Altermagnetism

A viable application for the Quantum Fluctuations Approach?

Tim Kalsberger

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CAU-Kiel

Overview

Motivation

Task

Find an application for the Quantum Fluctuations Approach (QFA).

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Find an application for the Quantum Fluctuations Approach (QFA).

A few criteria:

1. Interesting and relevant physical system
2. System should be dynamical, within the regime of the polarization approximation and profit from large sizes
3. Not too hard to implement

Why Altermagnetism?

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- Since its first description in 2021 by Šmejkal, Sinova and Jungwirth, more than 500 papers have been published [1]
- Number of publications is growing fast from year to year

PHYSICAL REVIEW X **12,** 031042 (2022)

Featured in Physics

Beyond Conventional Ferromagnetism and Antiferromagnetism: A Phase with Nonrelativistic Spin and Crystal Rotation Symmetry

Libor Šmejkal,[1,2] Jairo Sinova,[1,2] and Tomas Jungwirth[2,3]

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(Received 6 February 2022; revised 6 April 2022; accepted 11 August 2022; published 23 September 2022)

Recent series of theoretical and experimental reports have driven attention to time-reversal symmetry-breaking spintronic and spin-splitting phenomena in materials with collinear-compensated magnetic order incompatible with conventional ferromagnetism or antiferromagnetism. Here we employ an approach based on nonrelativistic spin-symmetry groups that resolves the conflicting notions of unconventional ferromagnetism or antiferromagnetism by delimiting a third basic collinear magnetic phase. We derive that all materials hosting this collinear-compensated magnetic phase are characterized by crystal-rotation symmetries connecting opposite-spin sublattices separated in the real space and opposite-spin electronic states separated in the momentum space. We describe prominent extraordinary characteristics of the phase,

[1]According to `https://www.semanticscholar.org`

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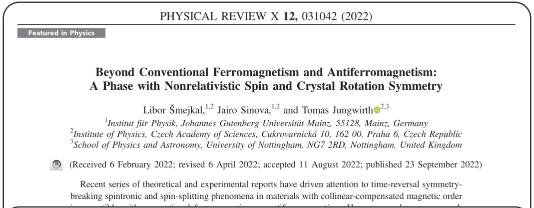
- Since its first description in 2021 by Šmejkal, Sinova and Jungwirth, more than 500 papers have been published [1]
- Number of publications is growing fast from year to year

$\Rightarrow$ Altermagnetism is a fairly new topic with potential use cases in active research fields like spintronics, ultrafast photomagnetism, superconductivity, etc. [1].



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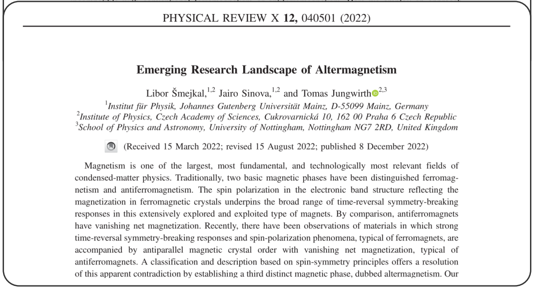

PHYSICAL REVIEW X **12**, 040501 (2022)

Emerging Research Landscape of Altermagnetism

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Magnetism is one of the largest, most fundamental, and technologically most relevant fields of condensed-matter physics. Traditionally, two basic magnetic phases have been distinguished ferromagnetism and antiferromagnetism. The spin polarization in the electronic band structure reflecting the magnetization in ferromagnetic crystals underpins the broad range of time-reversal symmetry-breaking responses in this extensively explored and exploited type of magnets. By comparison, antiferromagnets have vanishing net magnetization. Recently, there have been observations of materials in which strong time-reversal symmetry-breaking responses and spin-polarization phenomena, typical of ferromagnets, are accompanied by antiparallel magnetic crystal order with vanishing net magnetization, typical of antiferromagnets. A classification and description based on spin-symmetry principles offers a resolution of this apparent contradiction by establishing a third distinct magnetic phase, dubbed altermagnetism. Our

[1] According to `https://www.semanticscholar.org`

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Definition

Altermagnetism is a magnetic phase characterized by a vanishing net magnetization and spin-splitting of the bands, thus combining the characteristics of ferromagnets and antiferromagnets.

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Contradiction: How can a material be both ferromagnetic and antiferromagnetic?

$\Rightarrow$ Solved by a general symmetry formalism for spin arrangements on crystals, the so called **Spin-groups** [1, 2]

Traditional magnetic phases

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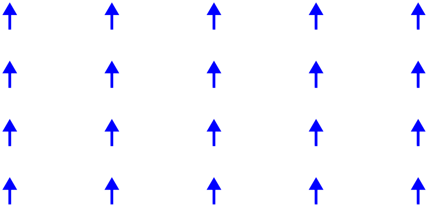
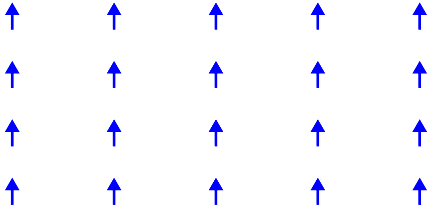
Ferromagnetism



- Net magnetization ($M \neq 0$)
- Spin polarized bands

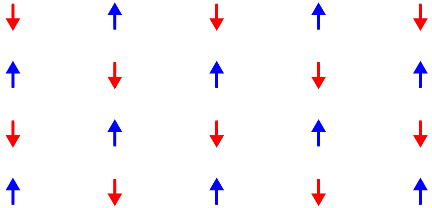
Traditional magnetic phases

Ferromagnetism



- Net magnetization ($M \neq 0$)
- Spin polarized bands

Anitferromagnetism



- No net magnetization ($M = 0$)
- No spin polarization

Definition

Non-relativistic spin groups use pairs of transformations $[R_i||R_j]$ where R_i only acts in spin space and R_j simultaneously acts only in real space [2].

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- Non-relativistic: real and spin space are decoupled $\Rightarrow$ much larger symmetry landscape than the relativistic magnetic groups

The non-relativistic spin-group

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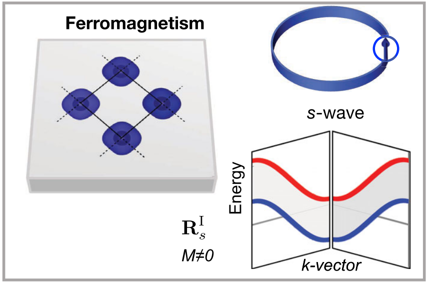
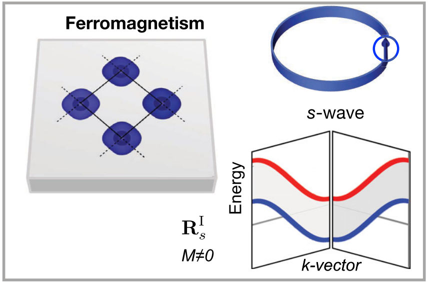
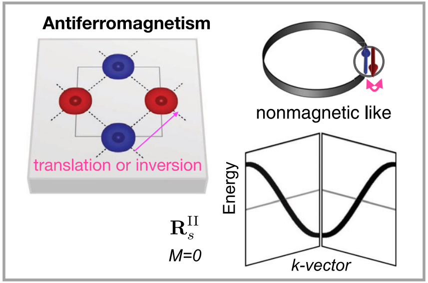
- Non-relativistic: real and spin space are decoupled $\Rightarrow$ much larger symmetry landscape than the relativistic magnetic groups
- Leads to three distinct phases of collinear magnetism:
    1. Opposite spin-sublattices are not connected by any symmetry transformations
    2. Opposite spin-sublattices are connected by translation or inversion
    3. Opposite spin-sublattices are connected by Rotation (proper or improper)

The non-relativistic spin-group



Figures adapted from [1], fig. 1





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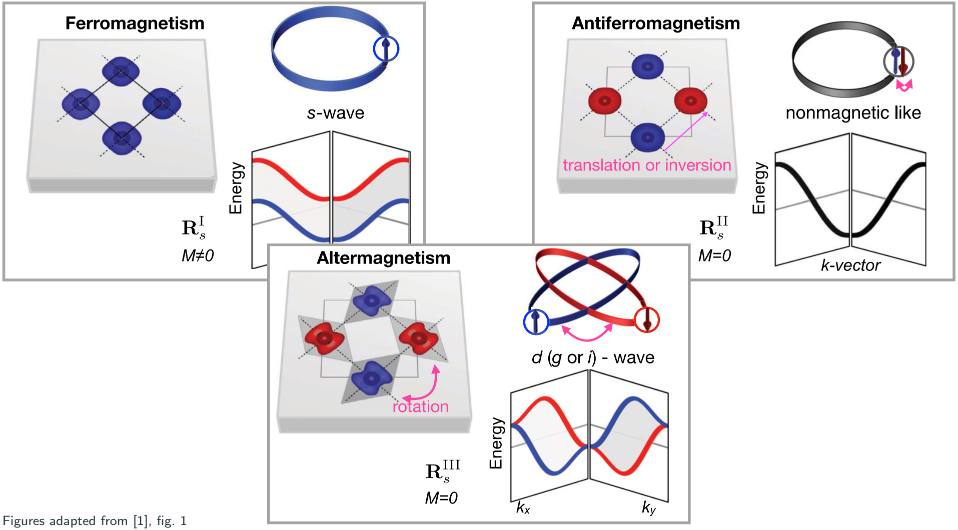

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Spin-group symmetries	Magnetic crystal structure	Nonrelativistic band structure
$[\boldsymbol{C}_\infty \| E]$	Collinear	Spin is a good quantum number and $\boldsymbol{k}$ independent
$[\bar{C}_2 \| \mathcal{T}]$	Coplanar	Invariance under inversion of $\boldsymbol{k}$
$[E \| \boldsymbol{H}]$	Sublattice spin-density anisotropy	Spin-Fermi-surface anisotropy
$[C_2 \| A\boldsymbol{H}]$	Compensated	Broken $\mathcal{T}$ and spin splitting at general $\boldsymbol{k}$
	$\vdots$	

Ferromagnetism: $\boldsymbol{R}_s^{\mathrm{I}} = [E||\boldsymbol{G}]$

Anitferromagnetism: $\boldsymbol{R}_s^{\mathrm{II}} = [E||\boldsymbol{G}] + [C_2||\boldsymbol{G}]$

Altermagnetism: $\boldsymbol{R}_s^{\mathrm{III}} = [E||\boldsymbol{H}] + [C_2||\boldsymbol{G} - \boldsymbol{H}] = [E||\boldsymbol{H}] + [C_2||A\boldsymbol{H}]$

Table from [1], tab. 2

Simulating Altermagnetism with the Quantum Fluctuations Approach

- QFA is based on quantum fluctuations:

$$\delta \hat{G}_{ij}(t) = \hat{G}^{<}_{ij}(t) - G^{<}_{ij}(t)$$

- EOM within SPA and Hubbard-Model:

$$\begin{aligned}
\mathrm{i}\hbar \frac{\mathrm{d}}{\mathrm{d}t} G^{<,\sigma}_{ij}(t) &= [h^{(1),\sigma}, G^{<,\sigma}]_{ij}(t) + \frac{1}{N_{\mathrm{samples}}} \sum_{\lambda=1}^{N_{\mathrm{samples}}} [\Delta\Sigma^{\sigma,(\lambda)}, \Delta G^{\sigma,(\lambda)}]_{ij}(t) \\
\mathrm{i}\hbar \frac{\mathrm{d}}{\mathrm{d}t} \Delta G^{\sigma,(\lambda)}_{ij}(t) &= [h^{(1),\sigma}, \Delta G^{\sigma,(\lambda)}]_{ij}(t) + [\Delta\Sigma^{\sigma,(\lambda)}, G^{<,\sigma}]_{ij}(t) \\
\Delta\Sigma^{\sigma,(\lambda)}_{ij}(t) &= -\mathrm{i}\hbar\delta_{ij} \Delta G^{\bar{\sigma},(\lambda)}_{ij}(t) U \\
h^{(1),\sigma}_{ij}(t) &= -J_{ij} - \mathrm{i}\hbar\delta_{ij} U G^{<,\bar{\sigma}}_{ii}(t)
\end{aligned}$$

The (altermagnetic) Hubbard-Model

- The altermagnetic Hubbard-Model was first introduced to realize Altermagnetism in ultracold atoms [3]

$$\hat{H} = -\sum_{i,j} J_{ij}\left(\hat{c}^{\dagger}_{is}\hat{c}_{js} + \hat{c}^{\dagger}_{js}\hat{c}_{is}\right) + U\sum_{i} \hat{n}_{i\uparrow}\hat{n}_{j\downarrow}$$

PHYSICAL REVIEW LETTERS **132,** 263402 (2024)

Realizing Altermagnetism in Fermi-Hubbard Models with Ultracold Atoms

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Altermagnetism represents a type of collinear magnetism, that is in some aspects distinct from ferromagnetism and from conventional antiferromagnetism. In contrast to the latter, sublattices of opposite spin are related by spatial rotations and not only by translations and inversions. As a result, altermagnets have spin-split bands leading to unique experimental signatures. Here, we show theoretically how a *d*-wave altermagnetic phase can be realized with ultracold fermionic atoms in optical lattices. We propose an altermagnetic Hubbard model with anisotropic next-nearest neighbor hopping and obtain the Hartree-Fock phase diagram. The altermagnetic phase separates in a metallic and an insulating phase and is robust over a large parameter regime. We show that one of the defining characteristics of altermagnetism, the anisotropic spin transport, can be probed with trap-expansion experiments.

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$$\hat{H} = -\sum_{i,j} J_{ij}\left(\hat{c}^{\dagger}_{is}\hat{c}_{js} + \hat{c}^{\dagger}_{js}\hat{c}_{is}\right) + U\sum_{i} \hat{n}_{i\uparrow}\hat{n}_{j\downarrow}$$

if $i,j \in A$:

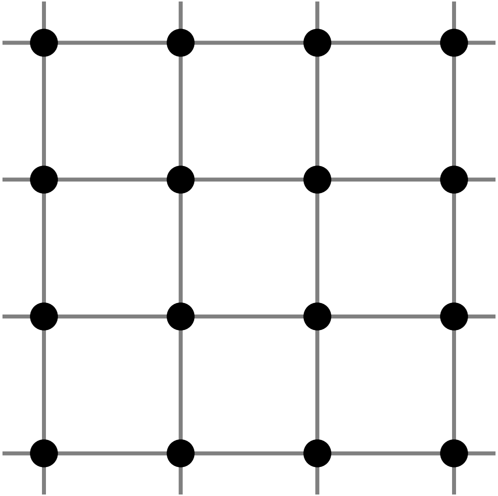
$$J_{ij} = \begin{cases} t_- & \text{if } x_j - x_i = 1, y_j - y_i = 1 \\ t_+ & \text{if } x_j - x_i = 1, y_j - y_i = -1 \end{cases}$$

if $i,j \in B$:

$$J_{ij} = \begin{cases} t_+ & \text{if } x_j - x_i = 1, y_j - y_i = 1 \\ t_- & \text{if } x_j - x_i = 1, y_j - y_i = -1 \end{cases}$$

$$t_{\pm} = t'(1 \pm \delta)$$

The (altermagnetic) Hubbard-Model



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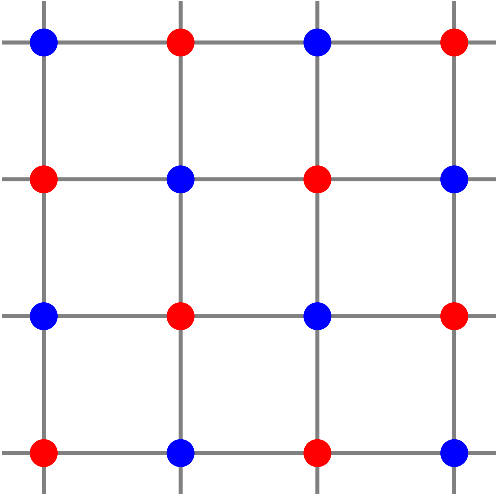

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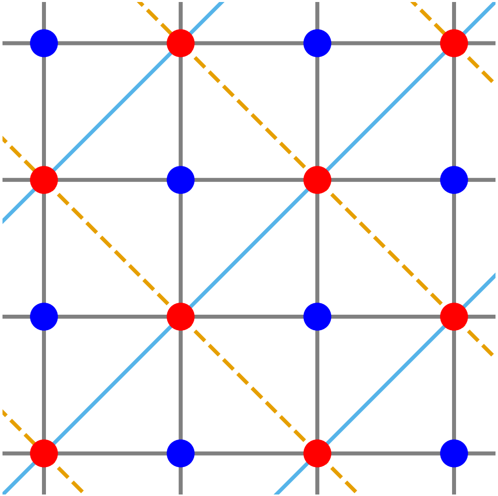
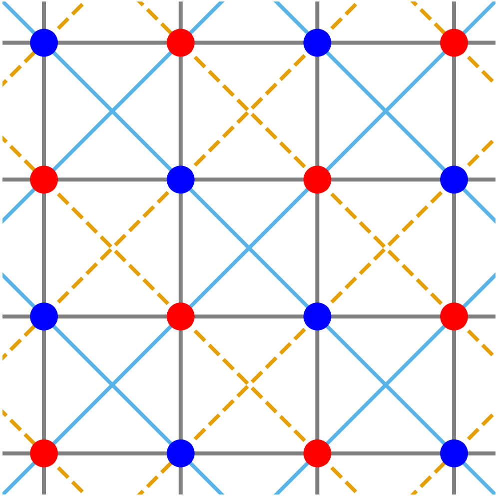




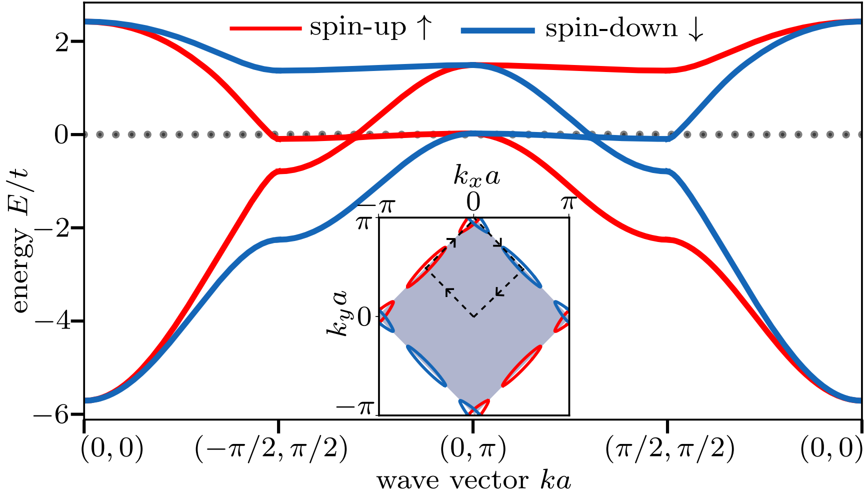


Figure from [3], fig. 1

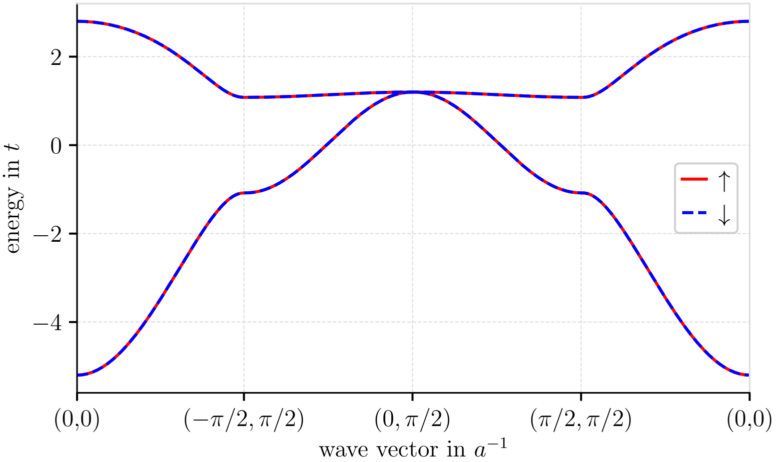
- The Tight-Binding Eigenstates and -values are not spin-polarized:

$$\hat{H}^{\mathrm{TB}} = \begin{pmatrix} \hat{a}^\dagger_{\boldsymbol{k},\uparrow} \\ \hat{b}^\dagger_{\boldsymbol{k},\uparrow} \\ \hat{a}^\dagger_{\boldsymbol{k},\downarrow} \\ \hat{b}^\dagger_{\boldsymbol{k},\downarrow} \end{pmatrix}^T \begin{pmatrix} h_{AA} & h_{AB} & 0 & 0 \\ h_{BA} & h_{BB} & 0 & 0 \\ 0 & 0 & h_{AA} & h_{AB} \\ 0 & 0 & h_{BA} & h_{BB} \end{pmatrix} \begin{pmatrix} \hat{a}_{\boldsymbol{k},\uparrow} \\ \hat{b}_{\boldsymbol{k},\uparrow} \\ \hat{a}_{\boldsymbol{k},\downarrow} \\ \hat{b}_{\boldsymbol{k},\downarrow} \end{pmatrix}$$

$$h_{AA} = -2t_- \cos(\boldsymbol{k} \cdot \boldsymbol{a}_1) - 2t_+ \cos(\boldsymbol{k} \cdot \boldsymbol{a}_2) \qquad h_{BB} = -2t_- \cos(\boldsymbol{k} \cdot \boldsymbol{a}_2) - 2t_+ \cos(\boldsymbol{k} \cdot \boldsymbol{a}_1)$$

$$h_{AB} = h_{BA} = -2t \cos(k_x a) - 2t \cos(k_y a)$$

$$\epsilon_{\sigma,a}(\boldsymbol{k}) = -\frac{1}{2}(h_{AA} + h_{BB}) \pm \sqrt{\frac{1}{4}(h_{AA} - h_{BB})^2 + h_{AB} h_{BA}}$$


2
0
−2
−4
energy in t
(0,0)
$(-\pi/2, \pi/2)$
$(0, \pi/2)$
$(\pi/2, \pi/2)$
(0,0)
wave vector in a^{-1}
↑
↓

- Order parameter:

$$\delta m = \frac{1}{4N} \sum_i \langle n_{iA\uparrow} - n_{iA\downarrow} - n_{iB\uparrow} + n_{iB\downarrow} \rangle$$

- Leads to effective interaction

$$-U\delta m \sum_{\boldsymbol{k}} (n_{\boldsymbol{k}A\uparrow} - n_{\boldsymbol{k}B\uparrow} - n_{\boldsymbol{k}A\downarrow} + n_{\boldsymbol{k}B\downarrow})$$

as an additive term in h_{AA} and h_{BB}, making them spin dependent.

$\Rightarrow$ Possible solution to problem: Apply a sublattice and spin dependent single-particle potential:

$$f_{i,\sigma} = \begin{cases} c^{\uparrow}, & \text{if } i \in A, \sigma = \uparrow \\ c^{\downarrow}, & \text{if } i \in B, \sigma = \downarrow \\ 0, & \text{else} \end{cases}$$

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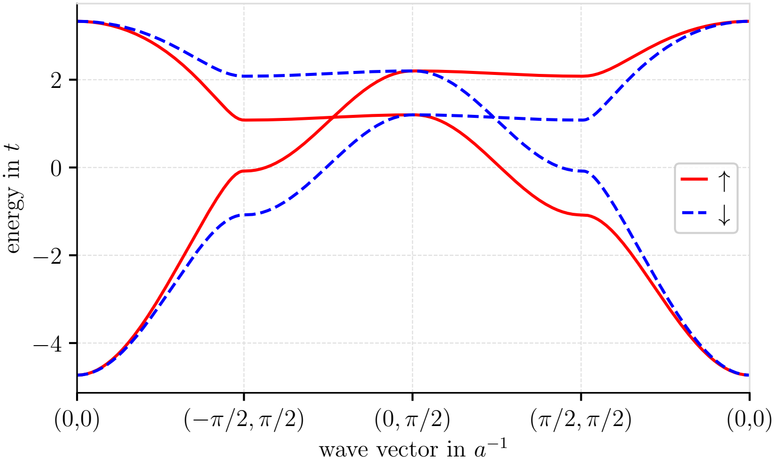
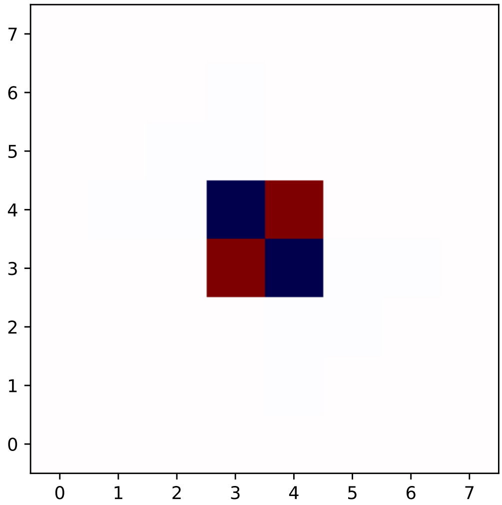


Simulation results



Conclusion

- Altermagnetism is an interesting and active topic in modern research
- Multiple dynamical (and large) systems are needed to describe different characteristics of Altermagnetism
- The altermagnetic Hubbard-Model allows for an easy implementation with problems only for the generation of an interacting ground state

References

[1] Libor Šmejkal et al. **"Emerging Research Landscape of Altermagnetism".** In: *Physical Review X* 12.4 (2022).

[2] Libor Šmejkal et al. **"Beyond Conventional Ferromagnetism and Antiferromagnetism: A Phase with Nonrelativistic Spin and Crystal Rotation Symmetry".** In: *Physical Review X* 12.3 (2022).

[3] Purnendu Das et al. **"Realizing Altermagnetism in Fermi-Hubbard Models with Ultracold Atoms".** In: *Physical Review Letters* 132.26 (2024).

Questions?