Relativistic Kadanoff-Baym equations for correlated initial states and baryogenesis

Mathias Garny (TU München)

KBE2010, CAU Kiel, 23–26.03.2010

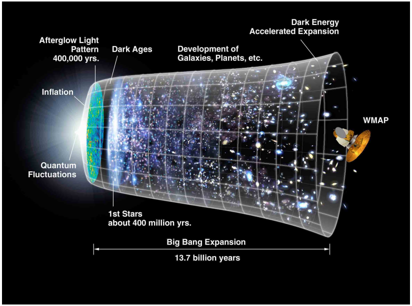
based on Phys.Rev.D80:085011,2009 with Markus Michael Müller
ongoing work with Urko Reinosa

Outline

Relativistic Kadanoff-Baym equations for correlated initial states and baryogenesis

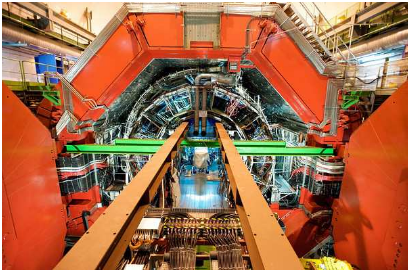
- Motivation: Baryogenesis
- Relativistic Kadanoff-Baym Equations
- UV divergences, renormalization and Non-Gaussian ICs
- Results for $\lambda\Phi^4$ 2PI 3-loop
- Numerical methods

Nonequilibrium dynamics at high energy



Heavy Ion Collisions

- LHC: ALICE
- RHIC



Early universe

- Reheating after Inflation
- Baryogenesis
- ...

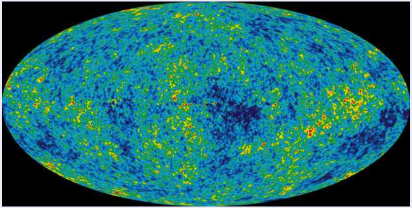
Baryogenesis

Baryon asymmetry of the universe

- Our galaxy consists of matter: $\bar{p}/p \lesssim 10^{-3}$
- No annihilations observed

Asymmetry parameter

$$\eta = \frac{n_b - n_{\bar{b}}}{s}$$



n_b = baryon density

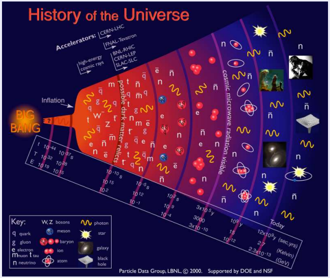
$n_{\bar{b}}$ = anti-baryon density

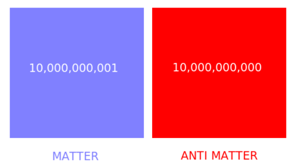
s = entropy density

$$4.7 \cdot 10^{-10} < \eta < 6.5 \cdot 10^{-10} \text{ (95\% CL)}$$

Baryogenesis

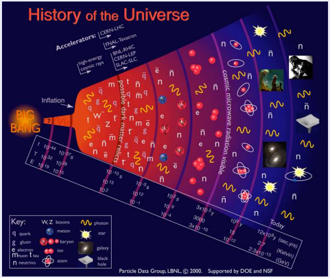
Baryon asymmetry of the universe

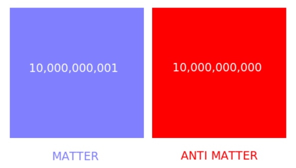




Baryogenesis

Baryon asymmetry of the universe





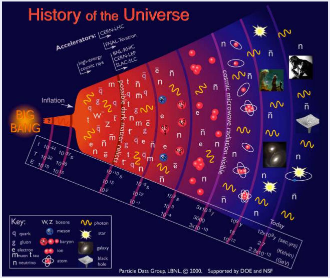
(1) Initial baryon asymmetry after Big Bang

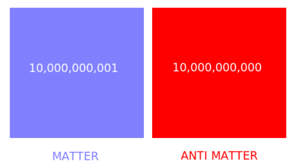
Problem:

- Diluted by inflation
- Washed out by $\Delta B \neq 0$ processes at high energy

Baryogenesis

Baryon asymmetry of the universe





(1) Initial baryon asymmetry after Big Bang

Problem:

- Diluted by inflation
- Washed out by $\Delta B \neq 0$ processes at high energy

(2) Dynamical creation: Baryogenesis

Baryogenesis

Baryogenesis: three Sakharov conditions

Sakharov 1967

- baryon number violation: $\langle B \rangle \neq$ const.
- CP violation: $\Gamma(i \to f) \neq \Gamma(\bar{i} \to \bar{f})$
- deviation from thermal equilibrium: $\Gamma(i \to f) \neq \Gamma(f \to i)$

Baryogenesis

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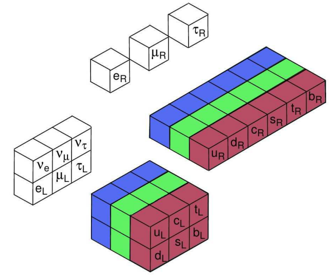
Baryogenesis within the Standard Model of Particles ?



- B-violation for $T > T_{EW}$

$$\Delta B = \Delta L$$

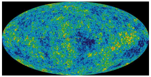
- CP-violation in quark mixing
  $\rightarrow \quad K^0/\bar{K}^0$ decay
- Expanding universe

But: much too weak...

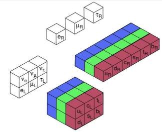
Baryogenesis

Collider exp.

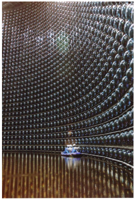
Baryogenesis



$4.7{\cdot}10^{-10} < \eta < 6.5{\cdot}10^{-10}$



$+\,?$



Superkamiokande

Neutrino Detector

Neutrino exp.

Dark matter

$$\frac{\rho_{\text{dark matter}}}{\rho_0} \simeq 0.23$$

Baryogenesis

Baryogenesis: three Sakharov conditions

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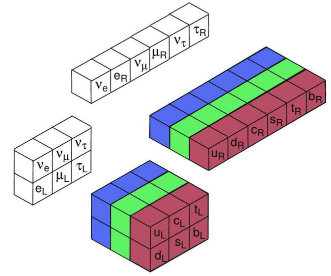
- deviation from thermal equilibrium: $\Gamma(i \to f) \neq \Gamma(f \to i)$

Baryogenesis in the SM + Right-handed neutrinos $(\nu_R)_{e,\mu,\tau}$



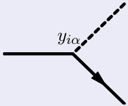
- B- via L-violation $M_R \bar{\nu}_R \nu_R^c$
  $\to 0\nu\beta\beta$: $(A,Z) \to (A,Z+2) + 2e^-$
  (Gerda, Nemo, Exo, ...)
- CP-violation in ν-mixing
  $\to \nu$-oscillation
  (Double Chooz, Daya Bay, ...)
- Expanding universe

Baryogenesis

Leptogenesis

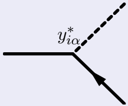
Out-of-equilibrium decay of heavy right-handed neutrino ν_R

$$\mathcal{M}_{\nu_{R,i}\to\ell_{L,\alpha}h^\dagger} =$$



$$+\ldots$$

$$\mathcal{M}_{\nu_{R,i}\to\ell^c_{L,\alpha}h} =$$



$$+\ldots$$

Baryogenesis

Leptogenesis

Out-of-equilibrium decay of heavy right-handed neutrino ν_R

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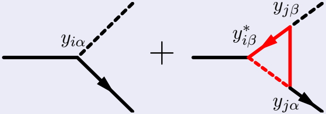
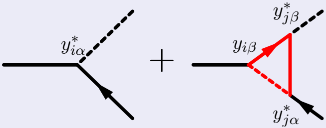


$+\dots$

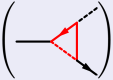
$\mathcal{M}_{\nu_{R,i}\to\ell^c_{L,\alpha}h} =$



$+\dots$

CP violation in decay described by loop process

$\Gamma(\nu_{R,i}\to\ell_{L,\alpha}h^\dagger)-\Gamma(\nu_{R,i}\to\ell^c_{L,\alpha}h) \sim \text{Im}(y_{i\alpha}y_{i\beta}y^*_{j\alpha}y^*_{j\beta})\cdot\text{Im}\Big($



$\Big)$

Baryogenesis

Baryogenesis via Leptogenesis

- CP violation in decay described by loop process
- deviation from thermal equilibrium

Quantum nonequilibrium effects ?

The semi-classical approach

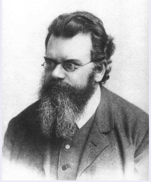
Boltzmann equation for leptogenesis

$$p^\alpha \mathcal{D}_\alpha f_{\ell_L}(t,\mathbf{x},\mathbf{p}) \;=\; \int d\Pi_{\nu_R} d\Pi_h$$

$$\times\, (2\pi)^4 \delta(p_{\ell_L} + p_h - p_{\nu_R})$$

$$\times \Big[|\mathcal{M}|^2_{\nu_R \to \ell_L h^\dagger}\; f_{\nu_R}(1 - f_{\ell_L})(1 + f_h)$$

$$-\; |\mathcal{M}|^2_{\ell_L h^\dagger \to \nu_R}\; f_{\ell_L} f_h (1 - f_{\nu_R}) \Big]$$



$|\mathcal{M}|^2$: microscopic interactions, off-shell processes

$f(t,\mathbf{x},\mathbf{p})$: macroscopic propagation of on-shell particles

$$\Delta x_{interaction} \ll \lambda_{mfp}, \quad \lambda_{de-Broglie} \ll \lambda_{mfp}$$

$$1/M \ll 1/\Gamma, \quad 1/T \ll 1/(y^2 T)$$

Corrections within Boltzmann picture

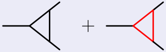
Bose-enhancement, Pauli-Blocking; kinetic (non-)equilibrium

- quantum statistical factors $1\pm f_k$
- non-integrated Boltzmann equations

Hannsestad, Basbøll 06; Garayoa, Pastor, Pinto, Rius, Vives 09; Hahn-Woernle, Plümacher, Wong 09

Medium corrections

- medium correction to decay rates



$$\epsilon = \frac{\Gamma(\nu_R \to \ell h^\dagger) - \Gamma(\nu_R \to \ell^c h)}{\Gamma(\nu_R \to \ell h^\dagger) + \Gamma(\nu_R \to \ell^c h)} = \epsilon^{vac} + \delta\epsilon^{th}(T, \ldots)$$

- thermal masses

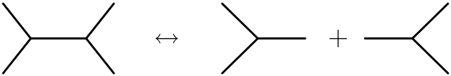
MG, Hohenegger, Kartavtsev, Lindner 09; Kiessig, Plümacher 09; Giudice, Notari, Raidal, Riotto, Stumia 04; Covi, Rius, Roulet, Vissani 98;...

Flavour effects

Nardi, Nir, Roulet, Racker 06; Adaba, Davidson, Ibarra, Josse-Micheaux, Losada, Riotto 06; Blanchet, diBari 06...

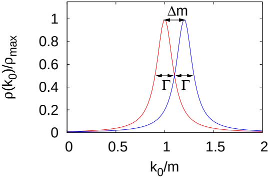
Limitations of the Boltzmann approach

- Unstable particles lead to double counting problems
  [real intermediate state subtraction]



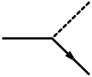
- Resonant Leptogenesis: $\Gamma \sim \Delta M$

Pilaftsis, Underwood, . . .

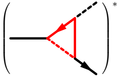


- Quantum interference out of equilibrium

$$\epsilon = \frac{\Gamma(\nu_R \to \ell h^\dagger) - \Gamma(\nu_R \to \ell^c h)}{\Gamma(\nu_R \to \ell h^\dagger) + \Gamma(\nu_R \to \ell^c h)} \sim$$



$\times$



$\sim 10^{-7}$

Going beyond the Boltzmann picture

Statistical propagator $G_F^{ij}(x,y) = \langle \Phi^i(x)\Phi^j(y) + \Phi^j(y)\Phi^i(x)\rangle/2$

Spectral function $G_\rho^{ij}(x,y) = i\langle \Phi^i(x)\Phi^j(y) - \Phi^j(y)\Phi^i(x)\rangle$

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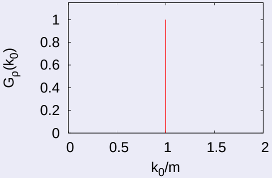
Boltzmann limit

- on-shell quasi-stable particles



$$G_\rho^{ij}(k) \sim \delta^{ij}\delta(k^2 - m_i^2)$$

- equilibrium-like fluctuation-dissipation relation

$$G_F^{ij}(t,k) = \left(f_k^i(t) + \frac{1}{2}\right) G_\rho^{ij}(k)$$

Going beyond the Boltzmann picture

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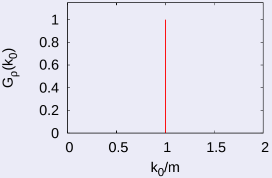

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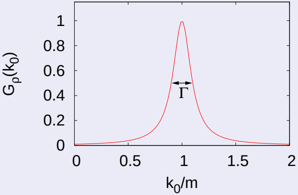
Propagation beyond Boltzmann

- spectrum with (thermal) width



$$G_\rho^{ij}(t,k) \sim \frac{\delta^{ij} 2k_0 \Gamma_i(t)}{(k^2 - m_{th,i}^2(t))^2 + k_0^2 \Gamma_i(t)^2} + \dots$$

- on-/off-shell, cross-correlations

$$G_F^{ij}(t,k) = \begin{pmatrix} G_F^{11} & G_F^{12} \\ G_F^{21} & G_F^{22} \end{pmatrix}$$

Relativistic Kadanoff-Baym equations

Relativistic Kadanoff-Baym equations

$$\left(\partial_{x^0}^2 - \nabla_x^2 + m_i^2(x)\right) G_F^{ij}(x,y) \quad = \quad \int_0^{y^0} d^4z\, \Pi_F^{ik}(x,z) G_\rho^{kj}(z,y) - \int_0^{x^0} d^4z\, \Pi_\rho^{ik}(x,z) G_F^{kj}(z,y)$$

$$\left(\partial_{x^0}^2 - \nabla_x^2 + m_i^2(x)\right) G_\rho^{ij}(x,y) \quad = \quad \int_{x_0}^{y^0} d^4z\, \Pi_\rho^{ik}(x,z) G_\rho^{kj}(z,y)$$

Statistical propagator $G_F^{ij}(x,y) = \langle \Phi^i(x)\Phi^j(y) + \Phi^j(y)\Phi^i(x)\rangle/2$

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Relativistic Kadanoff-Baym equations

Obtained from stationarity condition of the 2PI effective action

Cornwall, Jackiw, Tomboulis (1974)

$$\frac{\delta\Gamma[\phi, G]}{\delta G} = 0 \quad \Leftrightarrow \quad G^{-1} = G_0^{-1} - \Pi[G]$$

where $G(x,y) = G_F(x,y) - i/2\,\mathrm{sign}_{\mathcal{C}}(x^0 - y^0) G_\rho(x,y)$

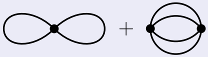
Controlled approximation...

... by truncation of the 2PI functional $\Gamma_2[\phi, G]$

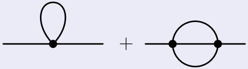
Relativistic Kadanoff-Baym equations

Example: Three-loop truncation in $\lambda\Phi^4$-theory (for $\langle\Phi\rangle = 0$)

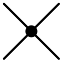
$$\Gamma_2[G] \quad = $$



$$\Pi[G] \;=\; \frac{2i\delta\Gamma_2}{\delta G} \quad = $$



$$S \;=\; \int d^4x \left(\frac{1}{2}\partial_\mu\phi\partial^\mu\phi - \frac{1}{2}m^2\phi^2 - \frac{\lambda}{4!}\phi^4 \right)$$

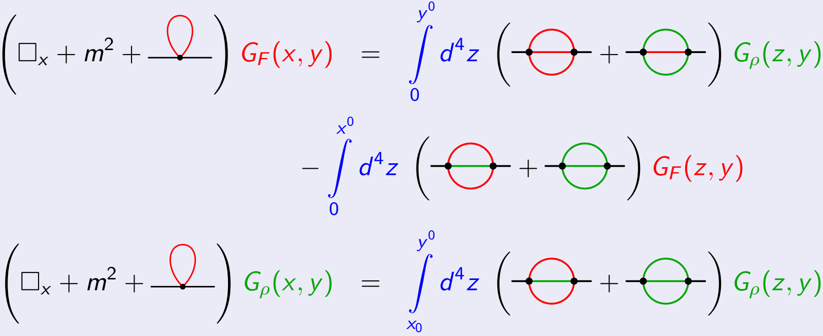


$$= \quad -i\lambda\delta(x_1 - x_2)\delta(x_1 - x_3)\delta(x_1 - x_3)$$

$$= \quad G(x, y)$$

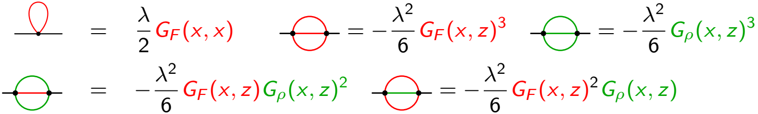
Relativistic Kadanoff-Baym equations

Setting-sun approximation for $\lambda\Phi^4$-theory



$$\left(\Box_x + m^2 + \right) G_F(x,y) = \int_0^{y^0} d^4z \left(+ \right) G_\rho(z,y) - \int_0^{x^0} d^4z \left(+ \right) G_F(z,y)$$

$$\left(\Box_x + m^2 + \right) G_\rho(x,y) = \int_{x_0}^{y^0} d^4z \left(+ \right) G_\rho(z,y)$$



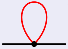
$$= \frac{\lambda}{2} G_F(x,x) \qquad = -\frac{\lambda^2}{6} G_F(x,z)^3 \qquad = -\frac{\lambda^2}{6} G_\rho(x,z)^3$$

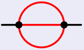
$$= -\frac{\lambda^2}{6} G_F(x,z) G_\rho(x,z)^2 \qquad = -\frac{\lambda^2}{6} G_F(x,z)^2 G_\rho(x,z)$$

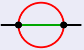
Relativistic Kadanoff-Baym equations

Homogeneous system

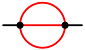
$$G(x,y) = G(x^0,y^0,\mathbf{x}-\mathbf{y}) \to G(x^0,y^0,\mathbf{k}), \quad \mathbf{k} = (k_x,k_y,k_z)$$

$\Big(\partial_{x^0}^2 + \mathbf{k}^2 + m^2 +$  $\Big)\, G_F(x^0,y^0,\mathbf{k}) \quad =$

$\int_0^{y^0} dz^0 \, \Big($  $+$  $\Big)\, G_\rho(z^0,y^0,\mathbf{k})$

$-\int_0^{x^0} dz^0 \, \Big($  $+$  $\Big)\, G_F(z^0,y^0,\mathbf{k})$

 $= \frac{\lambda}{2}\int\frac{d^3p}{(2\pi)^3}\, G_F(x^0,x^0,\mathbf{p})$

 $= -\frac{\lambda^2}{6}\int\frac{d^3p}{(2\pi)^3}\int\frac{d^3q}{(2\pi)^3}\, G_F(x^0,z^0,\mathbf{p})G_F(x^0,z^0,\mathbf{q})G_F(x^0,z^0,\mathbf{k}-\mathbf{p}-\mathbf{q})$

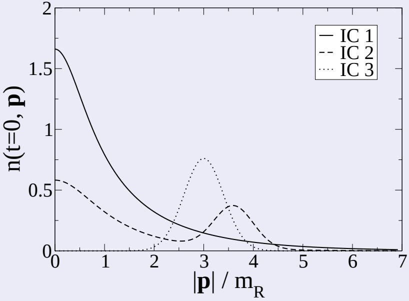
$= -\frac{\lambda^2}{6}\int d^3x\, e^{i\mathbf{kx}}\left[\int\frac{d^3p}{(2\pi)^3}\, e^{-i\mathbf{px}}G_F(x^0,z^0,\mathbf{p})\right]^3$ Danielewicz, Köhler,...

Relativistic Kadanoff-Baym equations

Initial conditions (Gaussian initial state)

Example: $\phi = \dot{\phi} = 0$,

$$\begin{aligned} G(x^0, y^0, \mathbf{p})|_{x^0=y^0=t_{init}} &= \frac{n_{\mathbf{p}}(t_{init}) + 1/2}{\omega_{\mathbf{p}}(t_{init})} \\ (\partial_{x^0} + \partial_{y^0}) G(x^0, y^0, \mathbf{p})|_{x^0=y^0=t_{init}} &= 0 \\ \partial_{x^0}\partial_{y^0} G(x^0, y^0, \mathbf{p})|_{x^0=y^0=t_{init}} &= \omega_{\mathbf{p}}(t_{init})\,(n_{\mathbf{p}}(t_{init}) + 1/2) \end{aligned}$$



$$\omega_{\mathbf{p}}(t_{init}) = \sqrt{m_R^2 + \mathbf{k}^2}$$

Relativistic Kadanoff-Baym equations

...very incomplete list...

NR nuclear coll: Danielewicz (1983); Köhler (1994,. . .); . . .

- Thermalization in relativistic scalar QFT

  Berges, Cox (2001); Berges (2002); Aarts, Berges (2002); Aarts, Resco (2003); Juchem, Cassing, Greiner (2004); Arrizabalaga, Smit, Tranberg (2005); Lindner, Müller (2006); Gasenzer, Pawlowski (2008); . . .

- Thermalization in relativistic fermionic QFT

  Berges, Borsanyi, Serreau (2003); Lindner, Müller (2008)

- Prethermalization

  Berges, Borsanyi, Wetterich (2004)

- Nonequilibrium Instabilities, Parametric Resonance

  Berges, Serreau (2003), Aarts, Tranberg (2007); Berges, Rothkopf, Schmidt (2008); Berges, Pruschke, Rothkopf (2009)

- 2PI renormalization *Borsanyi,Reinosa (2008); MG, Müller (2009)*

Leptogenesis/Baryogenesis [no two-time KBE analysis yet]

Buchmüller, Fredenhagen (2000); DeSimone, Riotto (2007); Anisimov, Buchmüller, Drewes, Mendizabal (2008,10); MG, Hohenegger, Kartavtsev, Lindner (2009,10); Gagnon (2009)

Relativistic Kadanoff-Baym equations

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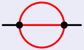
UV divergences, renormalization and non-Gaussian ICs

UV Divergences

$$= \frac{\lambda}{2}\int\frac{d^3p}{(2\pi)^3}G_{F,0}(x^0,x^0,p)$$

$$= \frac{\lambda}{2}\int\frac{d^3p}{(2\pi)^3}\frac{n_p+\frac{1}{2}}{\sqrt{m^2+\mathbf{p}^2}} \sim \Lambda^2$$

$$\int dz^0$$

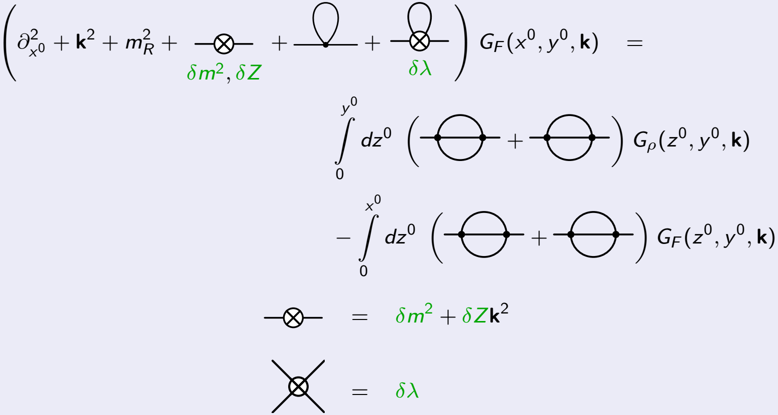


$$G(z^0,y^0,k) \sim \Lambda^2$$

UV cut-off $|\mathbf{p}| \leq k_{max} \equiv \Lambda$ ⇒ Quadratic divergence

UV divergences, renormalization and non-Gaussian ICs

Renormalization ?



Nonperturbative 2PI counterterms: *Hees, Knoll (2001, 2002); Blaizot, Iancu, Reinosa (2003); Berges, Borsanyi, Reinosa, Serreau (2004, 2005); Reinosa, Serreau (2006, 2007, 2009)*

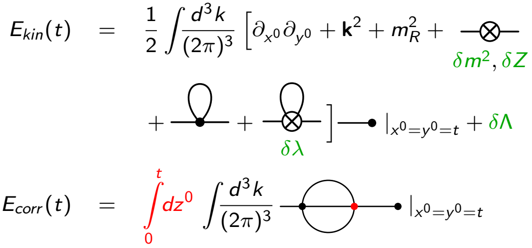
UV divergences, renormalization and non-Gaussian ICs

Problem

- Gaussian initial states
- ‘bare’ particles in the initial state
- Incompatible with renormalization

Why does the Gaussian initial state lead to singularities ?

$$E_{total} = E_{kin}(t) + E_{corr}(t)$$



$$E_{kin}(t) = \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\left[\partial_{x^0}\partial_{y^0} + \mathbf{k}^2 + m_R^2 + \underset{\delta m^2,\,\delta Z}{-\!\otimes\!-} + \quad + \underset{\delta\lambda}{\otimes}\right] \Big|_{x^0=y^0=t} + \delta\Lambda$$

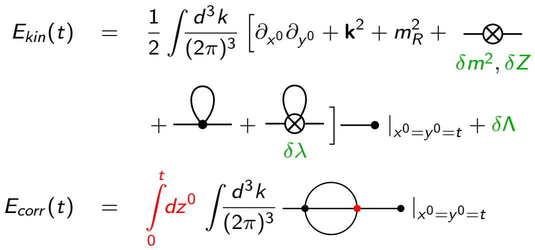
$$E_{corr}(t) = \int_0^t dz^0 \int\frac{d^3k}{(2\pi)^3} \Big|_{x^0=y^0=t}$$

- $E_{corr}(t)$ contains divergences
- $E_{kin}(t)$ contains 2PI counterterms

  Berges, Borsanyi, Reinosa, Serreau (2004, 2005)

Why does the Gaussian initial state lead to singularities ?

$$E_{total} = E_{kin}(t) + E_{corr}(t)$$

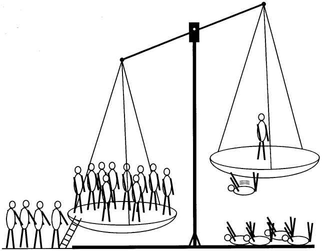


$$E_{kin}(t) = \frac{1}{2}\int\frac{d^3k}{(2\pi)^3}\left[\partial_{x^0}\partial_{y^0} + \mathbf{k}^2 + m_R^2 + \underset{\delta m^2,\,\delta Z}{-\!\otimes\!-} + \quad + \underset{\delta\lambda}{-\!\otimes\!-}\right]\!\!-\!\!\bullet\;\Big|_{x^0=y^0=t} + \delta\Lambda$$

$$E_{corr}(t) = \int_0^t dz^0 \int\frac{d^3k}{(2\pi)^3} \quad \Big|_{x^0=y^0=t}$$

- $E_{corr}(t)$ contains divergences
- $E_{kin}(t)$ contains 2PI counterterms

  Berges, Borsanyi, Reinosa, Serreau (2004, 2005)
- $E_{corr}(t)|_{t=0} = 0$ for Gaussian initial state
- $\Rightarrow$ unbalanced divergence at $t = 0$



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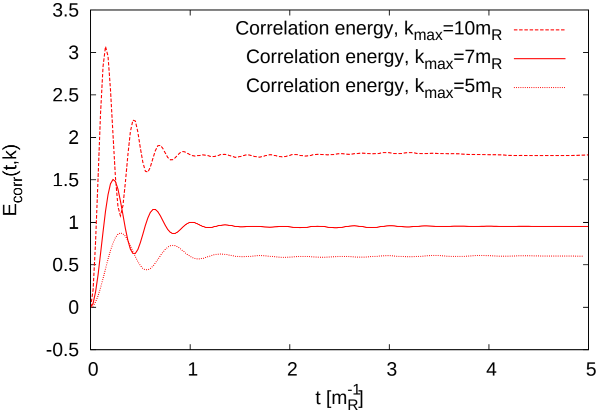


UV divergences, renormalization and non-Gaussian ICs

Non-Gaussian initial state

- *Explicit:* non-Gaussian density matrix ρ at $t = t_{init}$
  - imaginary time-stepping $\rho = \exp(-\mathcal{O})$
  - initial *n*-point correlation functions

$$\langle \varphi_+ | \rho | \varphi_- \rangle = \exp\left(i \sum_{n=0}^{\infty} \int_{x_i} \alpha_n^{\epsilon_1 \cdots \epsilon_n}(\mathbf{x_1}, \ldots, \mathbf{x_n}) \varphi_{\epsilon_1}(\mathbf{x_1}) \cdots \varphi_{\epsilon_n}(\mathbf{x_n}) \right)$$

where $\Phi(t_{init}, \mathbf{x}) | \varphi_\pm \rangle = \varphi_\pm(\mathbf{x}) | \varphi_\pm \rangle$ *[Calzetta, Hu]*

e.g. Hall, Kukharenkov, Tikhodeev, Danielewicz, Wagner, Schlanges, Bornath, Semkat, Kremp, Bonitz, Köhler, Morozov, Röpke,...

- *Implicit*
  - external two-point source $K(x, y)$ for $t < t_{init}$

    e.g. Borsanyi, Reinosa,...

UV divergences, renormalization and non-Gaussian ICs

Gaussian initial state

All n-point correlation functions vanish at $t = t_{init}$ for $n \geq 3$

$$\alpha_n(x_1, \ldots, x_n) = 0 \quad \text{for } n \geq 3$$

Renormalized initial state

Relevant n-point correlation functions asymptotically agree with vacuum correlations at short distances [for $n \leq 4$]

$$\alpha_n(x_1, \ldots, x_n) = \alpha_n^{vac/th}(x_1, \ldots, x_n) + \Delta\alpha_n(x_1, \ldots, x_n)$$

UV divergences, renormalization and non-Gaussian ICs

Initial *n*-point correlation functions:

Local standard vertices:

 $= -i\lambda_R,$  $= -i\delta\lambda$

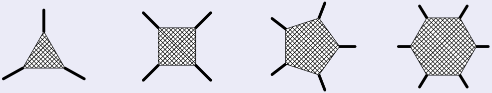
UV divergences, renormalization and non-Gaussian ICs

Initial *n*-point correlation functions:

Local standard vertices:

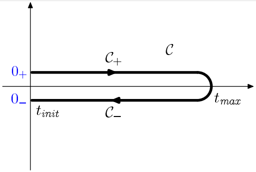
 $= -i\lambda_R$,  $= -i\delta\lambda$

Effective non-local vertices: $\alpha_n(x_1, \ldots, x_n)$



…encode the non-Gaussian initial correlations

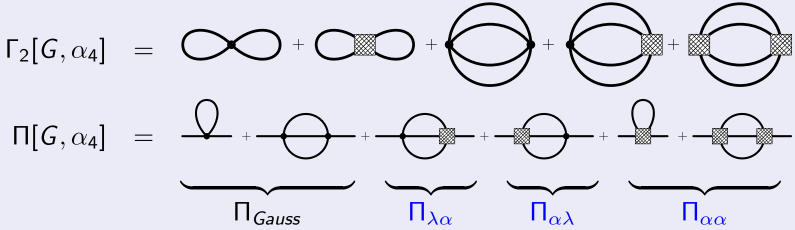
$$\alpha_n(x_1, \ldots, x_n) = \sum_{\epsilon_i \in \pm} \alpha_n^{\epsilon_1, \ldots, \epsilon_n}(\mathbf{x_1}, \ldots, \mathbf{x_n}) \times \delta(x_1^0 - 0_{\epsilon_1}) \cdots \delta(x_n^0 - 0_{\epsilon_n})$$



$\alpha_n^{\epsilon_1, \ldots, \epsilon_n}(\mathbf{x_1}, \ldots, \mathbf{x_n})$, $\epsilon_i \in \{+, -\}$ ⇔ IC for $(\partial_{x_1^0})^{c_1} \cdots (\partial_{x_n^0})^{c_n} V_n(x_1, \ldots, x_n)$, $c_i \in \{0, 1\}$

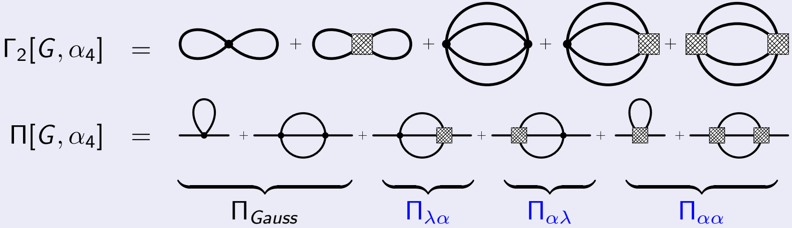
UV divergences, renormalization and non-Gaussian ICs

Example: Initial 4-point correlation, 2PI three-loop truncation



UV divergences, renormalization and non-Gaussian ICs

Example: Initial 4-point correlation, 2PI three-loop truncation



Initial-time surface contribution

e.g. Danielewicz, Semkat, Kremp, Bonitz,...

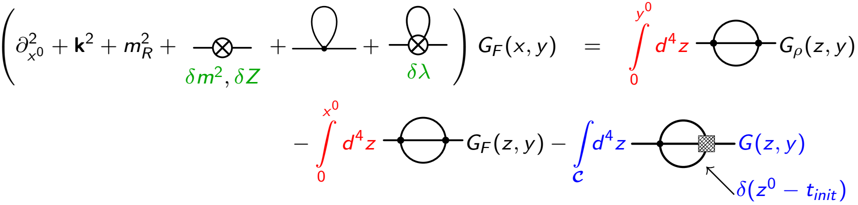
For the example:

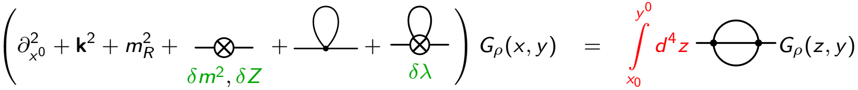
$$\Pi_{\lambda\alpha}(x,z) = -\frac{\lambda}{6}\int_{\mathcal{C}} d^4y_{123}\, G(x,y_1)G(x,y_2)G(x,y_3)\alpha_4(y_1,y_2,y_3,z)$$

In general:

$$\Pi_{\lambda\alpha}(x,z) = \Pi^{+}_{\lambda\alpha}(x,\mathbf{z})\delta(z^0-0_+) + \Pi^{-}_{\lambda\alpha}(x,\mathbf{z})\delta(z^0-0_-)$$

UV divergences, renormalization and non-Gaussian ICs



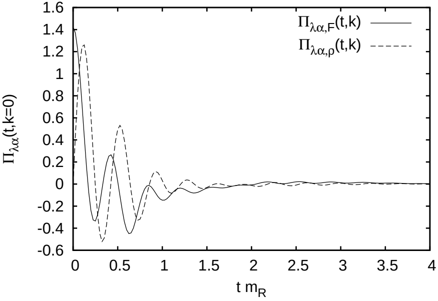


Non-Gaussian contribution on the right-hand side:

$$\int_{\mathcal{C}} d^4z \, \Pi_{\lambda\alpha}(x,z) G(z,y)$$

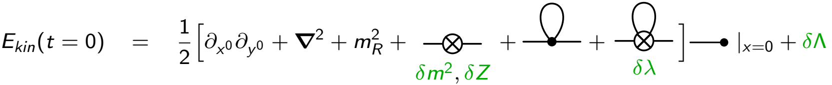
$$= \quad \int d^3z \, \Pi_{\lambda\alpha}(x,\mathbf{z}) G(z^0=0,\mathbf{z},y)$$

e.g. Danielewicz, Semkat, Kremp, Bonitz,...



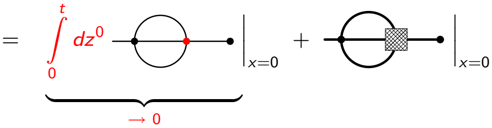
UV divergences, renormalization and non-Gaussian ICs

Correlation energy at initial time

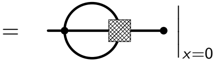


$$E_{kin}(t=0) = \frac{1}{2}\left[\partial_{x^0}\partial_{y^0} + \boldsymbol{\nabla}^2 + m_R^2 + \quad + \quad + \quad\right] \big|_{x=0} + \delta\Lambda$$

$$E_{corr}^{4-p.}(t=0) = \frac{i}{4}\int_{\mathcal{C}} d^4z\,[\Pi_{Gauss}(x,z) + \Pi_{non-Gauss}(x,z)]\,G(z,x)\Big|_{x=0}$$



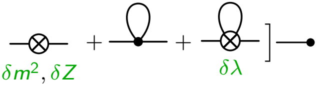
$$= \underbrace{\int_0^t dz^0 \quad \Big|_{x=0}}_{\to 0} + \quad \Big|_{x=0}$$



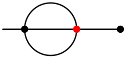
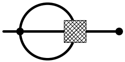
$$= \quad \Big|_{x=0}$$

UV divergences, renormalization and non-Gaussian ICs

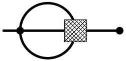
Correlation energy at initial time

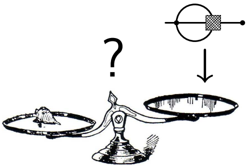
$E_{kin}(t=0) = \frac{1}{2}\Big[\partial_{x^0}\partial_{y^0} + \nabla^2 + m_R^2 +$  $|_{x=0} + \delta\Lambda$

$$E_{corr}^{4-p.}(t=0) = \frac{i}{4}\int_{\mathcal{C}} d^4z\,[\Pi_{Gauss}(x,z) + \Pi_{non-Gauss}(x,z)]\,G(z,x)\Big|_{x=0}$$

$= \int_0^t dz^0$  $\Big|_{x=0}$ $+$  $\Big|_{x=0}$

$\underbrace{\qquad\qquad\qquad\qquad}_{\to 0}$

$=$  $\Big|_{x=0}$



UV divergences, renormalization and non-Gaussian ICs

Questions

- Is it sufficient to include α_4, or does one need α_6 etc. ?
- How to choose α_n ?

Renormalized initial state

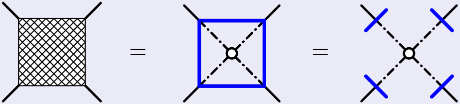
$$\alpha_n(x_1,\ldots,x_n) = \alpha_n^{vac/th}(x_1,\ldots,x_n) + \Delta\alpha_n(x_1,\ldots,x_n)$$

UV divergences, renormalization and non-Gaussian ICs

Thermal initial correlations

MG, Müller (2009)

$$\alpha_{4,3L}^{th,2PI}(z_1, z_2, z_3, z_4) \quad = \quad -i\lambda \int_{\mathcal{I}} d^4v \; \Delta(v, z_1)\Delta(v, z_2)\Delta(v, z_3)\Delta(v, z_4)$$



$$\Delta(-i\tau, z^0, \mathbf{k}) \quad = \quad$$

$$= \quad \Delta^+(-i\tau, \mathbf{k})\,\delta_{\mathcal{C}}(z^0 - 0_+) + \Delta^-(-i\tau, \mathbf{k})\,\delta_{\mathcal{C}}(z^0 - 0_-)$$

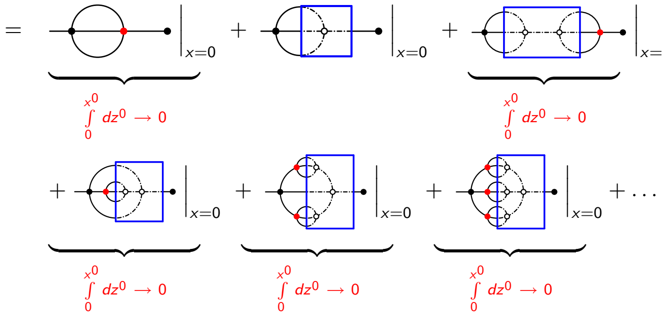
$$\Delta^+(-i\tau, \mathbf{k}) \quad = \quad \frac{G_{th}^{\mathcal{I}\mathcal{I}}(-i\tau, 0, \mathbf{k})}{2G_{th}(0, 0, \mathbf{k})} + \partial_\tau G_{th}^{\mathcal{I}\mathcal{I}}(-i\tau, 0, \mathbf{k})$$

$$\Delta^-(-i\tau, \mathbf{k}) \quad = \quad \frac{G_{th}^{\mathcal{I}\mathcal{I}}(-i\tau, 0, \mathbf{k})}{2G_{th}(0, 0, \mathbf{k})} - \partial_\tau G_{th}^{\mathcal{I}\mathcal{I}}(-i\tau, 0, \mathbf{k})$$

UV divergences, renormalization and non-Gaussian ICs

Full renormalized thermal correlation energy (computed at initial time):

$$E^{eq}_{corr}(t=0) = \frac{i}{4}\int_{\mathcal{C}} d^4z\,[\Pi_{Gauss}(x,z)+\Pi_{non-Gauss}(x,z)]\,G(z,x)\Big|_{x=0}$$



UV divergences, renormalization and non-Gaussian ICs

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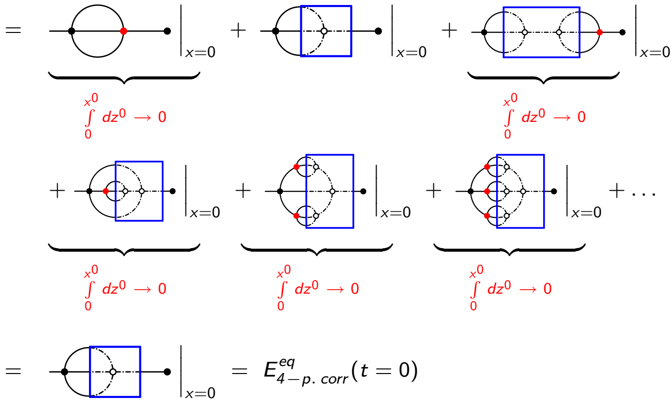


...only the thermal 4-point correlation contributes

$\Rightarrow$ truncate initial correlations with $n > 4$

Results for $\lambda\Phi^4$ 3-loop

Gaussian IC

$$\begin{aligned} G(x,y)|_{x^0,y^0=0} &= G_{th}(x,y)|_{x^0,y^0=0} \\ \alpha_4(x_1,\ldots,x_4) &= 0 \\ \alpha_n(x_1,\ldots,x_n) &= 0 \quad \text{for } n>4 \end{aligned}$$

Non-Gaussian IC with α_4^{th}

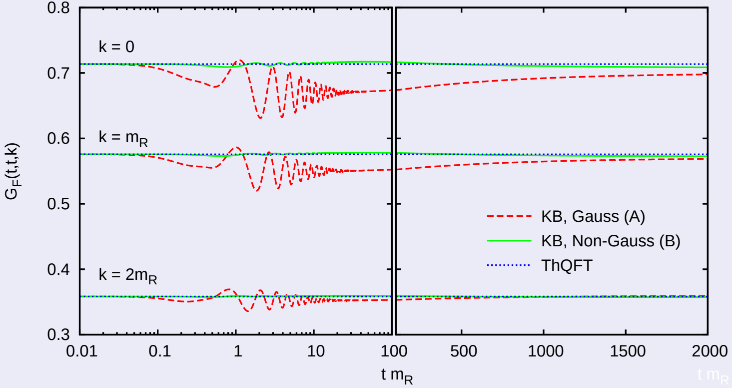
$$\begin{aligned} G(x,y)|_{x^0,y^0=0} &= G_{th}(x,y)|_{x^0,y^0=0} \\ \alpha_4(x_1,\ldots,x_4) &= \alpha_4^{th}(x_1,\ldots,x_4) \\ \alpha_n(x_1,\ldots,x_n) &= 0 \quad \text{for } n>4 \end{aligned}$$

- Truncate thermal initial correlations
- $\Rightarrow$ *nonequilibrium* initial states
- The upper states are ‘as thermal as possible’
- Expectation: Non-Gaussian state more close to equilibrium

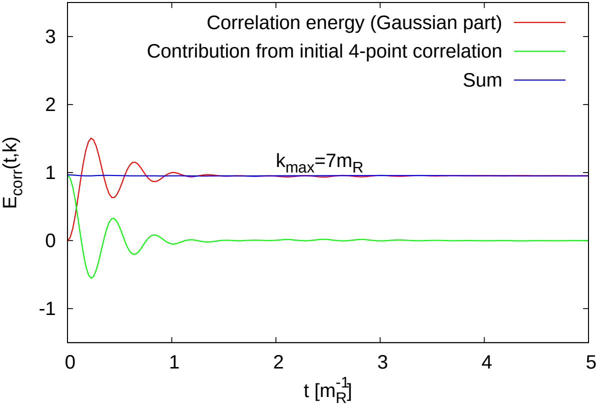
Results for $\lambda\Phi^4$ 3-loop

Minimal offset from thermal equilibrium

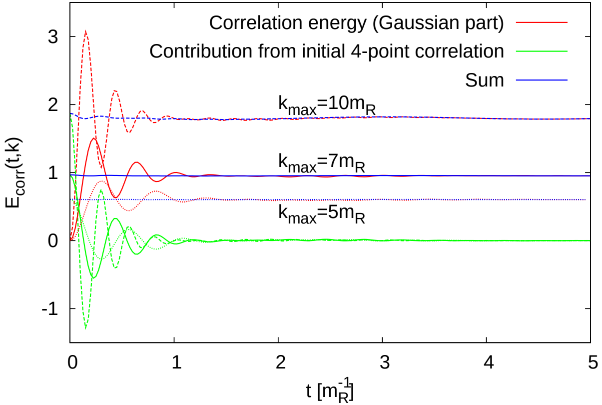
MG, Müller (2009)



Results for $\lambda\Phi^4$ 3-loop



Results for $\lambda\Phi^4$ 3-loop



Results for $\lambda\Phi^4$ 3-loop

Gaussian IC

$$\begin{aligned} E_{total} &= E_{kin}^{eq}(T_{init}) \\ &= E_{kin}^{eq}(T_{final}) + E_{corr}^{eq}(T_{final}) \end{aligned}$$

$\Rightarrow$ *e.g. Köhler, Morawetz, ...*

$$T_{init} \neq T_{final}$$

Cutoff-divergence $E_{corr}^{eq} \sim \Lambda^4 + T^2\Lambda^2 + \dots$

$$|T_{init} - T_{final}| \sim \Lambda^2$$

Results for $\lambda\Phi^4$ 3-loop

Gaussian IC

$$\begin{aligned} E_{total} &= E^{eq}_{kin}(T_{init}) \\ &= E^{eq}_{kin}(T_{final}) + E^{eq}_{corr}(T_{final}) \end{aligned}$$

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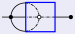
Cutoff-divergence $E^{eq}_{corr} \sim \Lambda^4 + T^2\Lambda^2 + \dots$

$$|T_{init} - T_{final}| \sim \Lambda^2$$

Non-Gaussian IC with α_4^{th}

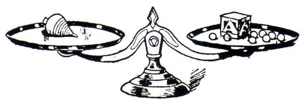
$$\begin{aligned} E_{total} &= E^{eq}_{kin}(T_{init}) + E^{eq}_{4-p.\,corr}(T_{init}) \\ &= E^{eq}_{kin}(T_{final}) + E^{eq}_{corr}(T_{final}) \end{aligned}$$

$E^{eq}_{4-p.\,corr} =$  $= E^{eq}_{corr} \Rightarrow$

$$T_{init} = T_{final}$$

No Cutoff-divergence

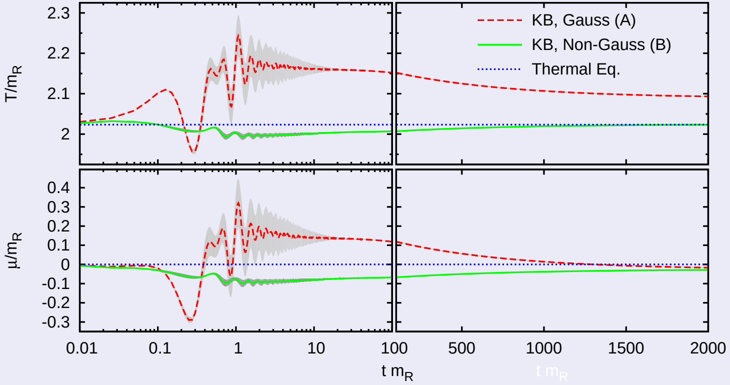
$$E_{total} = E^{eq}(T_{init}) = E^{eq}(T_{final}) = \text{finite}$$



Results for $\lambda\Phi^4$ 3-loop

No offset between initial and final temperature

MG, Müller (2009)



$\underbrace{\hphantom{xxxxxx}}_{\text{build-up}}$ $\underbrace{\hphantom{xxxxx}}_{\text{kinetic -}}$ $\underbrace{\hphantom{xxxxxxxxxxxx}}_{\text{chemical equilibration}}$

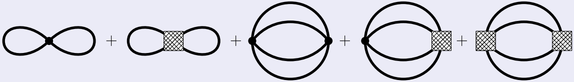
Results for $\lambda\Phi^4$ 3-loop

Renormalization of relativistic KBEs: status

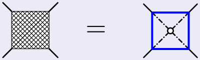
MG, Müller (2009)

- It is necessary to include initial 4-point correlation α_4

$\Gamma_2[G, \alpha_4] \quad =$



- Choosing $\alpha_4 = \alpha_4^{2PI,vac/th}$ renormalizes total (initial) energy



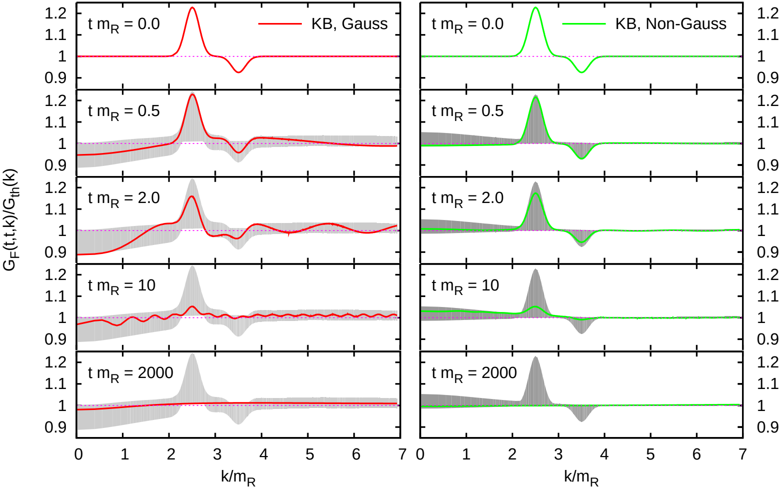
Restriction: Special class of ICs (close to equilibrium); $t \rightarrow 0, \infty$

Questions

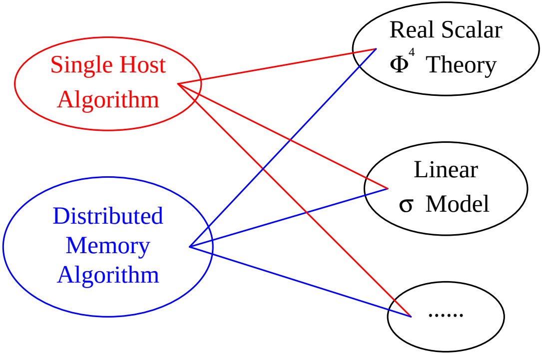
General conditions for renormalized initial states

Results for $\lambda\Phi^4$ 3-loop

Example: $G(x^0, y^0, \mathbf{k})|_{x^0=y^0=t_{init}} = G^{2PI}_{vac/th}(\mathbf{k}) + \underbrace{\Delta G(\mathbf{k})}_{\to 0 \text{ for } \mathbf{k}\to\infty}$, $\alpha_4 \in \{0, \alpha_4^{2PI,vac/th}\}$



Numerical methods



Single Host Algorithm

Distributed Memory Algorithm

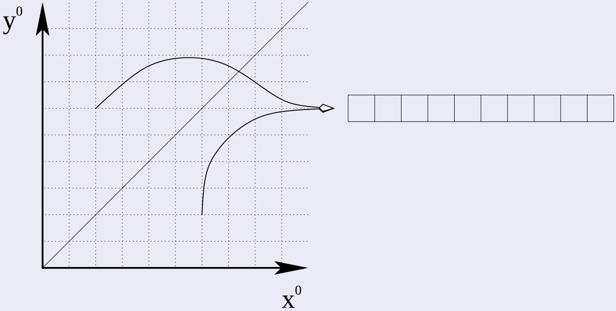
Real Scalar Φ^4 Theory

Linear σ Model

……

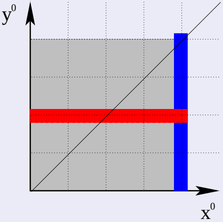
Numerical methods

Single host algorithm: memory layout



Numerical methods

Computation of self-energies and memory integrals



- Self-energy vector for $0 \leq y^0 \leq x^0$ using Fourier trf. *e.g. Danielewicz,Köhler,...*

$$\Pi(x^0, y^0, \hat{\mathbf{k}}) = -\frac{\lambda^2}{6} \sum_{\hat{\mathbf{x}}} e^{i\hat{\mathbf{k}}\hat{\mathbf{x}}} \left[\sum_{\hat{\mathbf{p}}} e^{-i\hat{\mathbf{p}}\hat{\mathbf{x}}} G(x^0, y^0, \hat{\mathbf{p}}) \right]^3$$

- Memory integrals = history matrix × self-energies

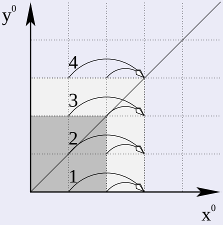
$$MEMINT\left(x^0, y^0, \hat{\mathbf{k}}\right) = \sum_{z^0} \Pi(x^0, z^0, \hat{\mathbf{k}}) G(z^0, y^0, \hat{\mathbf{k}})$$

Numerical methods

time-stepping

$$\begin{aligned}\partial^2_{x^0} G\left(x^0, y^0, k\right) \quad &\rightarrow \quad \Delta^b_0 \Delta^f_0 G\left(x^0, y^0, \hat{\mathbf{k}}\right) \\ = \quad &\frac{G\left(x^0 + a_t, y^0, \hat{\mathbf{k}}\right) - 2G\left(x^0, y^0, \hat{\mathbf{k}}\right) + G\left(x^0 - a_t, y^0, \hat{\mathbf{k}}\right)}{a_t^2}\end{aligned}$$

$$\begin{aligned}G\left(x^0 + a_t, y^0, \hat{\mathbf{k}}\right) &= 2G\left(x^0, y^0, \hat{\mathbf{k}}\right) - G\left(x^0 - a_t, y^0, \hat{\mathbf{k}}\right) \\ &+ a_t^2 \left[MEMINT\left(x^0, y^0, \hat{\mathbf{k}}\right) - \left(\hat{\mathbf{k}}^2 + M^2\left(x^0\right)\right) G\left(x^0, y^0, \hat{\mathbf{k}}\right)\right]\end{aligned}$$

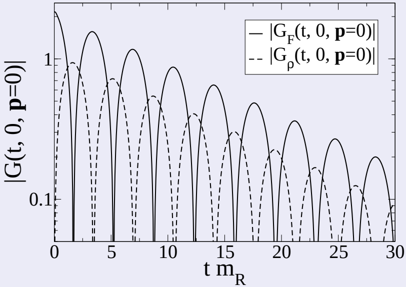


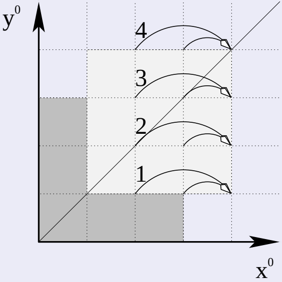
Step 4: Use

$$\begin{aligned}G_F(x^0, y^0, \hat{\mathbf{k}}) &= G_F(y^0, x^0, \hat{\mathbf{k}}) \\ G_\rho(x^0, y^0, \hat{\mathbf{k}}) &= -G_\rho(y^0, x^0, \hat{\mathbf{k}})\end{aligned}$$

Numerical methods

History cut-off



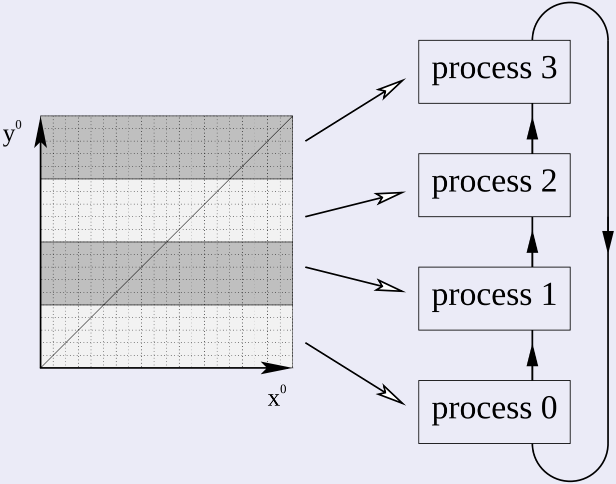


$$H = \{0, a_t, 2a_t, \ldots, (N_t - 1)\, a_t\}^2$$

$$G(x^0, y^0, \hat{\mathbf{k}}) \rightarrow G\left(x^0 \mathrm{mod} N_t, y^0 \mathrm{mod} N_t, \hat{\mathbf{k}}\right)$$

Numerical methods

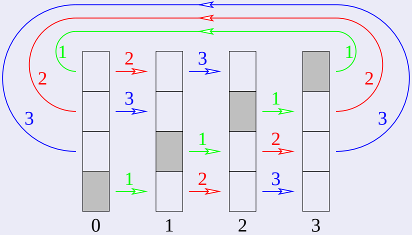
Parallelized distributed memory algorithm



Numerical methods

Parallelized distributed memory algorithm

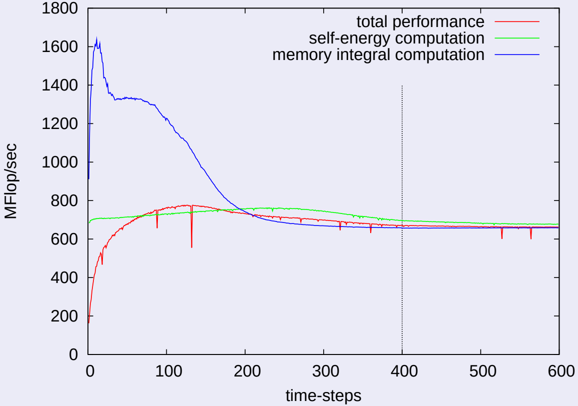
- compute self-energies on each stripe in parallel
- circulate self-energies



- compute memory integrals on each stripe in parallel
- time-stepping $(x^0, y^0) \longrightarrow (x^0 + a_t, y^0)$ on each stripe in parallel
- mirror history matrix
- time-stepping $(x^0, x^0 + a_t) \longrightarrow (x^0 + a_t, x^0 + a_t)$

Numerical methods

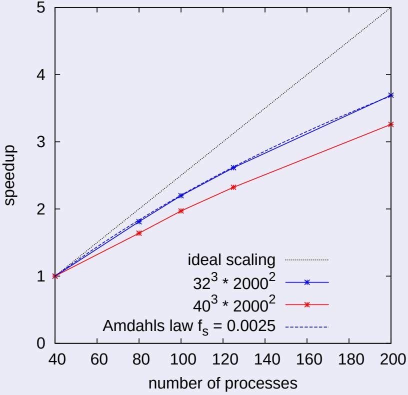
Performance



Intel Itanium2 Montecito Dual Core 1.6GHz, Peak performance 6.4 GFlop/s per core

Numerical methods

Scaling



$$s(N_p) = \frac{1}{f_s + \frac{f_p}{N_p} + k(N_p - 1)}$$

$$f_s : f_p \simeq 3 : 1000$$

LRZ, HLRB2 (SGI Altix 4700, 9728 cores, Total peak performance 62.3 TFlop/s)

Relativistic KBEs for correlated initial states and baryogenesis

thank you!

TECHNISCHE
UNIVERSITÄT
MÜNCHEN

Leibniz-Rechenzentrum
der Bayerischen Akademie der Wissenschaften

References

- *MG, Markus Michael Müller, Phys.Rev.D80:085011,2009*
- *Borsanyi, Reinosa Nucl.Phys.A820:147C,2009*
- *MG, Markus Michael Müller, proceedings of HLRB2 results workshop 2009*
- *MG, Hohenegger, Kartavtsev, Lindner, Phys.Rev.D80:125027,2009; arXiv:0911.4122; arXiv:1002.0331*

Thermal Initial Correlations

Thermal density matrix of the interacting theory

$$\rho_{th} = \frac{1}{Z} e^{-\beta H}, \qquad H = H_0 + H_{int}$$

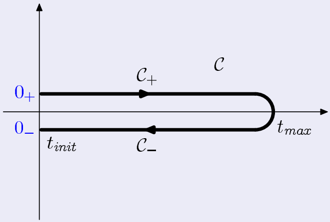
$\Rightarrow$ Compute the corresponding initial correlations

$$\langle \varphi_+ | \rho_{th} | \varphi_- \rangle = \exp\left(i \sum_{n=0}^{\infty} \int d^4 x_{12\dots n} \, \alpha_n^{th}(x_1, \dots, x_n) \varphi(x_1)\varphi(x_2)\cdots\varphi(x_n) \right)$$

- Can be done order-by-order in H_{int}
- Problem: Need approximations compatible with 2PI formalism
- Solution: Match 2PI on closed real-time path with 2PI thermal (imaginary-time) field theory *MG, Müller (2009)*

Thermal Initial Correlations

Closed time path $\mathcal{C}$ with α_n

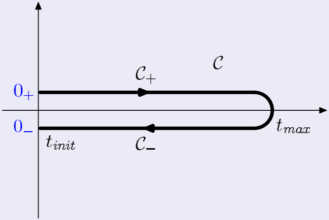


$$\langle\varphi_+|\rho_{th}|\varphi_-\rangle$$

$$= \quad \exp\left(i\sum_{n=0}^{\infty}\alpha^{th}_{12\cdots n}\varphi_1\varphi_2\cdots\varphi_n\right)$$

Thermal Initial Correlations

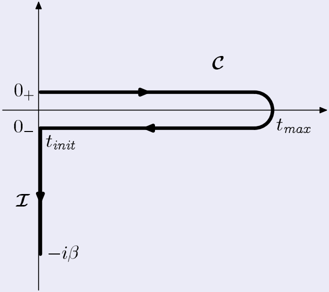
Closed time path $\mathcal{C}$ with α_n



$$\langle\varphi_+|\rho_{th}|\varphi_-\rangle$$

$$= \quad \exp\left(i\sum_{n=0}^{\infty}\alpha^{th}_{12\cdots n}\varphi_1\varphi_2\cdots\varphi_n\right)$$

Thermal time path $\mathcal{C}+\mathcal{I}$



$$\langle\varphi_+|\rho_{th}|\varphi_-\rangle$$

$$= \int_{\varphi(0,\mathbf{x})=\varphi_-(\mathbf{x})}^{\varphi(-i\beta,\mathbf{x})=\varphi_+(\mathbf{x})}\mathcal{D}\varphi\ \exp\left(i\int_{\mathcal{I}}d^4x\,\mathcal{L}(x)\right)$$

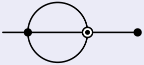
Thermal Initial Correlations

Thermal time path $\mathcal{C}+\mathcal{I}$

Self-consistent Schwinger-Dyson equation

$$G_{th}^{-1}(x,y) = G_{0,th}^{-1}(x,y) - \Pi_{th}(x,y) \qquad \Leftrightarrow$$

$$(\Box_x + m^2)G_{th}(x,y) = -i\delta_{\mathcal{C}+\mathcal{I}}(x-y) - i\underbrace{\int_{\mathcal{C}+\mathcal{I}} d^4z \Pi_{th}(x,z)G_{th}(z,y)}$$



Thermal Initial Correlations

Thermal time path $\mathcal{C}+\mathcal{I}$

Self-consistent Schwinger-Dyson equation

$$G_{th}^{-1}(x,y) = G_{0,th}^{-1}(x,y) - \Pi_{th}(x,y) \qquad \Leftrightarrow$$

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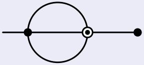


Closed time path $\mathcal{C}$ with initial correlations α

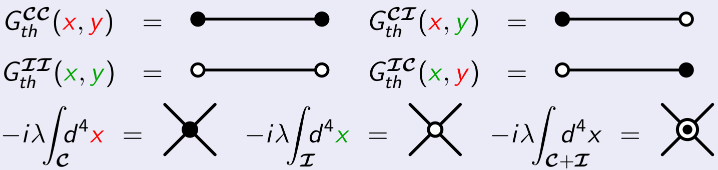
Kadanoff-Baym equation for a Non-Gaussian initial state

$$\begin{aligned}(\Box_x + m^2)G(x,y) &= -i\delta_{\mathcal{C}}(x-y) \\ &\quad - i\int_{\mathcal{C}} d^4z \left[\Pi_{Gauss}(x,z) + \Pi_{non-Gauss}(x,z)\right] G(z,y)\end{aligned}$$

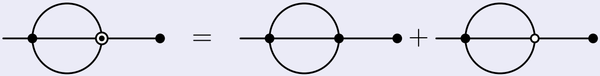
Thermal Initial Correlations

Thermal time path $\mathcal{C}+\mathcal{I}$

The lines represent the *complete* propagator



Example: 2PI three-loop truncation



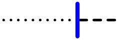
Thermal Initial Correlations: Perturbation Theory

$$G_{0,th}^{\mathcal{IC}}(-i\tau, y^0, \mathbf{k}) \quad = \quad \int_{\mathcal{C}} dt\, \Delta_0(-i\tau, t, \mathbf{k})\, G_{0,th}^{\mathcal{CC}}(t, y^0, \mathbf{k})$$

Free ‘connection’

$$\Delta_0(-i\tau, z^0, \mathbf{k}) \quad = \quad \Delta_0^+(-i\tau, \mathbf{k})\, \delta_{\mathcal{C}}(z^0 - 0_+) + \Delta_0^-(-i\tau, \mathbf{k})\, \delta_{\mathcal{C}}(z^0 - 0_-)$$

$$=$$



where

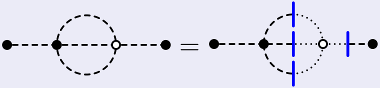
$$\Delta_0^+(-i\tau, \mathbf{k}) \quad = \quad \frac{\sinh(\omega_{\mathbf{k}}\tau)}{\sinh(\omega_{\mathbf{k}}\beta)} \qquad = \frac{G_{0,th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})}{2G_{0,th}(0,0,\mathbf{k})} + \partial_\tau\, G_{0,th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})$$

$$\Delta_0^-(-i\tau, \mathbf{k}) \quad = \quad \frac{\sinh(\omega_{\mathbf{k}}(\beta - \tau))}{\sinh(\omega_{\mathbf{k}}\beta)} = \frac{G_{0,th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})}{2G_{0,th}(0,0,\mathbf{k})} - \partial_\tau\, G_{0,th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})$$

Thermal Initial Correlations: Perturbation Theory

Example

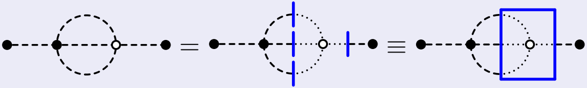
MG, Müller (2009)



Thermal Initial Correlations: Perturbation Theory

Example

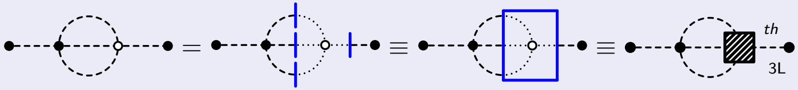
MG, Müller (2009)



Thermal Initial Correlations: Perturbation Theory

Example

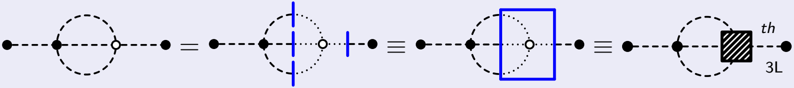
MG, Müller (2009)



Thermal Initial Correlations: Perturbation Theory

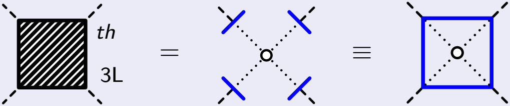
Example

MG, Müller (2009)



Perturbative initial 4-point correlation

$$\alpha^{th}_{4,3L}(z_1,z_2,z_3,z_4) \;=\; -i\lambda\int_{\mathcal{I}} d^4v\;\Delta_0(v,z_1)\Delta_0(v,z_2)\Delta_0(v,z_3)\Delta_0(v,z_4)$$



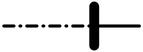
Thermal Initial Correlations: 2PI

$$G_{th}^{\mathcal{IC}}(-i\tau, y^0, \mathbf{k}) = \int_{\mathcal{C}} dt\, \tilde{\Delta}(-i\tau, t, \mathbf{k}) G_{th}^{\mathcal{CC}}(t, y^0, \mathbf{k})$$

$=$

Complete 'connection':

$\tilde{\Delta}(-i\tau, z^0, \mathbf{k}) =$



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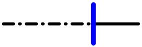
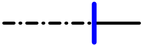


$+$

$\Delta(-i\tau, z^0, \mathbf{k}) =$



$$= \Delta^+(-i\tau, \mathbf{k})\, \delta_{\mathcal{C}}(z^0 - 0_+) + \Delta^-(-i\tau, \mathbf{k})\, \delta_{\mathcal{C}}(z^0 - 0_-)$$

where

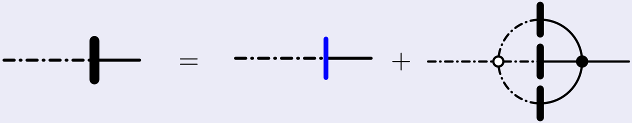
$$\Delta^+(-i\tau, \mathbf{k}) = \frac{G_{th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})}{2G_{th}(0, 0, \mathbf{k})} + \partial_\tau G_{th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})$$

$$\Delta^-(-i\tau, \mathbf{k}) = \frac{G_{th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})}{2G_{th}(0, 0, \mathbf{k})} - \partial_\tau G_{th}^{\mathcal{II}}(-i\tau, 0, \mathbf{k})$$

Thermal Initial Correlations: 2PI

Example: 2PI three-loop truncation

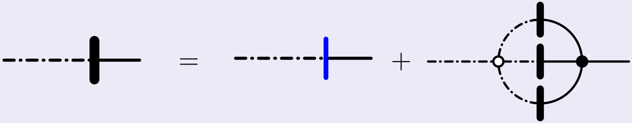
MG, Müller (2009)



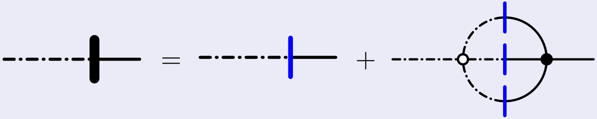
Thermal Initial Correlations: 2PI

Example: 2PI three-loop truncation

MG, Müller (2009)



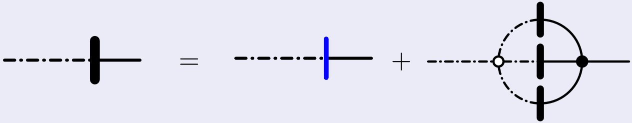
Iterative Solution:



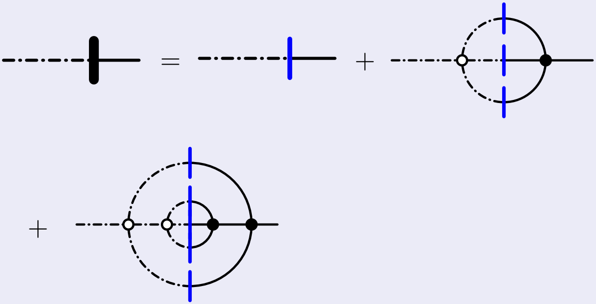
Thermal Initial Correlations: 2PI

Example: 2PI three-loop truncation

MG, Müller (2009)



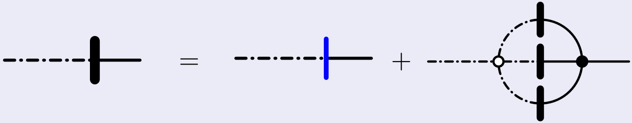
Iterative Solution:



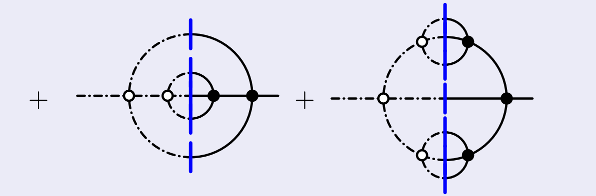
Thermal Initial Correlations: 2PI

Example: 2PI three-loop truncation

MG, Müller (2009)



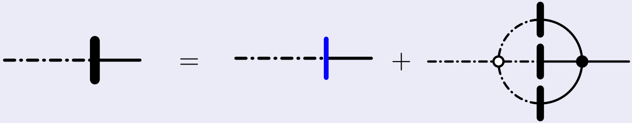
Iterative Solution:



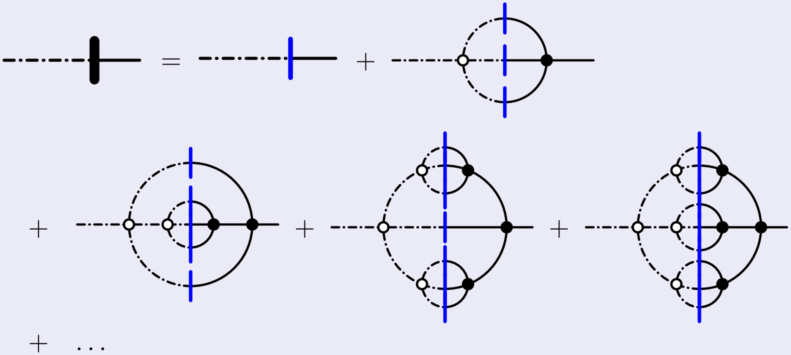
Thermal Initial Correlations: 2PI

Example: 2PI three-loop truncation

MG, Müller (2009)



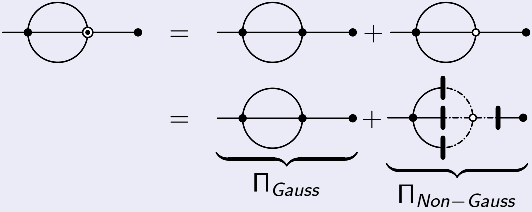
Iterative Solution:



Thermal Initial Correlations: 2PI

Example: 2PI three-loop truncation

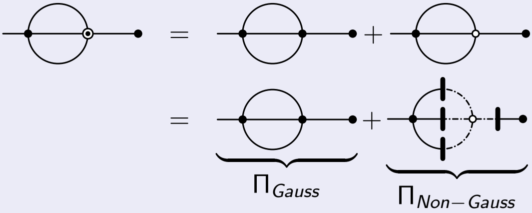
MG, Müller (2009)



Thermal Initial Correlations: 2PI

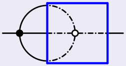
Example: 2PI three-loop truncation

MG, Müller (2009)



Non-Gaussian self-energy contains $\alpha_n^{th}(x_1, \ldots, x_n)$ for all $n \geq 4$

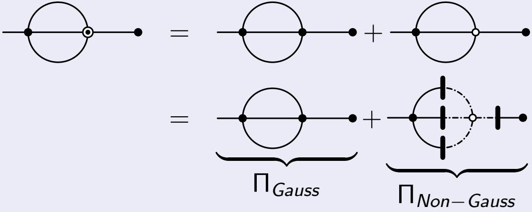
$\Pi_{non-Gauss}(x, z) \quad =$



Thermal Initial Correlations: 2PI

Example: 2PI three-loop truncation

MG, Müller (2009)



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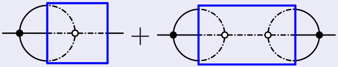

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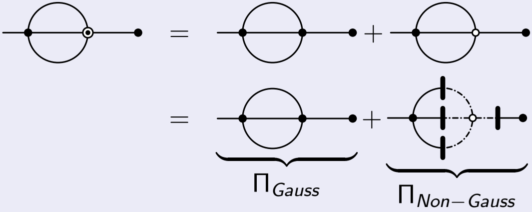

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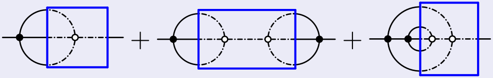

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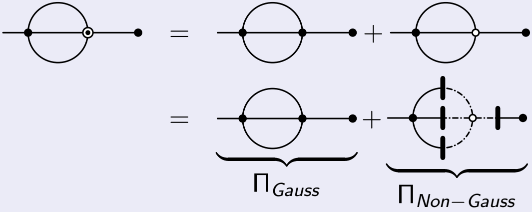

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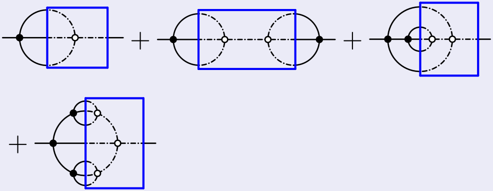

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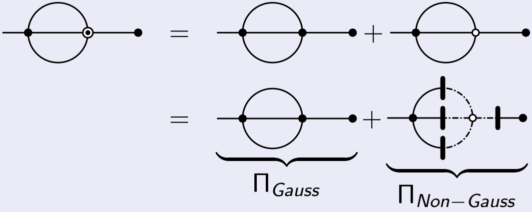

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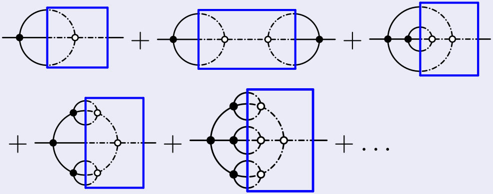
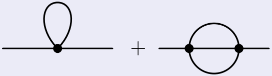


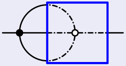
UV divergences, renormalization and non-Gaussian ICs

Result: Example for Setting-Sun Approximation

$\Pi_{Gauss} \quad =$



$\Pi^{th}_{non-Gauss} \quad =$



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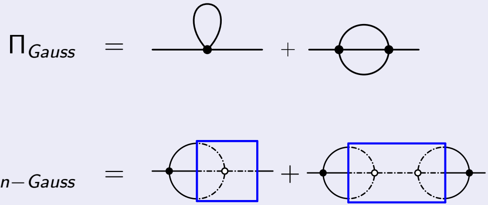

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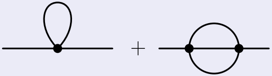

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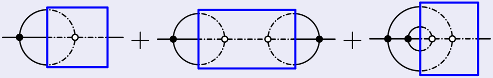

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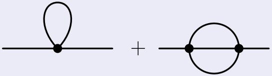

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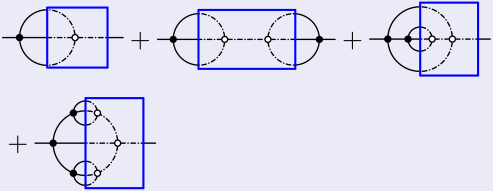

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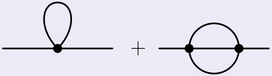

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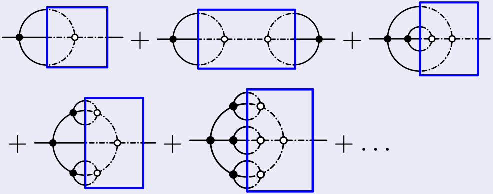

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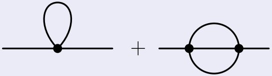

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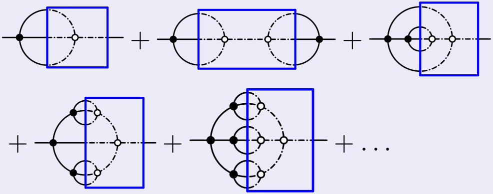

$+ \ldots$

Kadanoff-Baym equations with thermal initial correlations contain

$\alpha_n^{th}(x_1, \ldots, x_n)$ for all $n \geq 4$