

Atomistic Simulations with High-Dimensional Neural Network Potentials

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1

Overview

RUB

Topic of this talk

Machine Learning as a tool to extend the time and length scales of ab initio molecular dynamics

Goal 1: General method for all types of systems

Goal 2: Quality of reference method (here DFT)

Goal 3: Answer questions that cannot be answered with AIMD directly

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2

Introduction

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Goal:
Atomic-level understanding of complex systems in chemistry and materials science
⇒ Predictive computer simulations with first-principles quality

The accuracy of the obtained results depends on the quality of the atomic interaction

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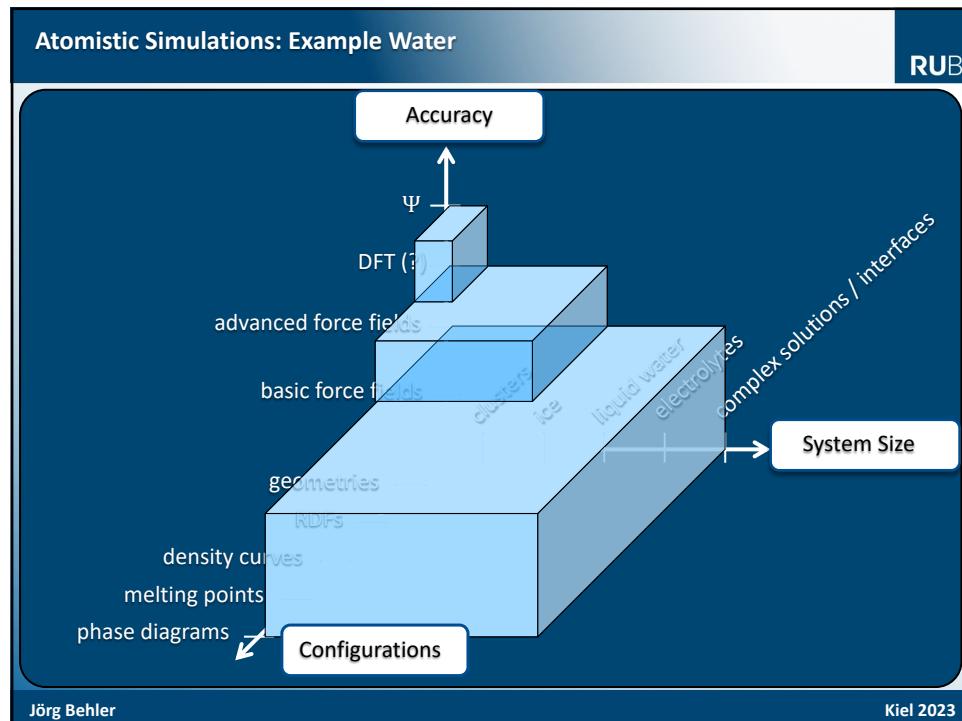
Potentials for Different Length and Time Scales

RUB

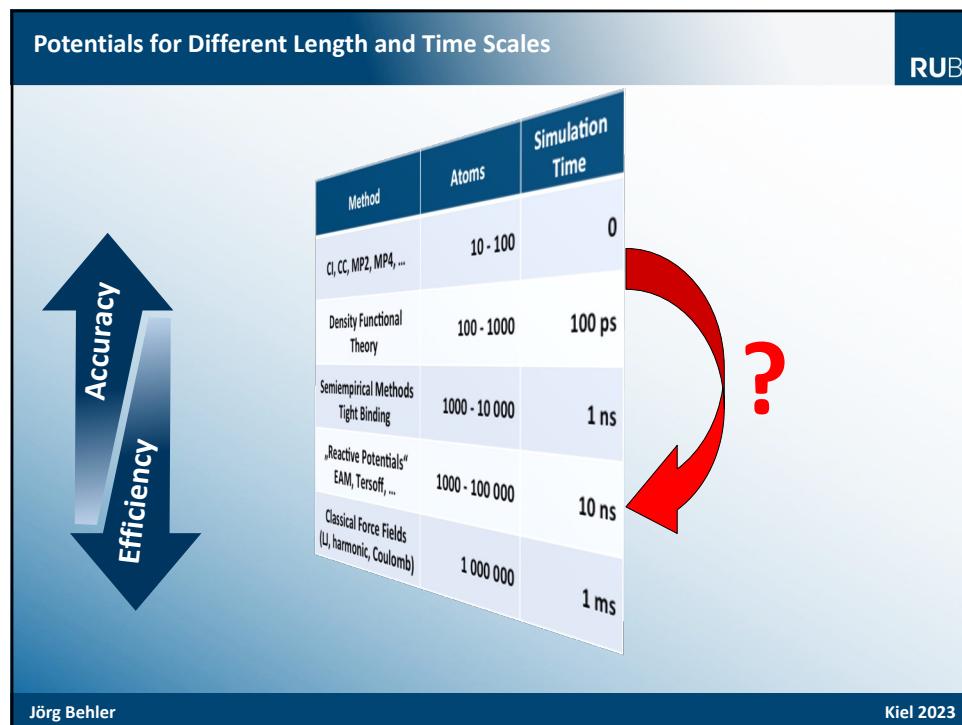
⇒ PES is the central quantity for atomistic simulations

Method	Atoms	Simulation Time
CI, CC, MP2, MP4, ...	10 - 100	0
Density Functional Theory	100 - 1000	100 ps
Semiempirical Methods Tight Binding	1000 - 10 000	1 ns
„Reactive Potentials“ EAM, Tersoff, ...	1000 - 100 000	10 ns
Classical Force Fields (LJ, harmonic, Coulomb)	1 000 000	1 ms

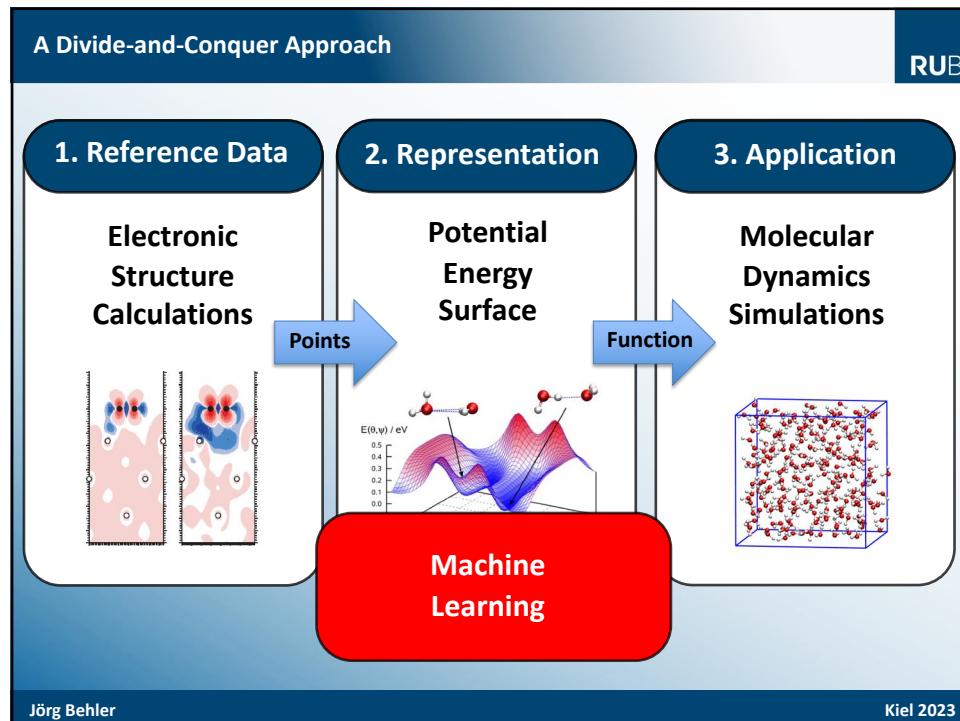
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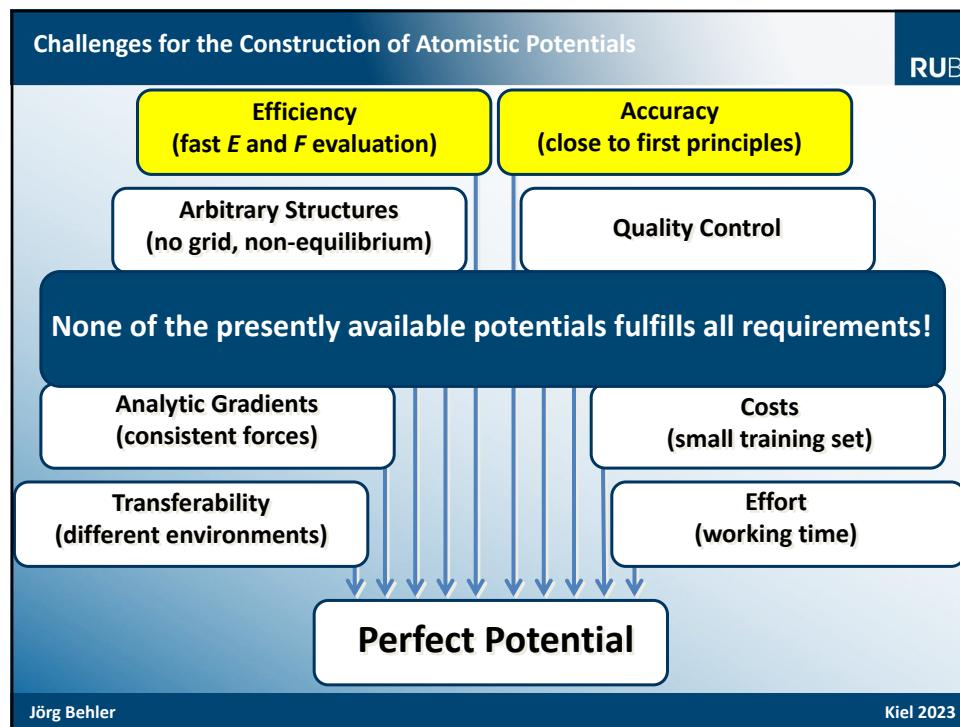
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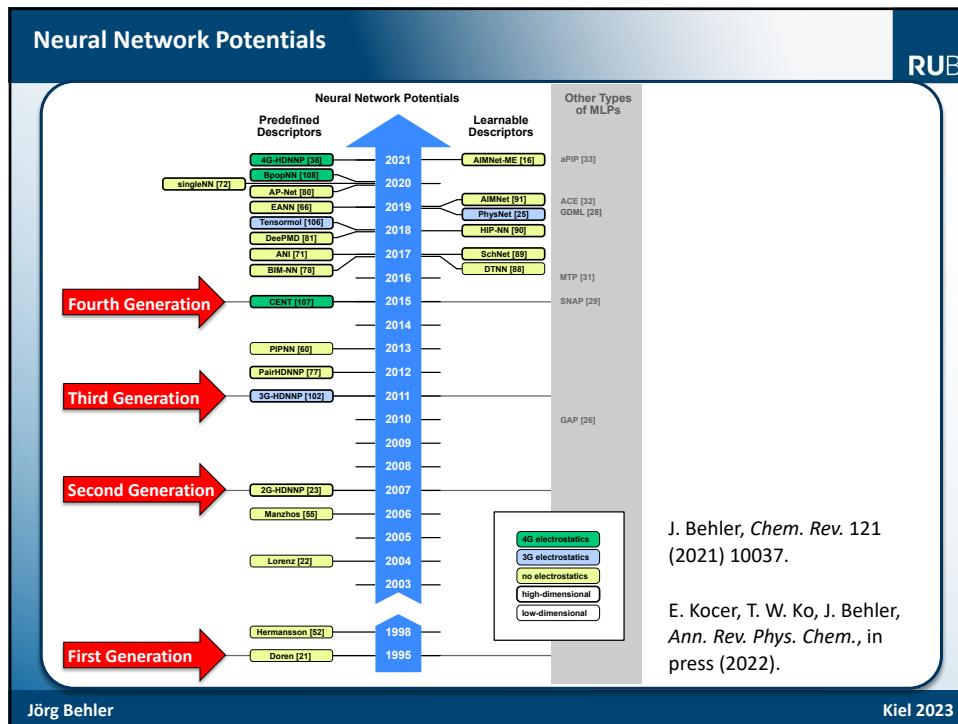
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9

First-Generation Neural Network Potentials 1995 – 2007

10

Machine Learning Potentials

RUB

1995 2000 2005 2010 2015 2020

↑

First Machine Learning Potential = First Neural Network Potential

T.B. Blank, S.D. Brown, A.W. Calhoun, and D.J. Doren, *J. Chem. Phys.* **103** (1995) 4129.

Input Layer Hidden Layer 1 Hidden Layer 2 Output Layer

Feed-forward neural network

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11

Machine Learning Potentials

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1995 2000 2005 2010 2015 2020

↑

First-Generation MLPs

- about 30 papers (1995 – 2007) from about 10 groups
- all methods in the first decade are based on neural networks

Examples:

T.B. Blank, S.D. Brown, A.W. Calhoun, and D.J. Doren, *J. Chem. Phys.* **103** (1995) 4129.
H. Gassner, M. Probst, A. Lauenstein, K. Hermansson, *J. Phys. Chem. A* **102** (1998) 4596.
S. Lorenz, A. Groß, M. Scheffler, *Chem. Phys. Lett.* **395** (2004) 210.
S. Manzhos, T. Carrington, Jr., *J. Chem. Phys.* **125** (2006) 194105.
J. Behler, S. Lorenz, K. Reuter, *J. Chem. Phys.* **127** (2007) 014705.

Focus:

Training
Symmetry
Surfaces
Spectroscopy
Surface Symmetry

⇒ Basic ideas and key concepts are well established

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12

Machine Learning Potentials

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1995 2000 2005 2010 2015 2020

First-Generation MLPs

Limitation: Applicable to low-dimensional systems only

Challenges:

- limited number of dimensions (up to ≈ 12)
- permutation symmetry of the system not included (change in order of atoms changes the energy)
- energy depends on rotation and translation
- potential is valid only for a given system size (number of atoms)

⇒ No generally applicable solution for all systems

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13

Machine Learning Potentials

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1995 2000 2005 2010 2015 2020

First-Generation MLPs

Limitation: Applicable to low-dimensional systems only

Challenges:

- limited number of dimensions (up to ≈ 12)
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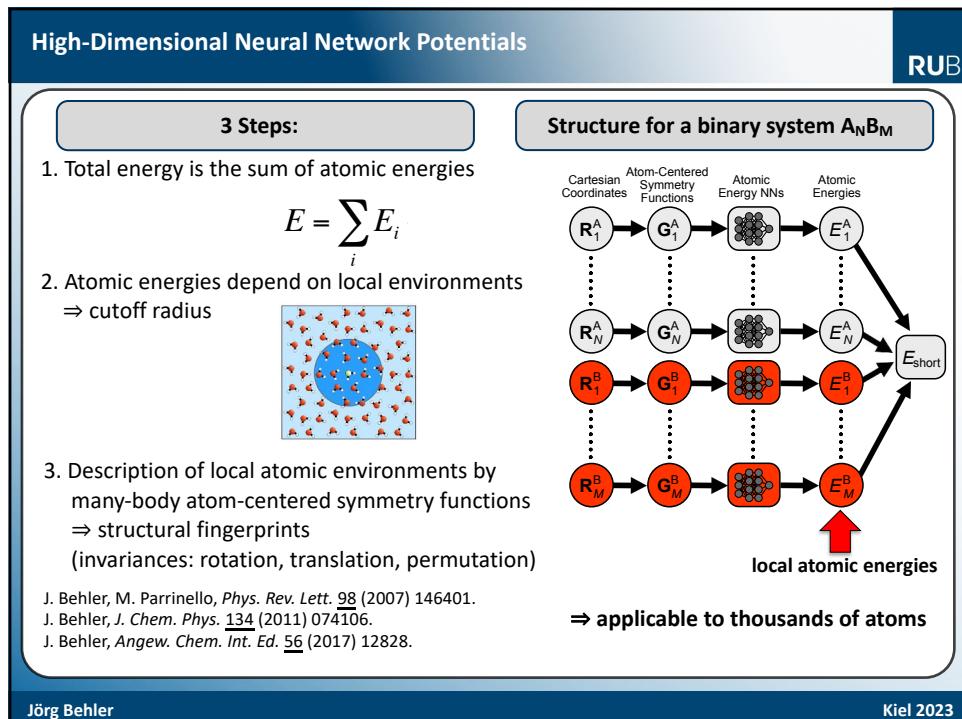
⇒ Another approach is required for high-dimensional systems

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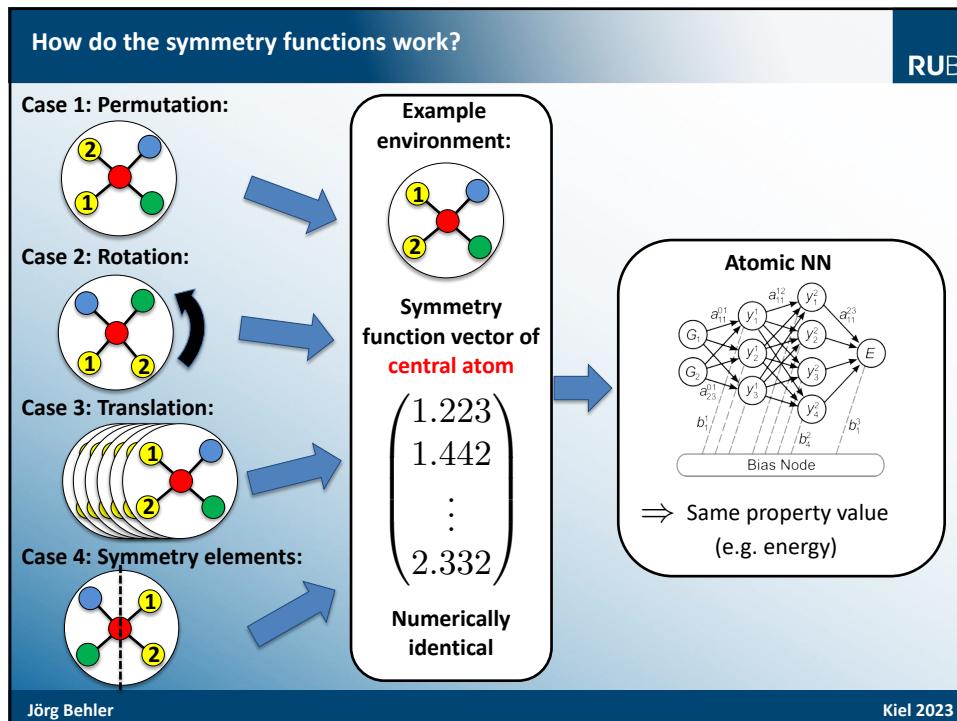
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Second-Generation Neural Network Potentials: Locality

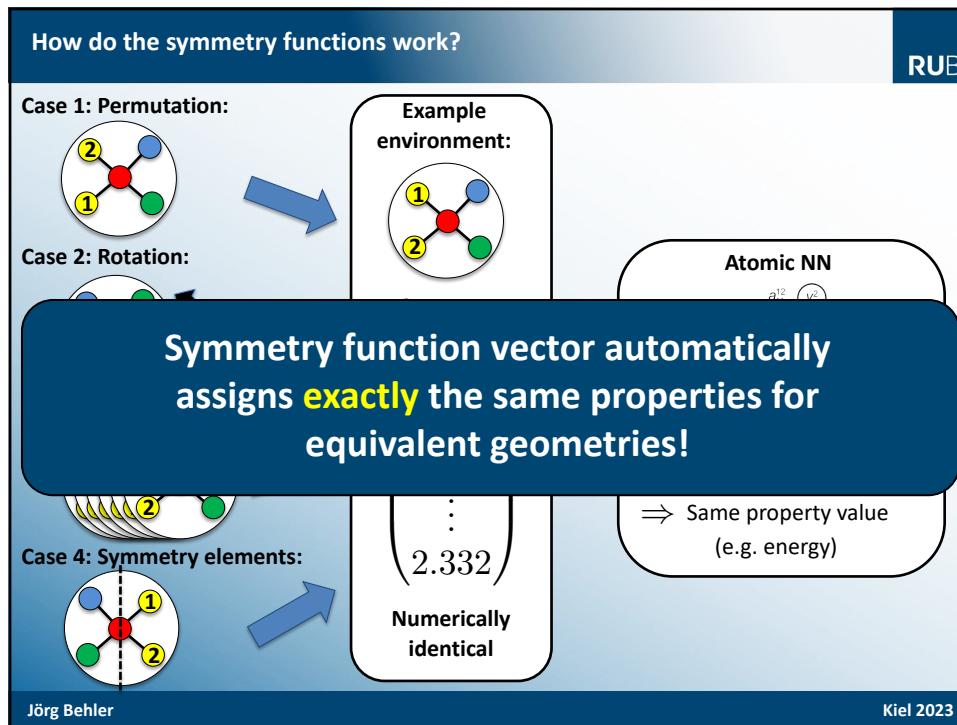
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16



17



18

Symmetry Functions: Cutoff Function RUB

Cutoff Function

- decays to zero in value and slope at R_c
- reflects decreasing chemical interaction
- central component of all symmetry functions
- R_c is increased until potential converges

$$f_c(R_{ij}) = \begin{cases} \frac{1}{2} \left[\cos\left(\frac{\pi R_{ij}}{R_c}\right) + 1 \right] & \text{for } R_{ij} < R_c \\ 0 & \text{for } R_{ij} > R_c \end{cases}$$

Typical cutoff radius: $R_c = 6 - 8 \text{ \AA}$

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19

Symmetry Functions: Radial Functions RUB

- decay with increasing distance \Rightarrow Gaussians
- summation over all neighbors
- many-body term, interpretation as coordination number
- one-to-one correspondence between function value and R_{ij}

$$G_i^{rad} = \sum_j e^{-\eta(R_{ij}-R_s)^2} f_c(R_{ij})$$

**Set of radial functions:
„Radial Fingerprint“**

J. Behler, J. Chem. Phys. 134 (2011) 074106.

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20

Symmetry Functions RUB

Angular symmetry functions

$$G_i^\mu = 2^{1-\xi} \sum_{j,k \neq i} (1 + \lambda \cos \theta_{ijk})^\xi \cdot e^{-\eta(R_{ij}^2 + R_{ik}^2 + R_{jk}^2)} f_c(R_{ij}) f_c(R_{ik}) f_c(R_{jk})$$

⇒ Several angular functions are used

- all symmetry functions are many-body terms
- typical number: about 10 radial and 40 angular functions

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21

Symmetry Functions: Combinatorial Growth RUB

For an N element system, there are N separate atomic NN types
⇒ no significant increase in complexity

For each radial function there are N functions for the possible neighboring elements
⇒ increase in complexity

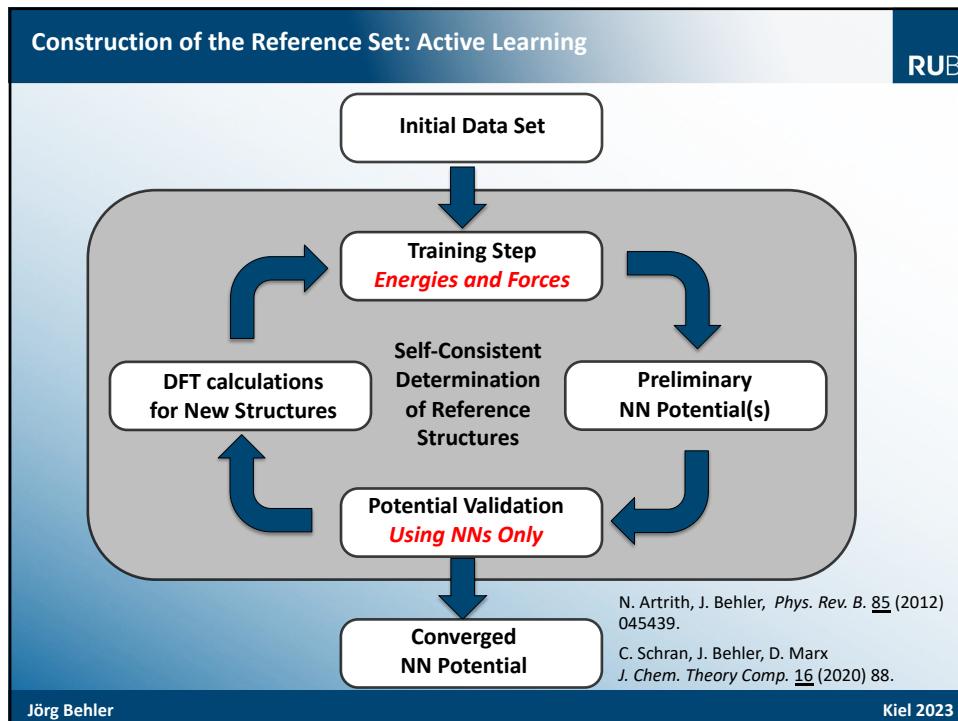
For each angular function there are $N(N+1)/2$ functions for the neighboring elements
⇒ strong increase in complexity

Elements	atomic NNs	multiplier radial/angular symmetry functions	typical total number of symmetry functions
1	1	1 / 1	6 + 25
2	2	2 / 3	12 + 75
3	3	3 / 6	18 + 150
4	4	4 / 10	24 + 250

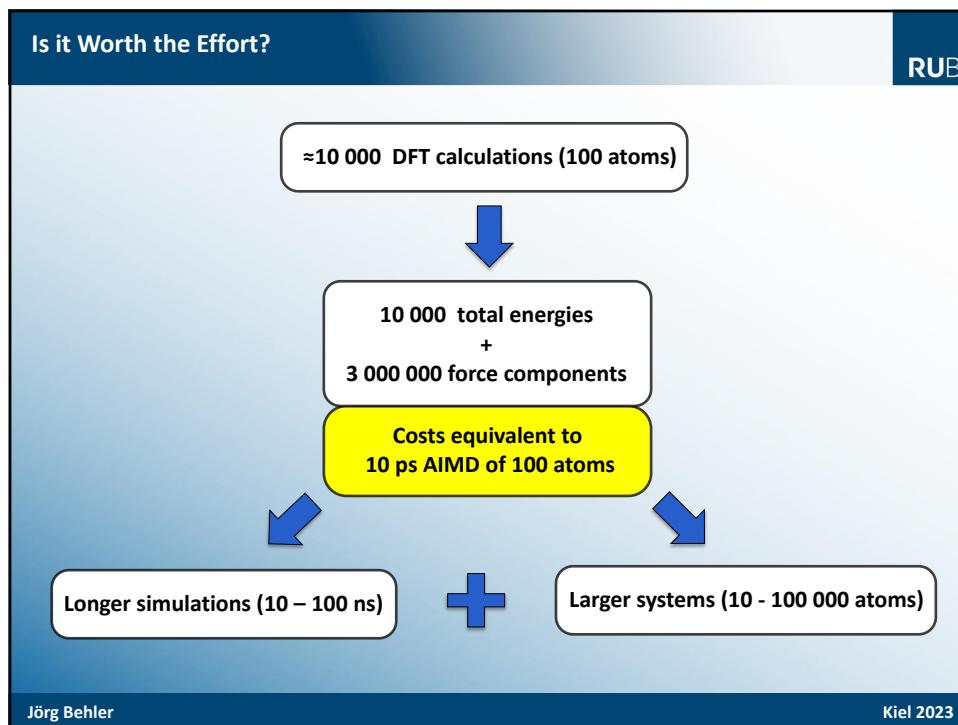
Often, a reduction is possible, if only some compositions or structures are relevant

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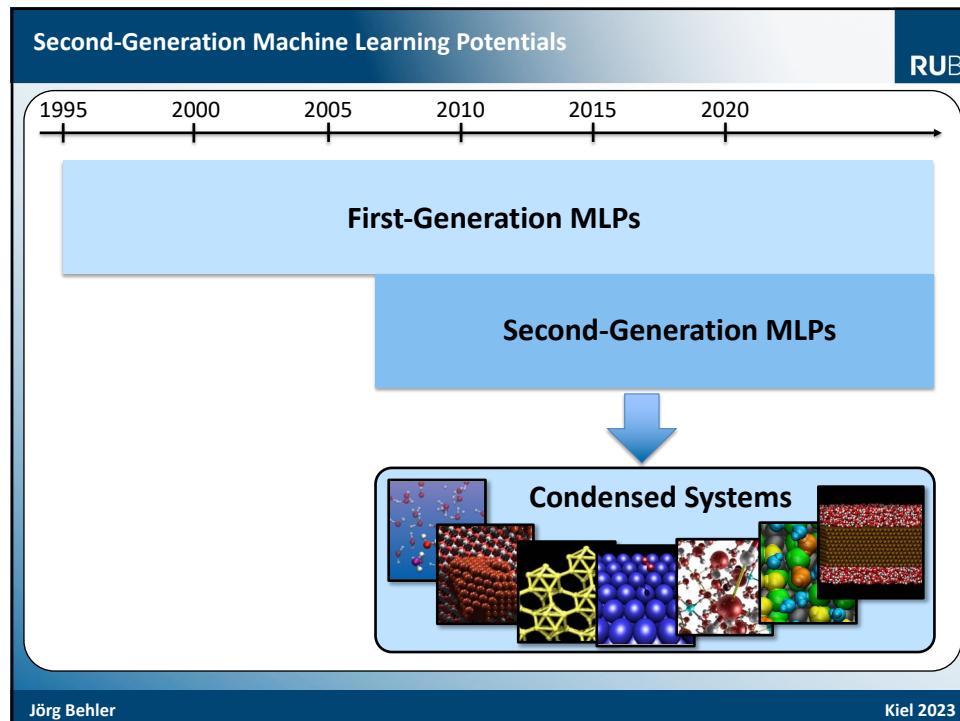
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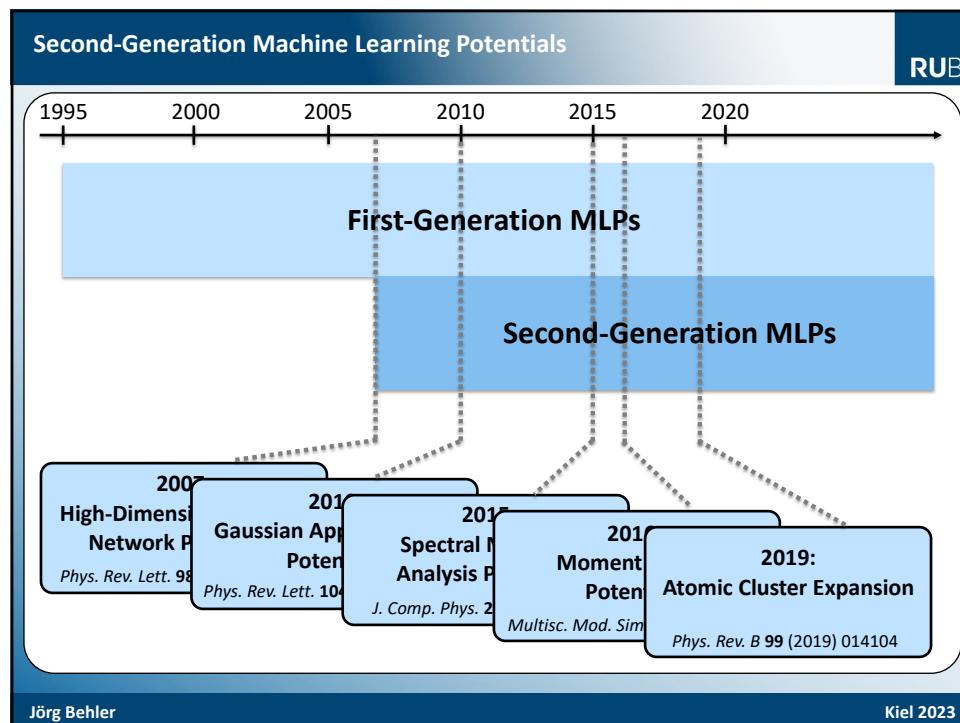
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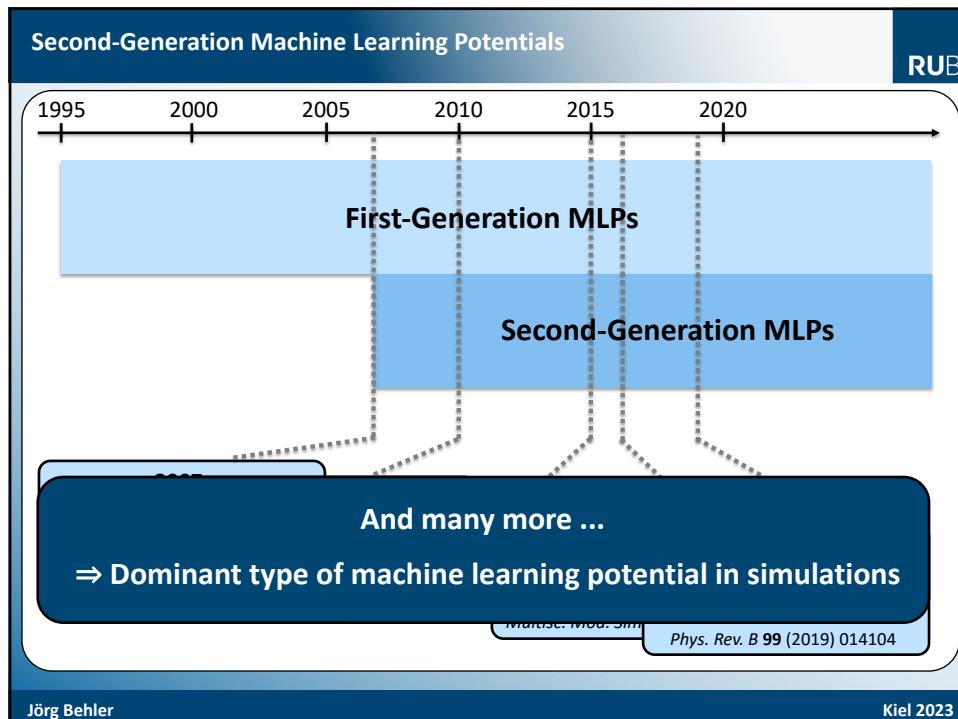
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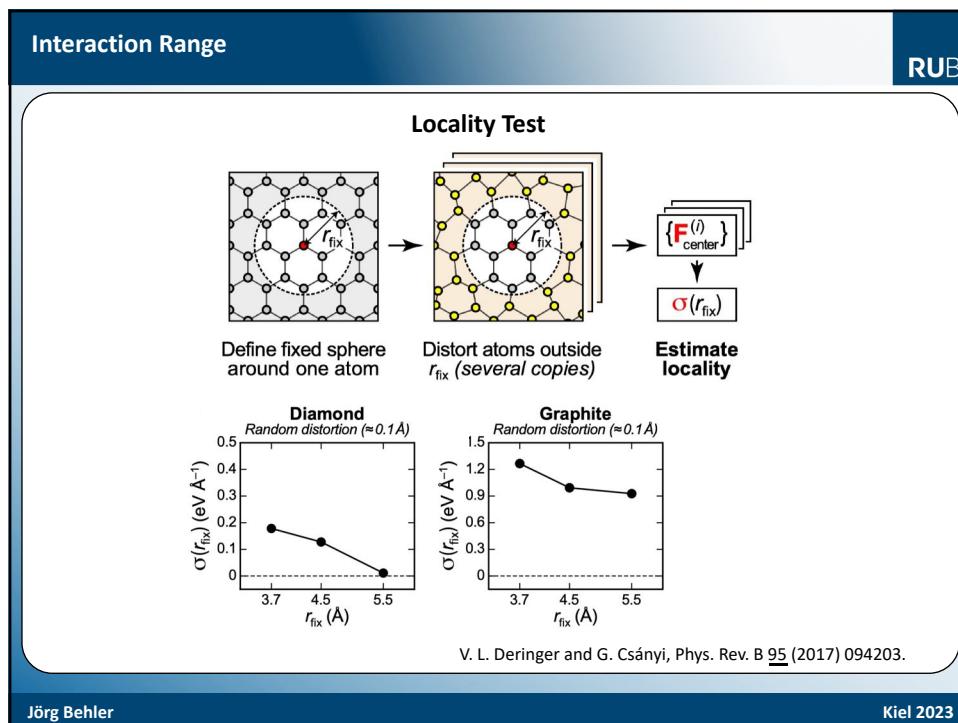
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26



27



28

Interaction Range

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A Hessian-Based Analytic Locality Test

Hessian $H_{A_\alpha B_\beta} = \frac{\partial^2 E}{\partial A_\alpha \partial B_\beta} = -\frac{\partial f_{B_\beta}}{\partial A_\alpha} = -\frac{\partial f_{A_\alpha}}{\partial B_\beta}$ $\alpha, \beta = \{x, y, z\}$

		B	1	2	3	4		
		x	y	z	x	y	z	
		x						
1		y						
		z						
2		x						
		y						
		z						
3		x						
		y						
		z						
4		x						
		y						
		z						
A	α	β	x	y	z	x	y	z

Hessian submatrix norm

$$\|\mathbf{h}_{AB}\| = \sqrt{\sum_{\alpha=x,y,z} \sum_{\beta=x,y,z} h_{A_\alpha B_\beta}^2}$$

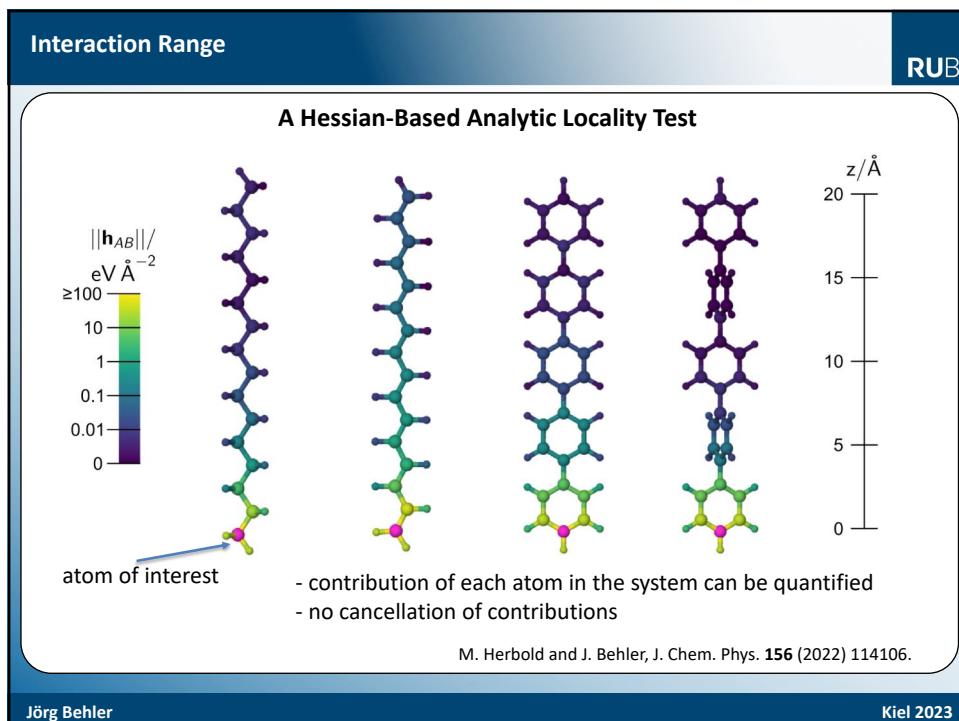
Dependence of force on each individual atom in the system can be quantified

M. Herbold and J. Behler, J. Chem. Phys. **156** (2022) 114106.

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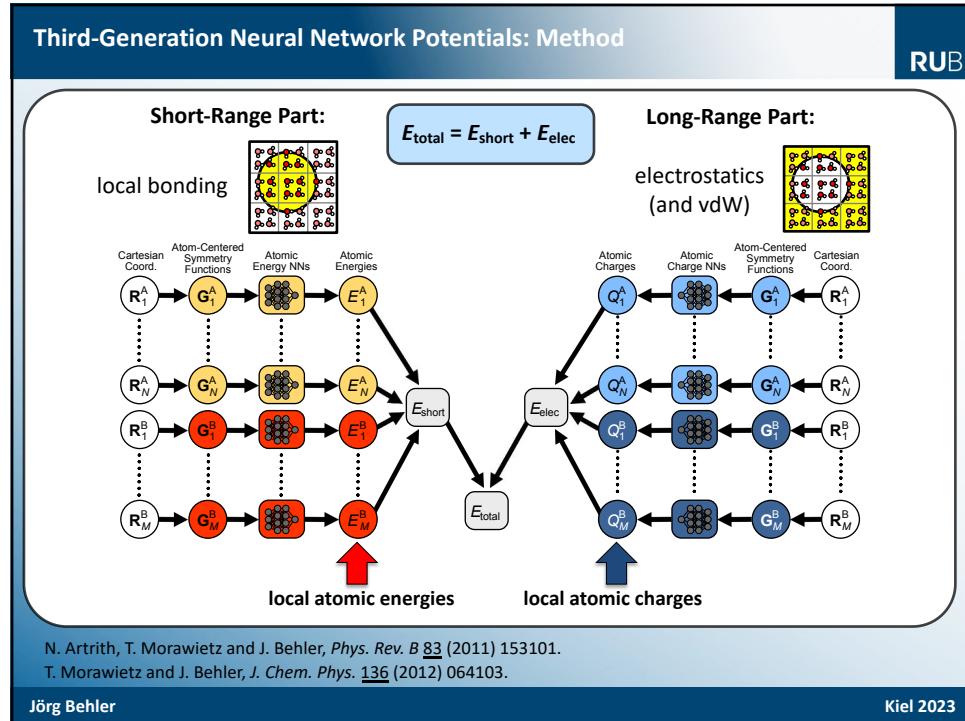
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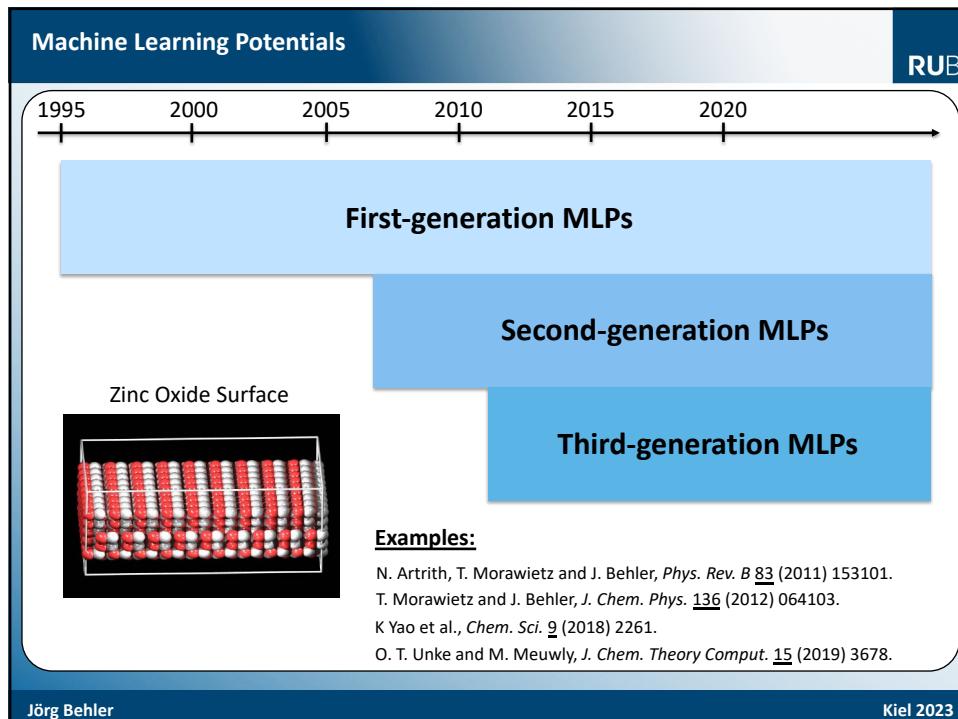
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Third-Generation Neural Network Potentials: Long-Range Interactions

31



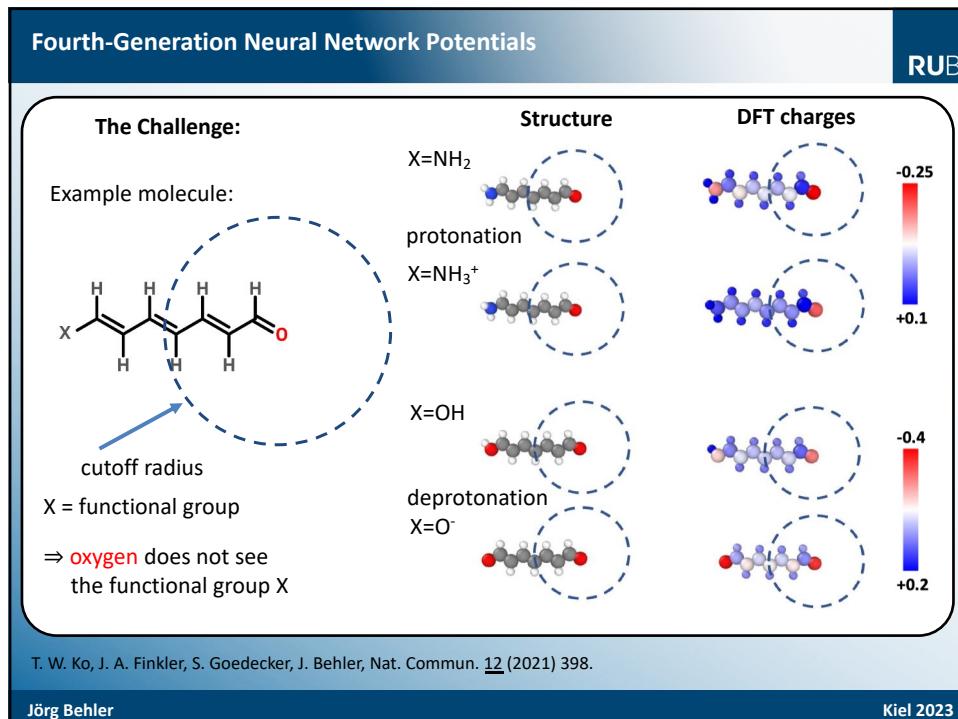
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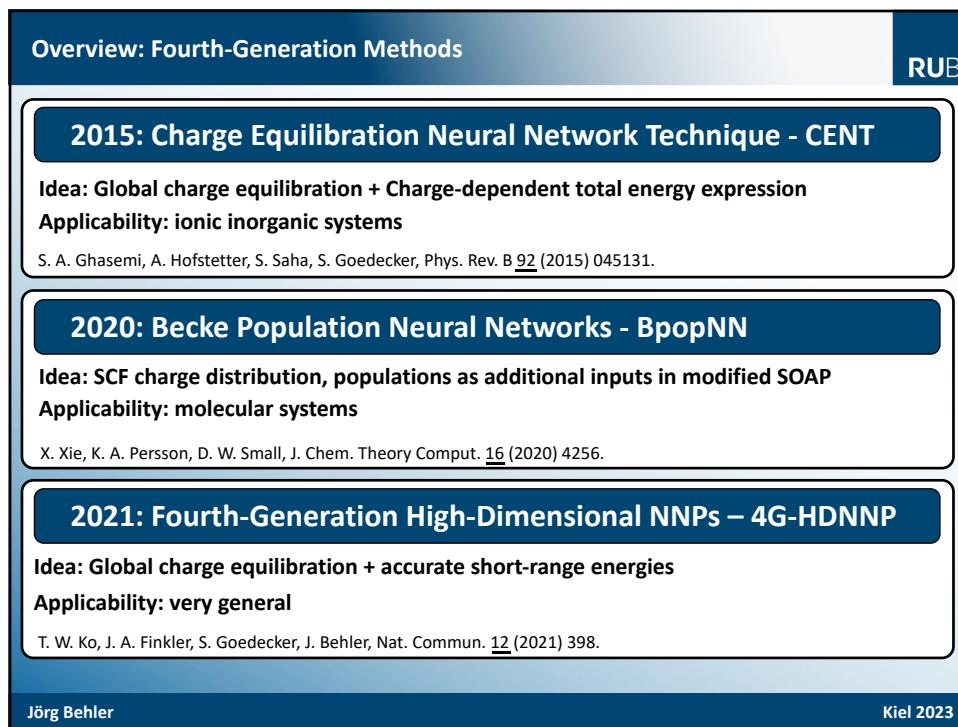
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Fourth-Generation Neural Network Potentials: Global Electronic Structure

34



35



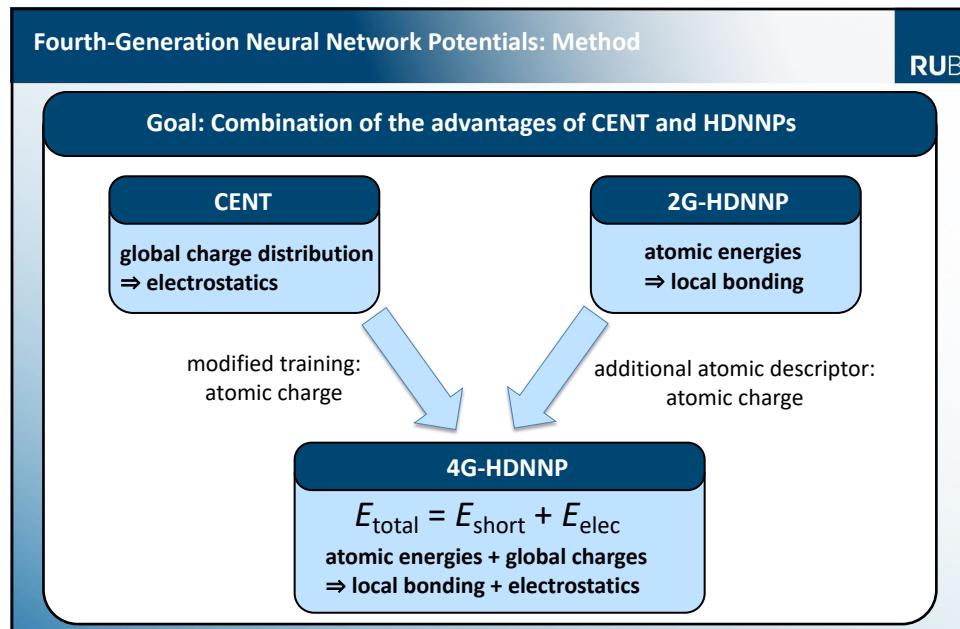
36

S. A. Ghasemi, A. Hofstetter, S. Saha, S. Goedecker, Phys. Rev. B **92** (2015) 045131.

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37



T. W. Ko, J. A. Finkler, S. Goedecker, J. Behler, *Nature Commun.* **12** (2021) 398.

T. W. Ko, J. A. Finkler, S. Goedecker, J. Behler, Acc. Chem. Res. 54 (2021) 808.

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38

Fourth-Generation Neural Network Potentials: Method

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Goal: Combination of the advantages of CENT and HDNNPs

Advantages:

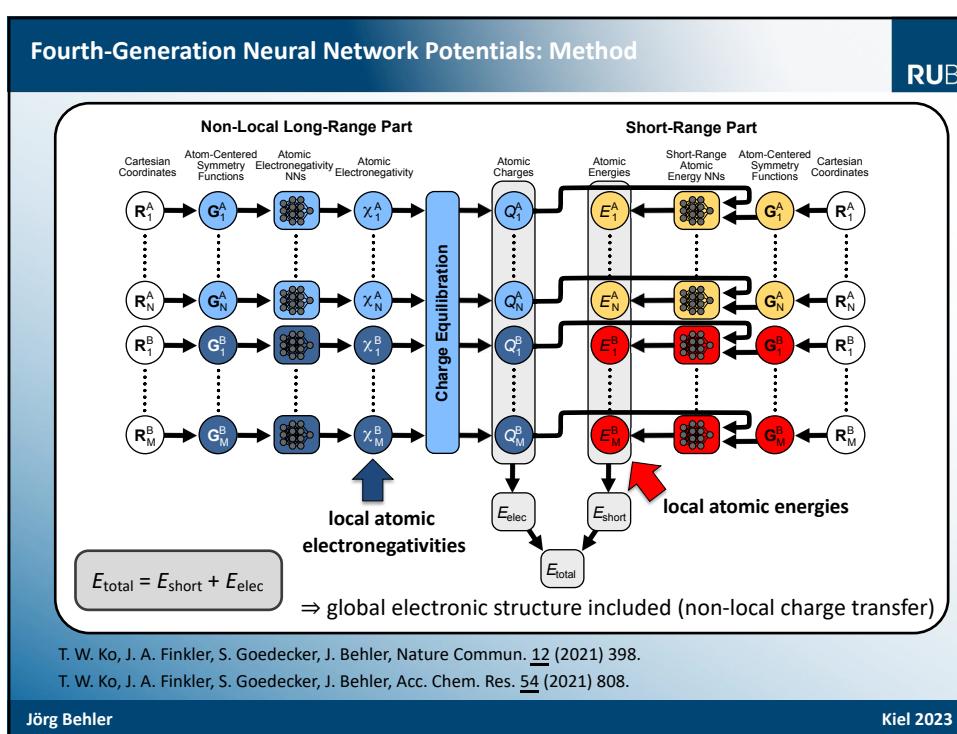
- applicable to all types of bonding and systems (ionic, covalent, metallic, ...)
- long-range electrostatic interactions (flexible charges)
- description of non-local charge transfer
- applicable to multiple global charge states

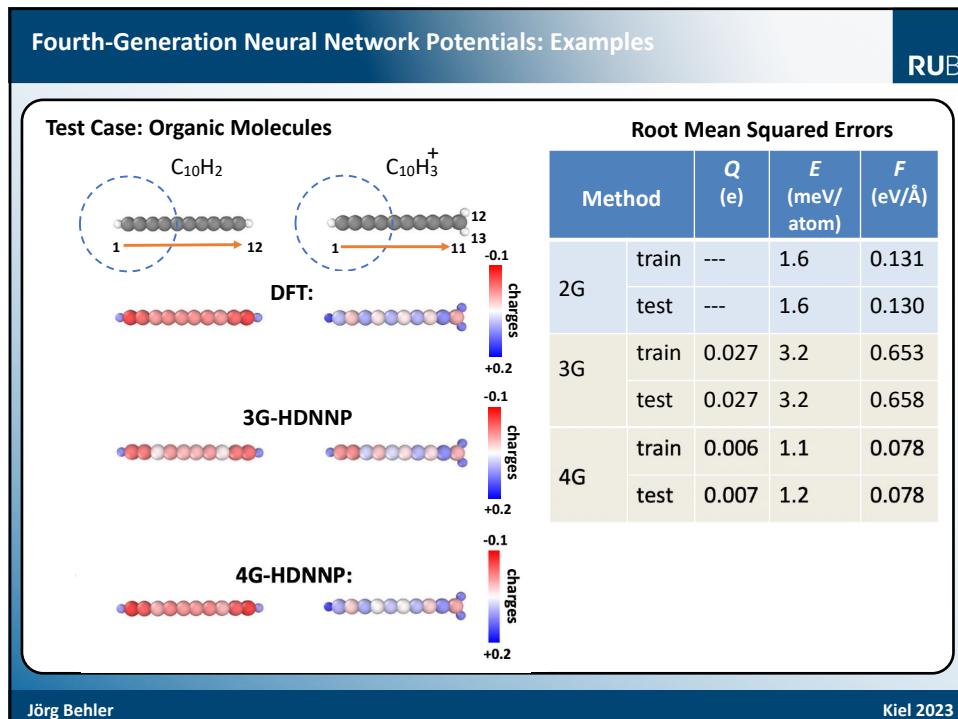
atomic energies + global charges
⇒ local bonding + electrostatics

T. W. Ko, J. A. Finkler, S. Goedecker, J. Behler, Nature Commun. 12 (2021) 398.
T. W. Ko, J. A. Finkler, S. Goedecker, J. Behler, Acc. Chem. Res. 54 (2021) 808.

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41



42

Magnetic High-Dimensional Neural Network Potentials RUB

Most descriptors do not depend on atomic spins

indistinguishable

antiferromagnetic ferromagnetic

⇒ spin-related energy changes are treated as noise
⇒ unreliable energy surfaces
⇒ improved descriptors are needed

M. Eckhoff and J. Behler, *npj Comput. Mater.* **7** (2021) 170.
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43

Magnetic High-Dimensional Neural Network Potentials RUB

Spin-Dependent Atom-Centered Symmetry Functions (sACSF)

Step 1: Atomic spin coordinates (collinear spin)

$$s_i = \begin{cases} 0 & \text{for } |M_S| < M_S^{\text{thres}} \\ \text{sgn}(M_S) & \text{otherwise} \end{cases} \quad \text{with} \quad M_S = \frac{1}{2}(n_\uparrow - n_\downarrow)$$

Step 2: Spin-augmentation functions (SAF), radial case

$$M^0(s_i, s_j) = 1, \quad M^+(s_i, s_j) = \frac{1}{2}|s_i s_j| \cdot |s_i + s_j|, \quad M^-(s_i, s_j) = \frac{1}{2}|s_i s_j| \cdot |s_i - s_j|$$

⇒ spin-sensitive filter

Step 3: sACSF, radial case

$$G_i^{\text{rad}} = \sum_j M^x(s_i, s_j) \cdot e^{-\eta R_{ij}^2} \cdot f_c(R_{ij})$$

⇒ Second-generation (environment-dependent) extension

⇒ Magnetic High-Dimensional Neural Network Potentials

M. Eckhoff and J. Behler, *npj Comput. Mater.* **7** (2021) 170.
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44

Magnetic High-Dimensional Neural Network Potentials

RUE

Application: Manganese Oxide (MnO)

- antiferromagnetic ground state (AFM-II) with rhombohedral distortion
- magnetic cell is $2 \times 2 \times 2$ supercell of geometric unit cell

Training

RMSE energy (3000 $2 \times 2 \times 2$ supercells, HSE06 functional):

HDNNP: 11 meV/atom mHDNNP: 1 meV/atom
 $(\text{AFM-II} = \text{FM})$ $(\text{AFM-II} \neq \text{FM})$

Results

$\Delta E(\text{AFM-II} / \text{FM})$ mHDNNP: 46.3 meV/atom (HSE06: 45.9 meV/atom)

Lattice parameters AFM-II mHDNNP: $a = 4.433 \text{ \AA}$, $\alpha = 90.77^\circ$
 HSE06: $a = 4.434 \text{ \AA}$, $\alpha = 90.89^\circ$
 Exp.: $a = 4.430 \text{ \AA}$, $\alpha = 90.62^\circ$

M. Eckhoff and J. Behler, *npj Comput. Mater.* **7** (2021) 170.

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How do we know if a Neural Network Potential is reliable?

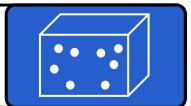
What can we do with Neural Network Potentials?

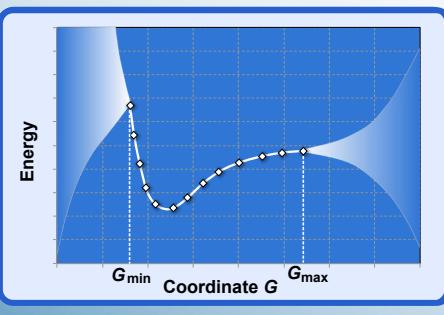
46

Quality Control

47

Multistep Quality Control RUB

1. Generate reasonable initial training set
⇒ preliminary NN potential 
2. Check for extrapolation

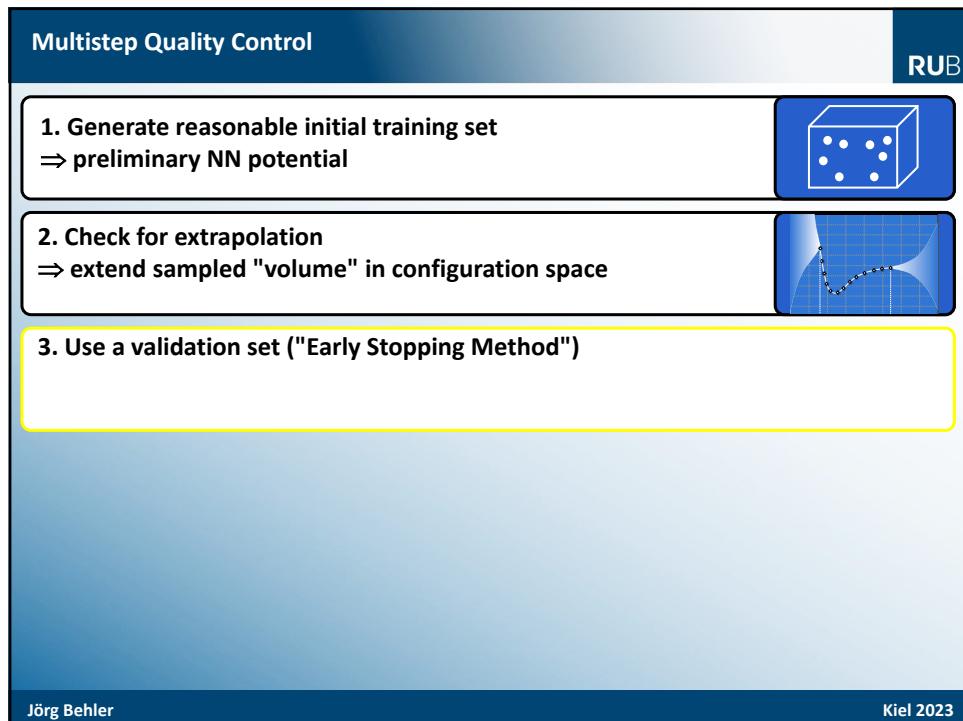


⇒ easy to detect

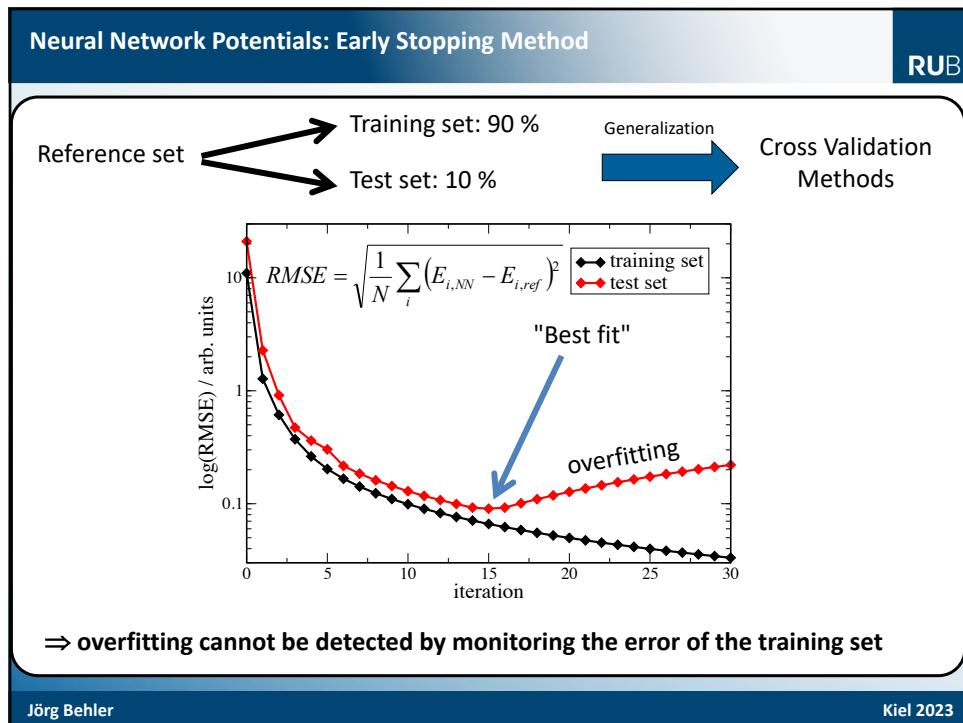
J. Behler, Angew. Chem. Int. Ed. 56 (2017) 12828.

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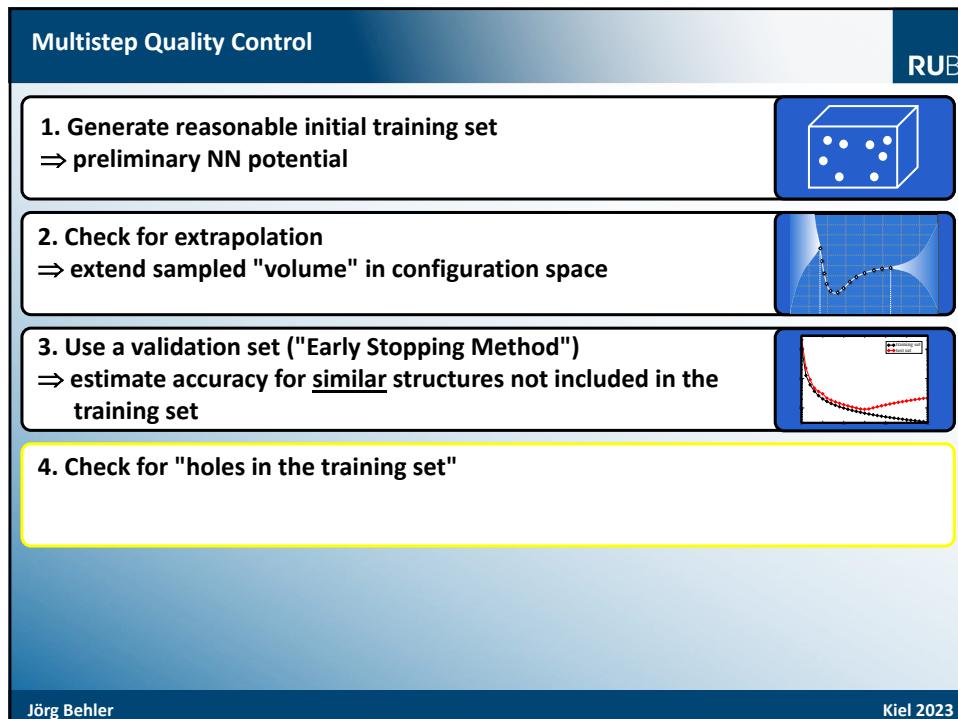
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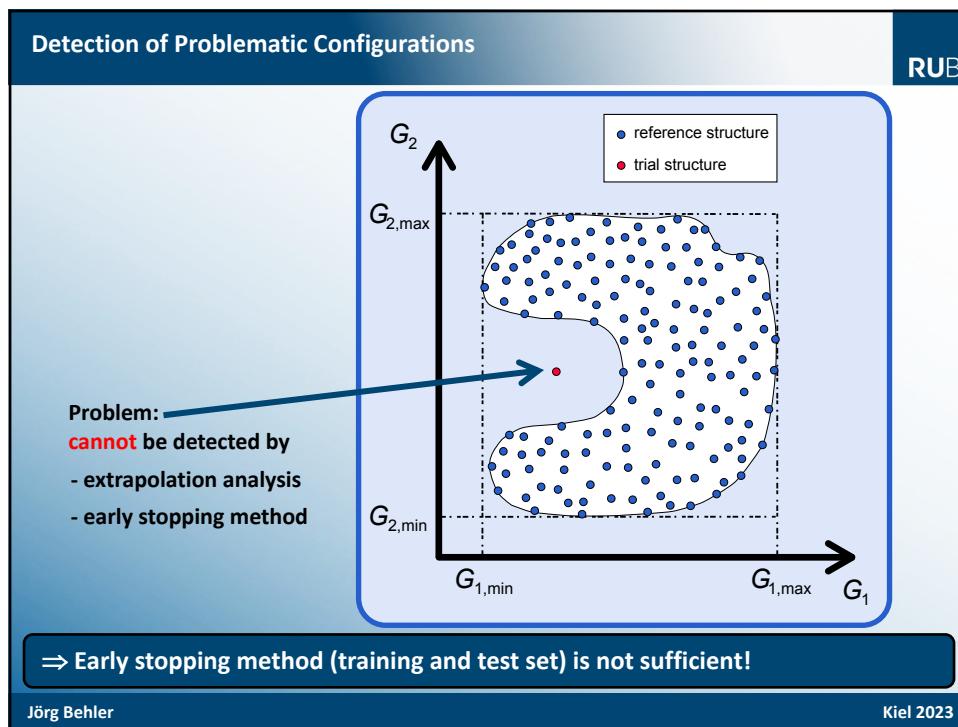
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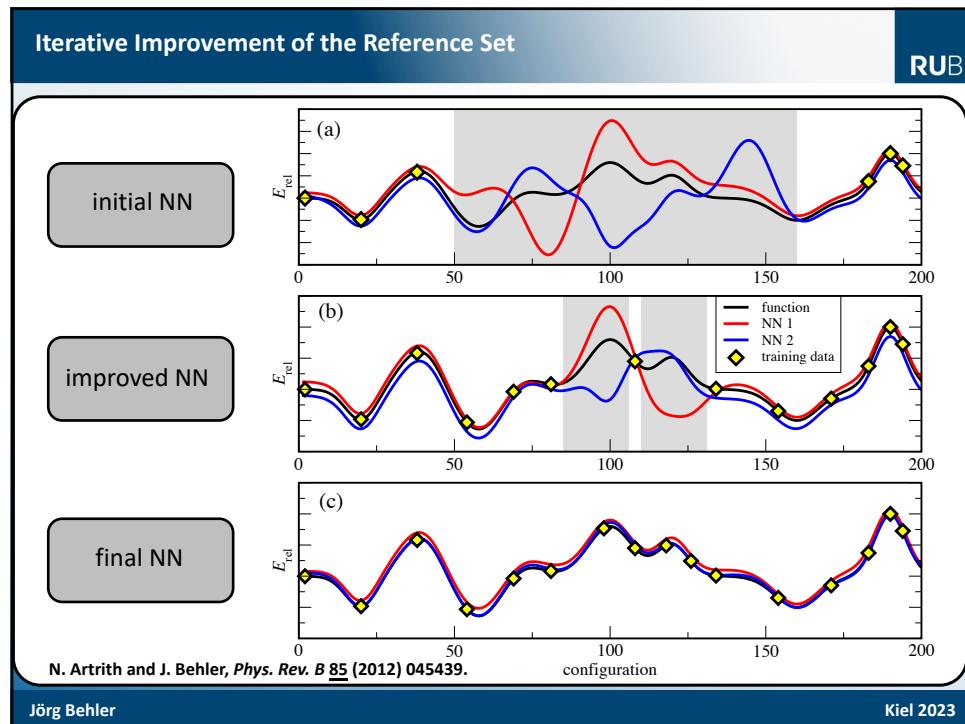
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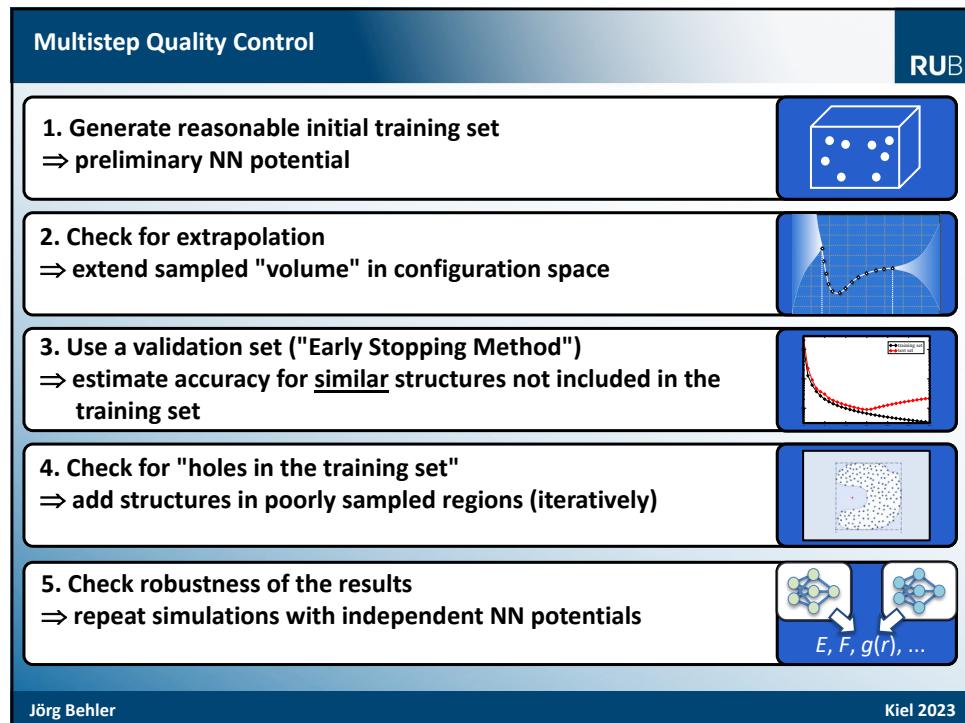
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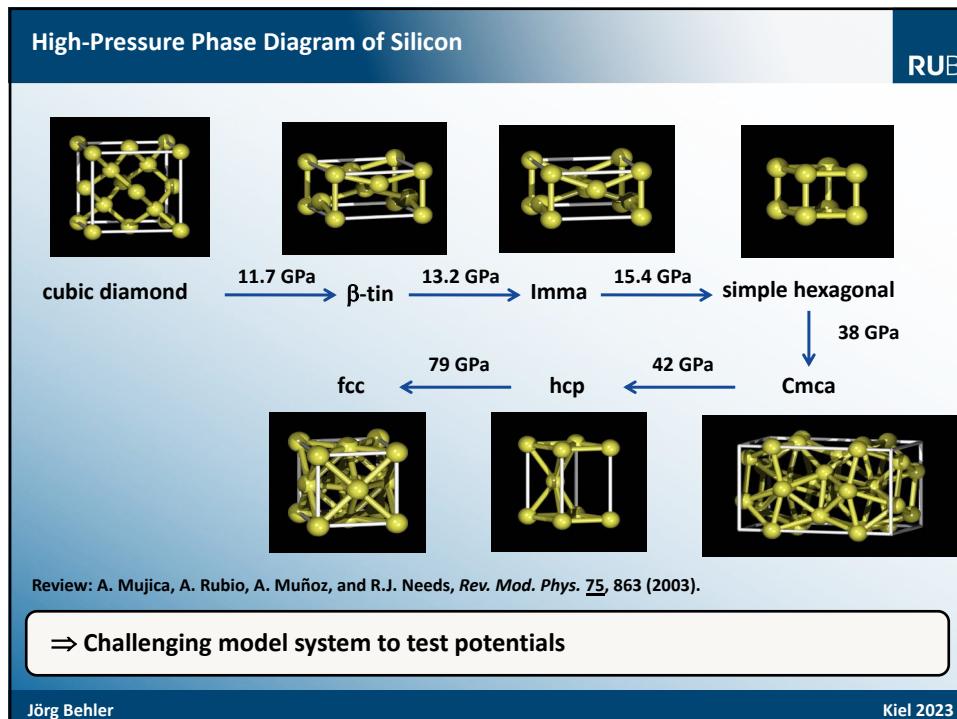
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Applications

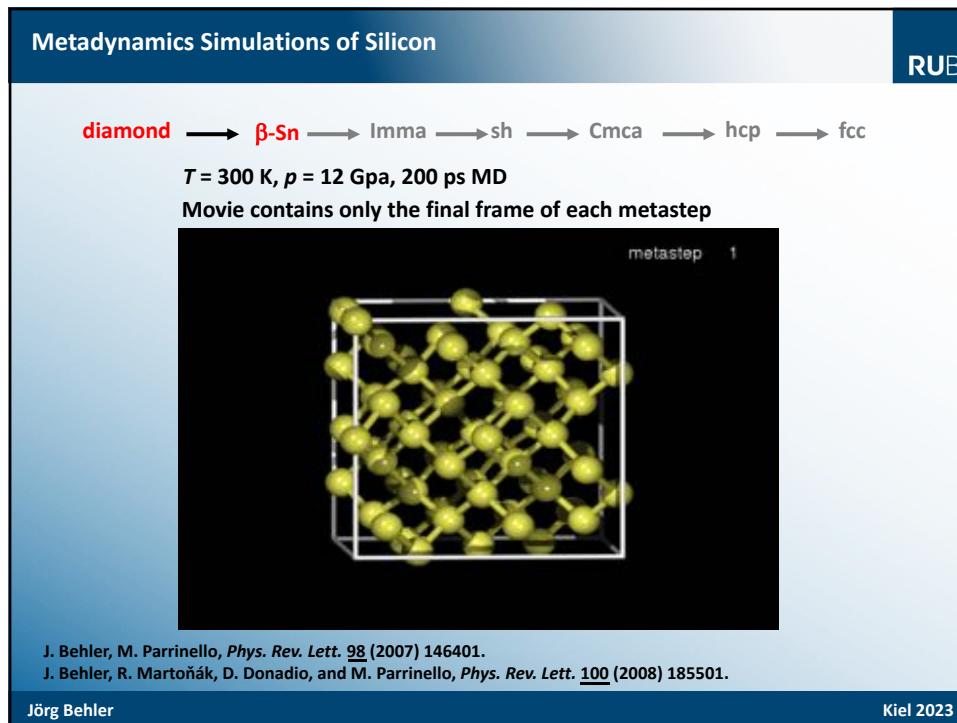
55

Materials Science: Crystal Structure Prediction

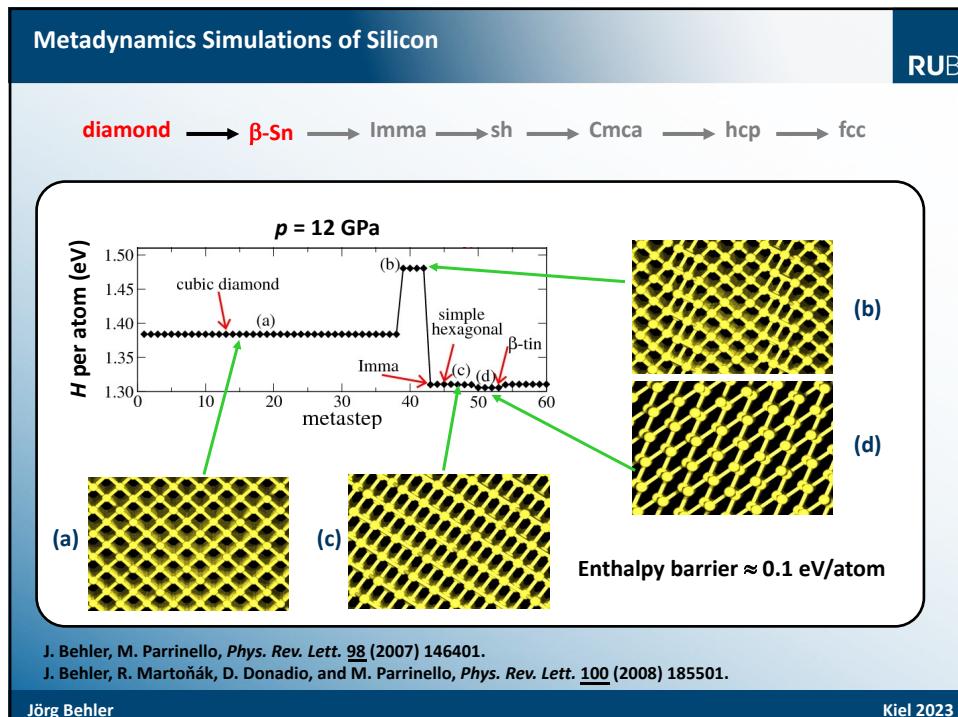
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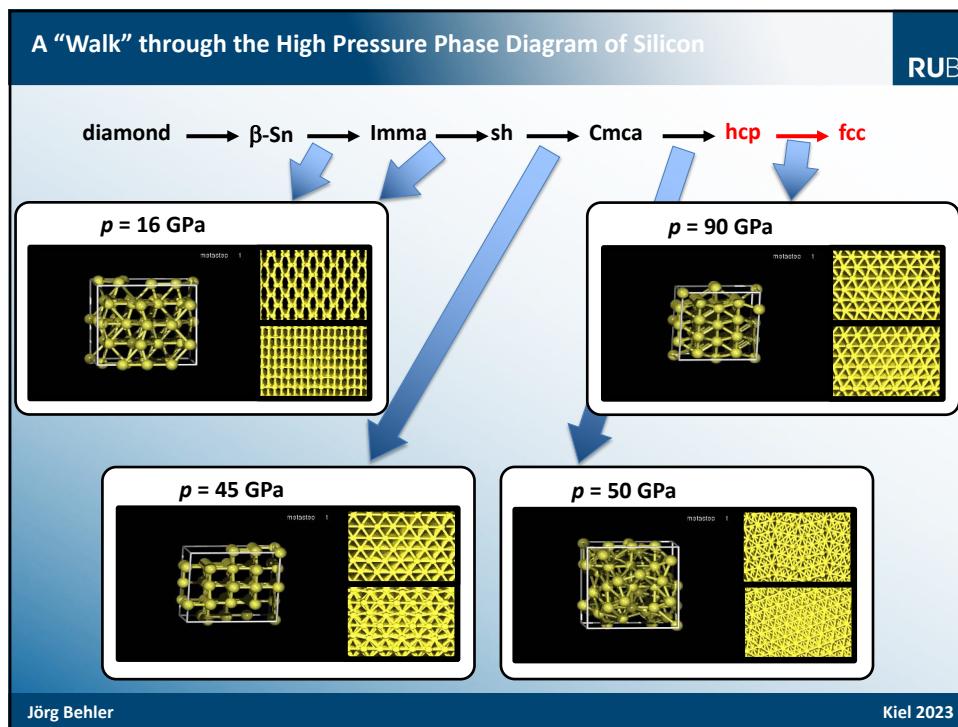
57



58



59



60

Prediction of the High-Pressure Phase Diagram of Silicon RUB

- ⇒ NN-based simulations have DFT quality
(and excellent agreement with experiment)
- ⇒ Predictive simulations impossible with DFT (costs!)
or other potentials like EAM, tight binding, ... (too low accuracy)

Simulation Costs

≈ 100 simulations x 100 metasteps x 2 ps MD (stress tensor)
 ⇒ 20 000 000 energy and force evaluations

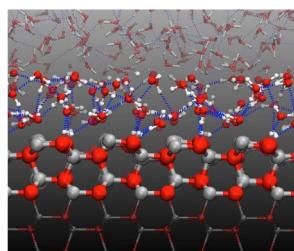
DFT:
 ⇒ ≈ 20 000 CPU years per phase diagram (8 M€ hardware "consumption")

NN:
 ⇒ ≈ 20 CPU years per phase diagram (8 k€ hardware "consumption")

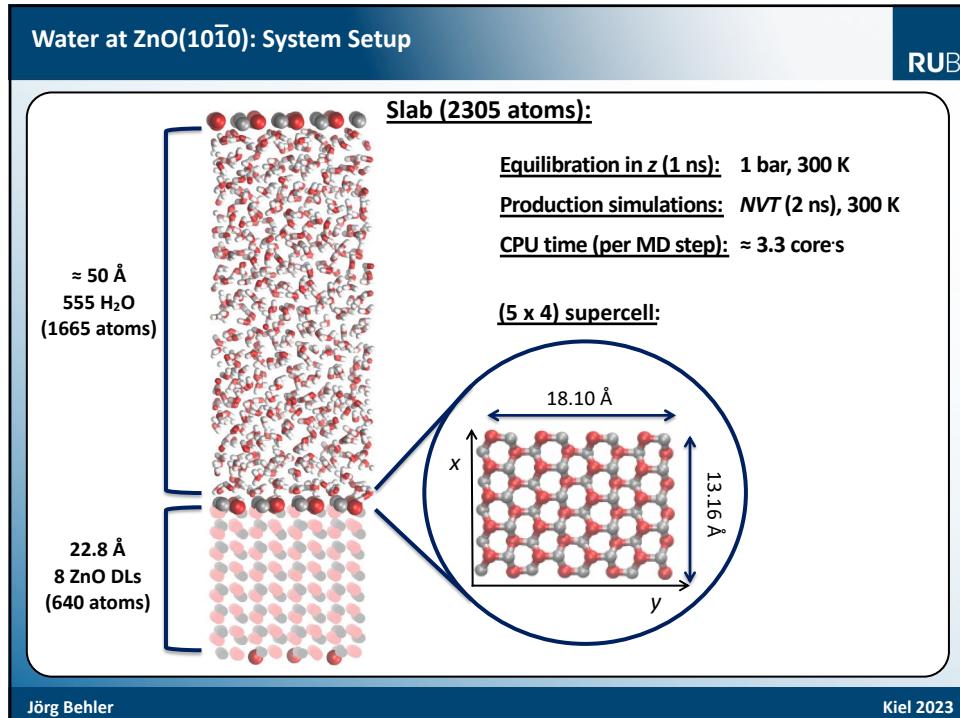
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61

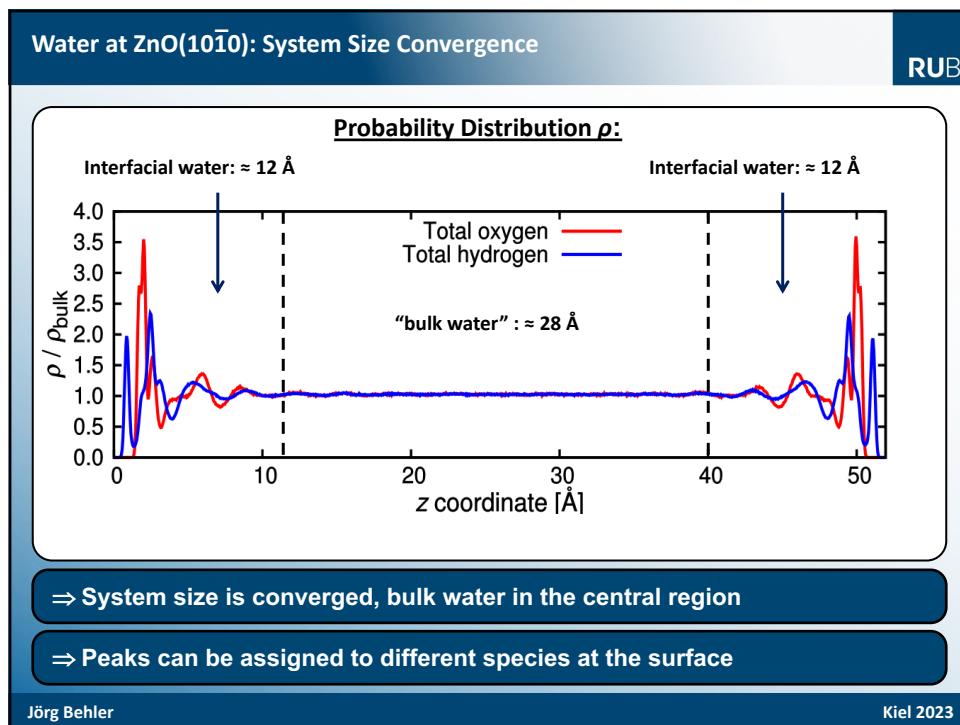
Solid-Liquid Interfaces: Water-Zinc Oxide



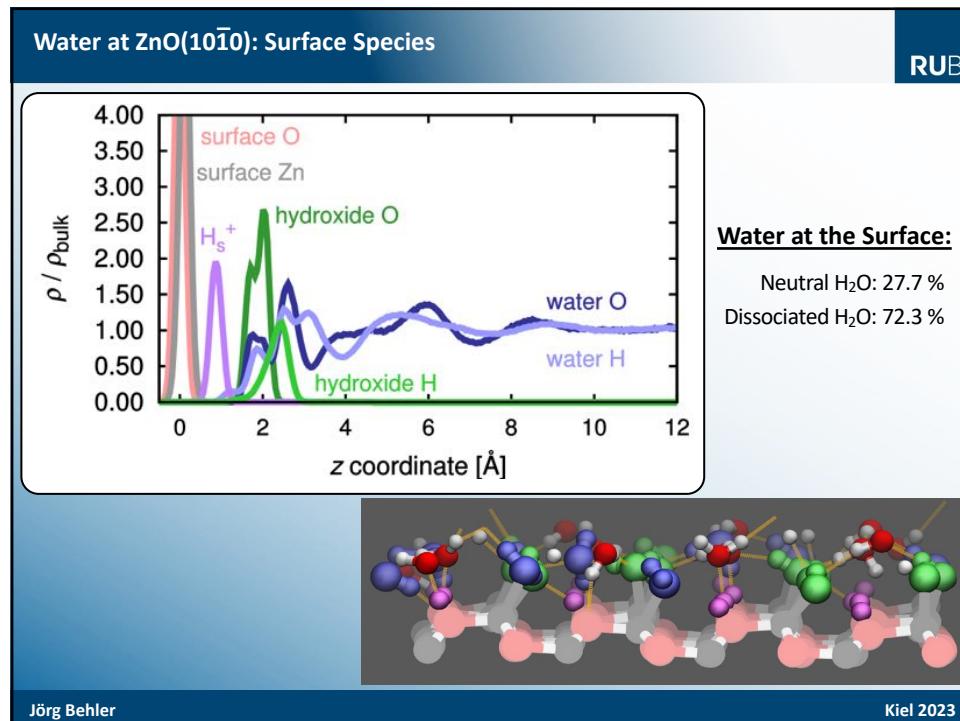
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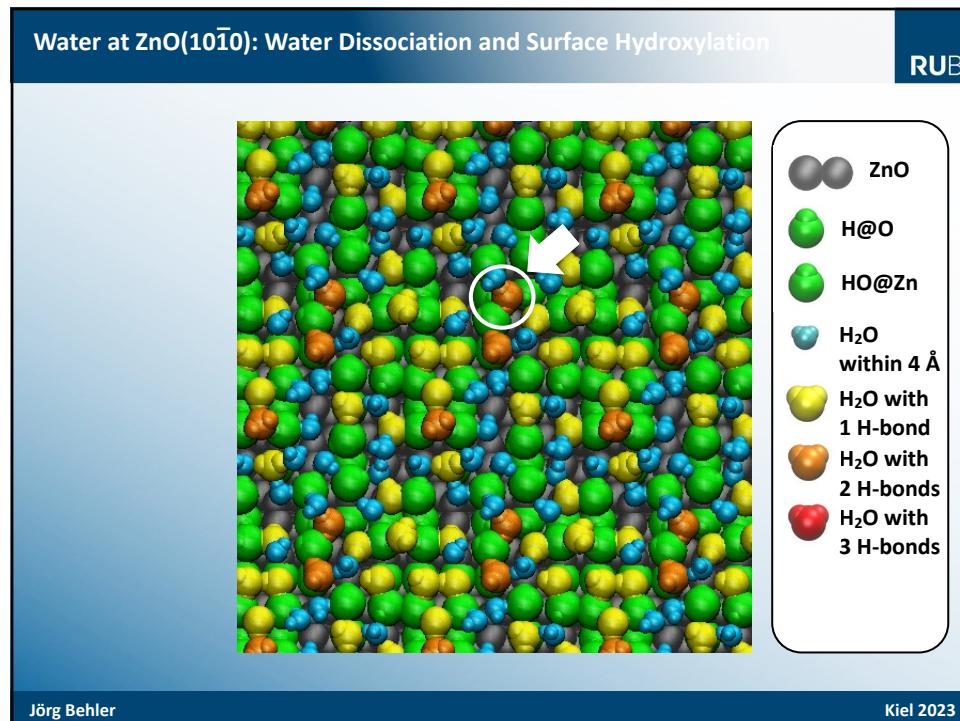
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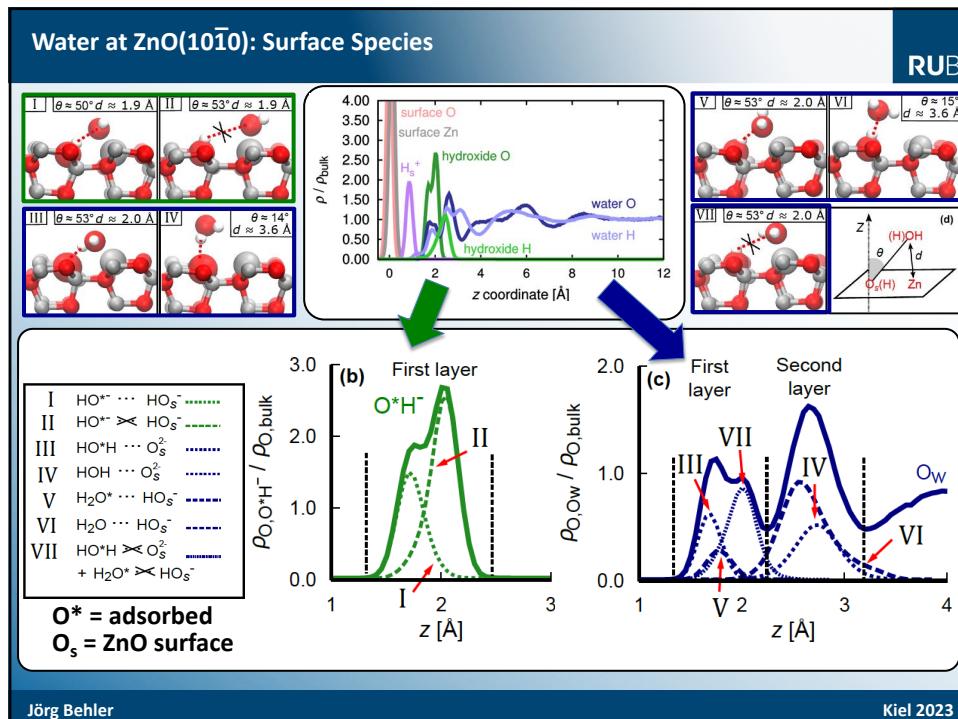
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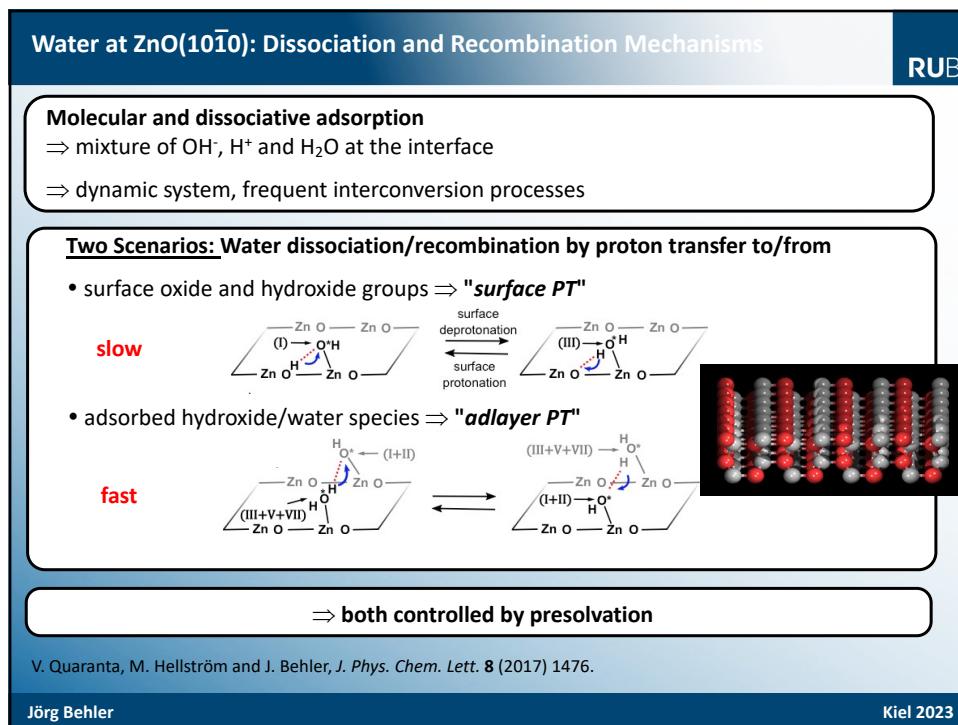
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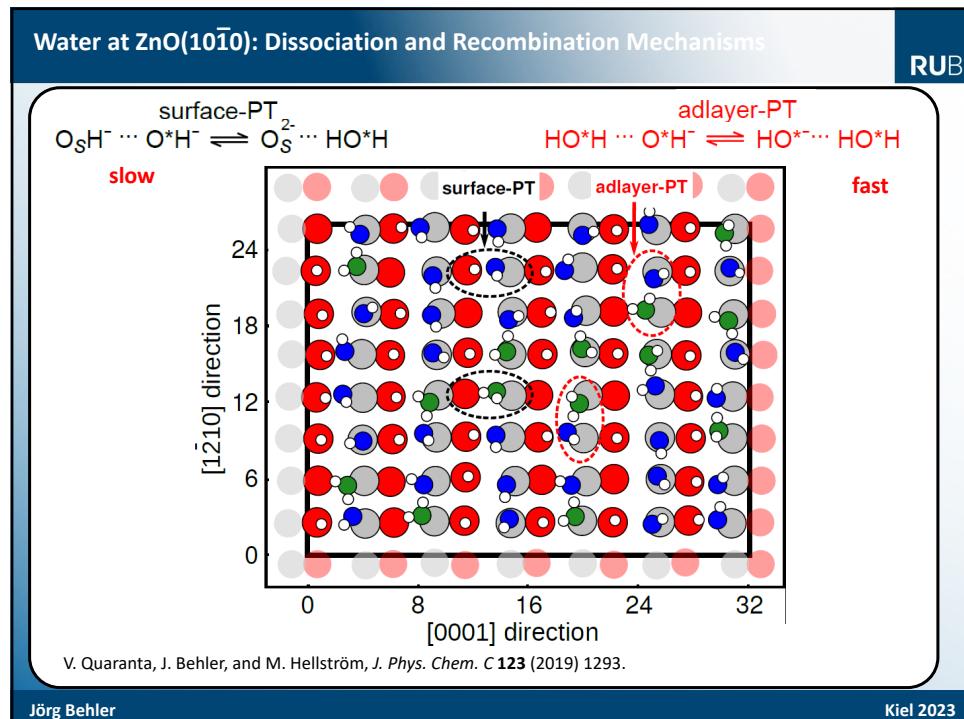
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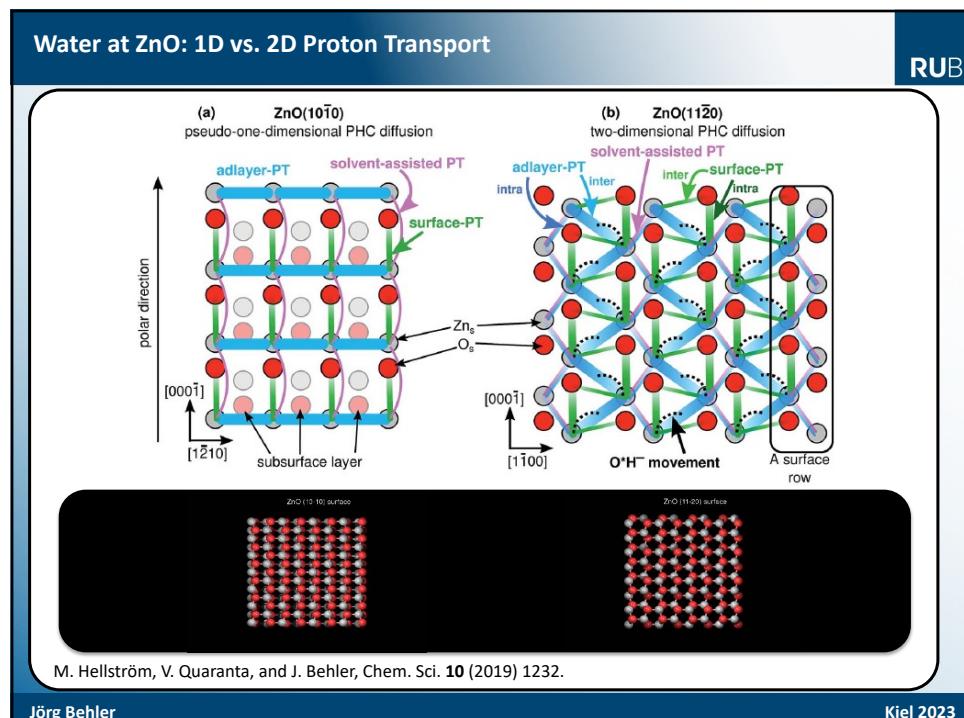
67



68



69



70

Summary: Four Generations of Neural Network Potentials

RUB

First-Generation Neural Network Potentials

Global description ⇒ low-dimensional systems

Second-Generation Neural Network Potentials

Short-range: **yes** Long-range: **no** Non-local: **no** Spin: **yes**

J. Behler, M. Parrinello, *Phys. Rev. Lett.* **98** (2007) 146401.

Third-Generation Neural Network Potentials

Short-range: **yes** Long-range: **yes** Non-local: **no** Spin: **yes**

N. Artrith, T. Morawietz and J. Behler, *Phys. Rev. B* **83** (2011) 153101.

Fourth-Generation Neural Network Potentials

Short-range: **yes** Long-range: **yes** Non-local: **yes** Spin: **yes**

T. W. Ko, J. A. Finkler, S. Goedecker, J. Behler, *Nature Commun.* **12** (2021) 398.

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71

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T. W. Ko, J. A. Finkler, S. Goedecker, J. Behler, *Nature Commun.* **12** (2021) 398.

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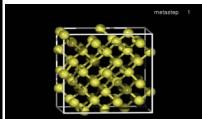
*Neural Network Potentials fill a gap
in the toolbox of chemistry, physics and materials science*

72

Summary: Applications of Neural Network Potentials

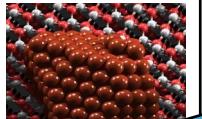
RUB

Bulk Materials



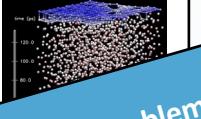
PRL **98** (2007) 146401.
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Nanostructures



PRB **95** (2017) 105102.

Nucleation



time [ns]
120.0
100.0
80.0

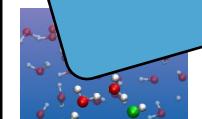
JCP **147** (2017) 100103.

Phase Transitions



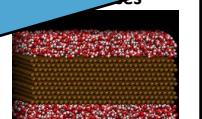
Science Materials **10** (2011) 693.

Polymers



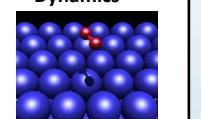
PNAS **113** (2016) 8368.

Proteins



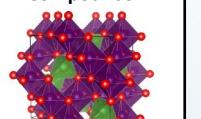
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JPCL **10** (2019) 1763.

Li-Intercalation Compounds



PRB **102** (2020) 174102.
JCP **153** (2020) 164107.

Neural Networks Help Solving Problems

and many more ...

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Kiel 2023