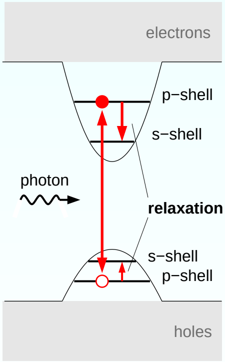
CdTe, $\alpha = 0.31$ GaAs, $\alpha = 0.06$



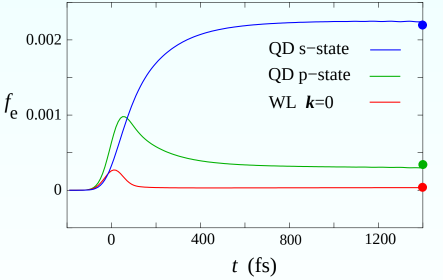
P. Gartner, J. Seebeck, F. Jahnke, Phys. Rev. **B 73**, 115307 (2006)

Kubo-Martin-Schwinger test in quantum dots

- CdTe $\alpha = 0.31$ $\Delta E = 2.4\hbar\omega_{\text{LO}}$ p-level excitation



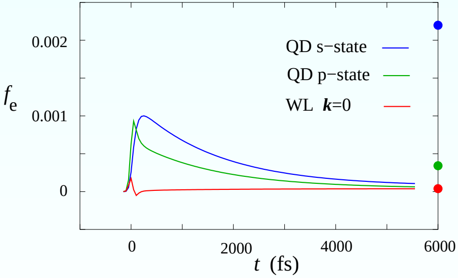
2-time calculation



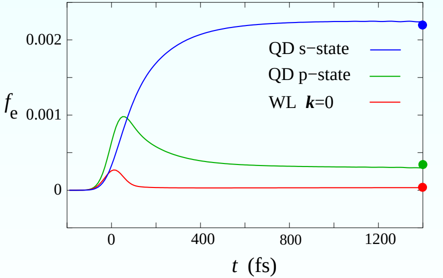
Kubo-Martin-Schwinger test in quantum dots

- CdTe $\alpha = 0.31$ $\Delta E = 2.4\hbar\omega_{\mathrm{LO}}$ p-level excitation

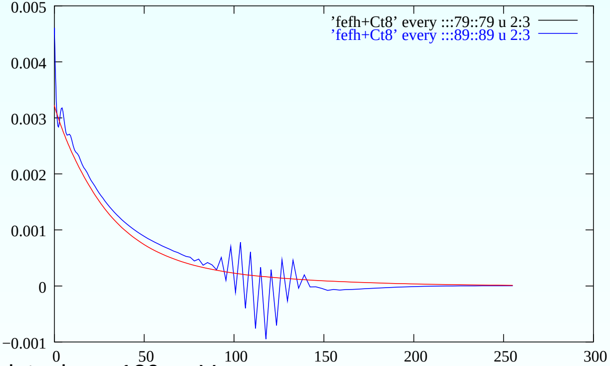
1-time calculation



2-time calculation



Numerics: instabilities



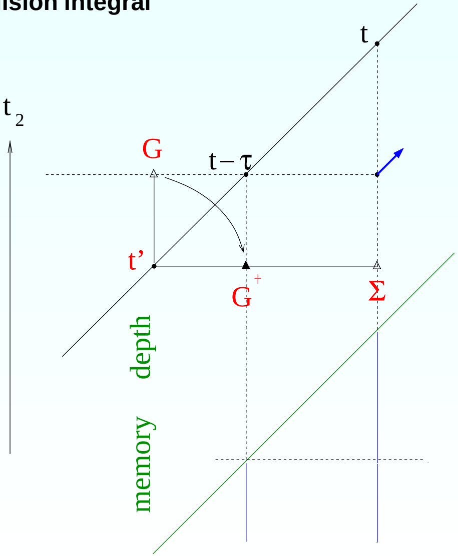
detuning = 120 meV

t = 550fs t = 650fs

abrupt cut of the collision integrals at memory **depth = 500 fs**

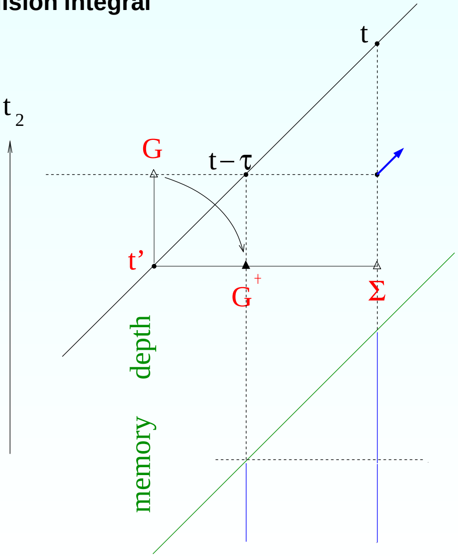
Numerics: instabilities

Collision integral



Numerics: instabilities

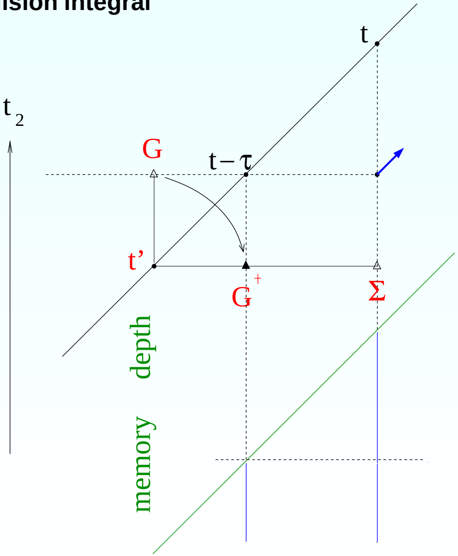
Collision integral



- integral abruptly cut at memory depth

Numerics: instabilities

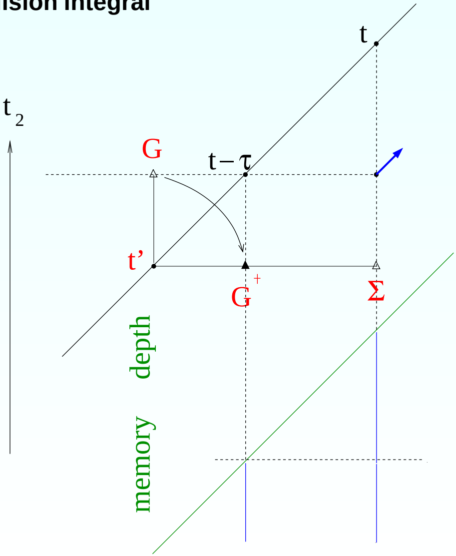
Collision integral



- integral abruptly cut at memory depth
- beyond the memory depth extend with equilibrium Σ , G

Numerics: instabilities

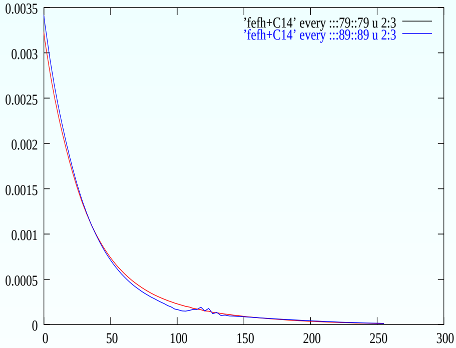
Collision integral

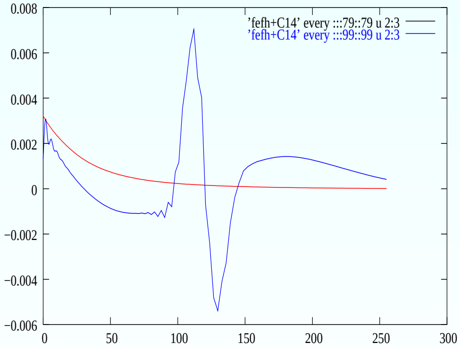


- integral abruptly cut at memory depth
- beyond the memory depth extend with equilibrium Σ , G
- smooth interpolation to equilibrium Σ , G

Numerics: instabilities

Collision integrals extended with equilibrium GFs



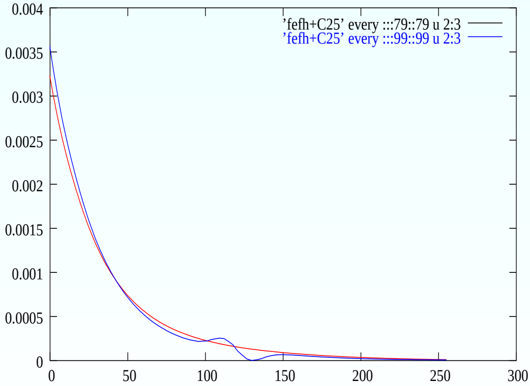


t = 550fs t = 650fs t = 550fs t = 750fs

Numerics: instabilities

Collision integrals extended with equilibrium GFs

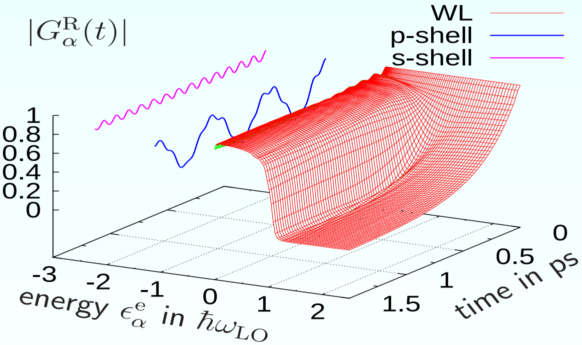
- smooth change using an interpolating (Fermi-like) function



t = 550fs t = 750fs

Low temperatures

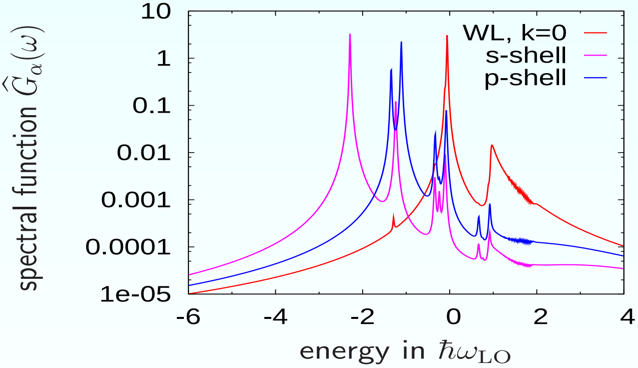
The polaron



Polaronic GF: GaAs at T=10K

Low temperatures

The polaron



Spectral function: GaAs at T=10K

Low temperatures

Quantum kinetics 1-time

$$\frac{d}{dt} f_\alpha(t) = 2\mathrm{Re} \int_{-\infty}^{t} dt' \sum_{\beta\, q} |M_{\alpha,\beta}(q)|^2 \; G^{\mathrm{R}}_\beta(t-t') \left[G^{\mathrm{R}}_\alpha(t-t')\right]^* \times \Big\{ -f_\alpha(t') \left[1 - f_\beta(t')\right] D^{>}_q(t-t') + \left[1 - f_\alpha(t')\right] f_\beta(t') D^{<}_q(t-t') \Big\}$$

Low temperatures

Quantum kinetics 1-time

$$\frac{d}{dt} f_\alpha(t) = 2\mathrm{Re} \int_{-\infty}^{t} dt' \sum_{\beta\, q} |M_{\alpha,\beta}(q)|^2 \; G_\beta^{\mathrm{R}}(t-t') \left[G_\alpha^{\mathrm{R}}(t-t') \right]^* \times \left\{ -f_\alpha(t') \; \left[1 - f_\beta(t') \right] D_q^{>}(t-t') + \left[1 - f_\alpha(t') \right] \; f_\beta(t') D_q^{<}(t-t') \right\}$$

$$G_\alpha^{\mathrm{R}}(t) = \frac{1}{i\hbar} \Theta(t) \left[\sum_j e^{-\frac{i}{\hbar}(\epsilon_\alpha - \Delta_{\alpha,j})t} + \mathcal{G}_\alpha^R(t) \right]$$

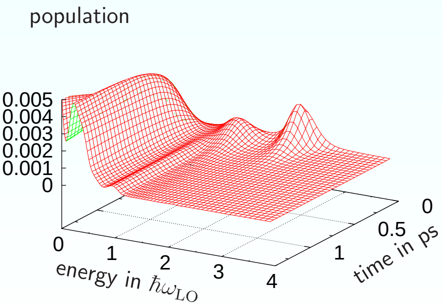
$$\mathcal{G}_\alpha^R(t) \to 0 \quad \text{for} \quad t \to \infty$$

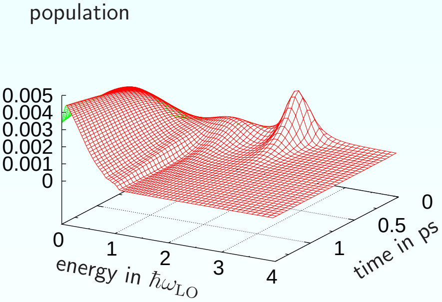
Low temperatures

WL electron population

2-time

⇓



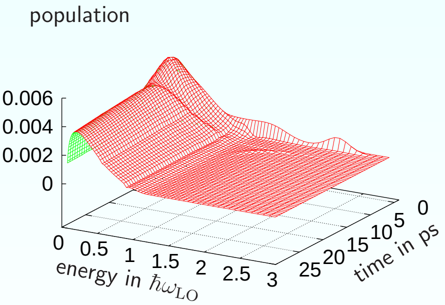


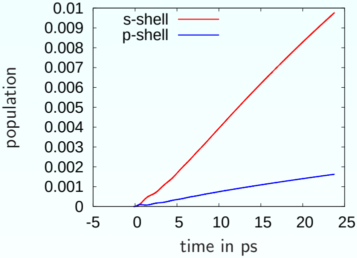
⇑

1-time

Low temperatures

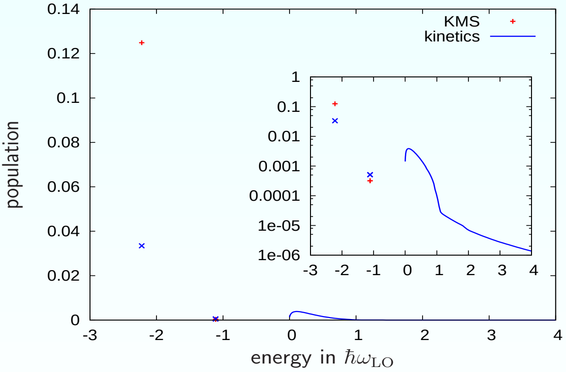
Electron population (long time)





Low temperatures

KMS test of the steady state



no thermalization (asymptotic bottleneck)

Summary and conclusions

- Carrier-LO-phonon interaction role - twofold:
  - equilibrium: polaron
  - kinetics: polaron-phonon scattering
- Relaxation properties (room temp.):
  - Steady state reached both in 2-time and in 1-time kinetics
  - Thermalization:
    - 2-time kinetics: Yes!
    - 1-time kinetics, only low α
- Problems with low temperatures:
  - 2-time - only the early kinetics
  - 1-time - no thermalization (asymptotic bottleneck ?)