

Electronic excitations in correlated finite 2D materials generated by ion impact

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detailed version of talk at:
www.theo-physik.uni-kiel.de/bonitz/talks.html

Fermionic atoms in optical lattices tunable lattice depth and interaction

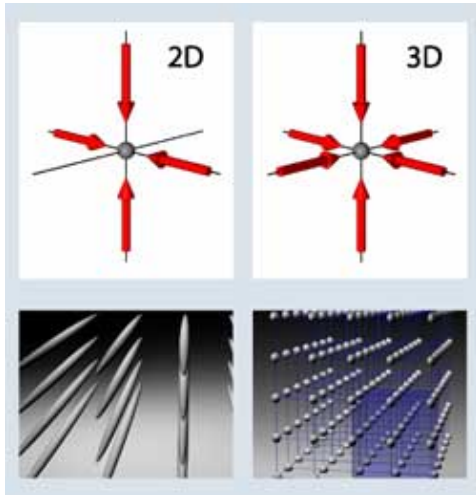
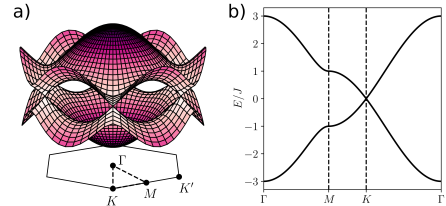
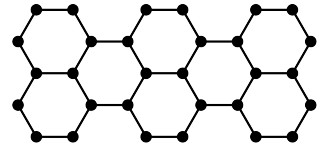


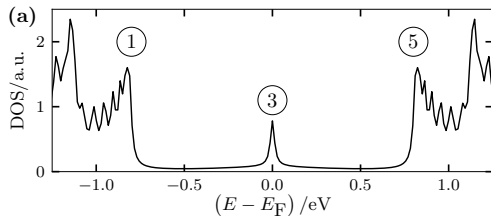
Fig.: M. Greiner (Harvard)

Graphene: high mobility, no bandgap

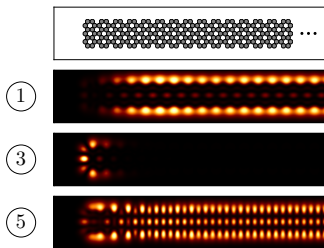


Graphene nanoribbons: finite tunable bandgap





(b)



7 armchair GNR of 504 atoms, GW-NEGF ground state simulation,

J.-P. Joost, A.-P. Jauho, and M. Bonitz, Nano Letters (2019)

- top: total density of states (DOS)
- DOS size and shape dependent
- importance of e-e interactions
- topological states, strong Coulomb correlations

Consider nonequilibrium electron dynamics following ion impact

⇒ very strong localized excitation

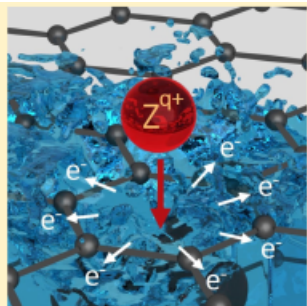
⇒ energy gain, charge transfer

⇒ creation of doublons^a

⇒ ions sensitive non-destructive local diagnostic of electronic properties

^aK. Balzer et al., PRL (2018)

ABSTRACT: Low-energy electrons (LEEs) are of great relevance for ion-induced radiation damage in cells and genes. We show that charge exchange of ions leads to LEE emission upon impact on condensed matter. By using a graphene monolayer as a simple model system for condensed organic matter and utilizing slow highly charged ions (HCIs) as projectiles, we highlight the importance of charge exchange alone for LEE emission. We find a large number of ejected electrons resulting from individual ion impacts (up to 80 electrons/ion for Xe^{40+}). More than 90% of emitted electrons have energies well below 15 eV. This “splash” of low-energy electrons is interpreted as the consequence of ion deexcitation via an interatomic Coulombic decay (ICD) process.



- ultrafast electronic processes (1...5 fs)
 - strong excitation: nonlinear and non-adiabatic response
 - rapid intralayer electron transport
 - emission of low-energy electrons, strong dependence on target properties
- ⇒ **Needed: nonequilibrium quantum many-body theory for inhomogeneous systems**

¹Schweska *et al.*, J. Phys. Chem. Lett. **10**, 4805 (2019)

2nd quantization

- Fock space $\mathcal{F} \ni |n_1, n_2 \dots\rangle$, $\mathcal{F} = \bigoplus_{N_0 \in \mathbb{N}} \mathcal{F}^{N_0}$, $\mathcal{F}^{N_0} \subset \mathcal{H}^{N_0}$
- $\hat{c}_i, \hat{c}_i^\dagger$ creates/annihilates a particle in single-particle orbital ϕ_i
- spin accounted for by canonical (anti-)commutator relations

$$\left[\hat{c}_i^{(\dagger)}, \hat{c}_j^{(\dagger)} \right]_{\mp} = 0, \quad \left[\hat{c}_i, \hat{c}_j^\dagger \right]_{\mp} = \delta_{i,j}$$
- Hamiltonian:
$$\hat{H}(t) = \underbrace{\sum_{k,m} h_{km}^0 \hat{c}_k^\dagger \hat{c}_m}_{\hat{H}_0} + \frac{1}{2} \underbrace{\sum_{k,l,m,n} w_{klmn} \hat{c}_k^\dagger \hat{c}_l^\dagger \hat{c}_n \hat{c}_m}_{\hat{W}} + \hat{F}(t)$$

Particle interaction w_{klmn}

- Coulomb interaction
- electronic correlations

Time-dependent excitation $\hat{F}(t)$

- single-particle type
- em field, quench, particles

Nonequilibrium Green Functions (NEGF)

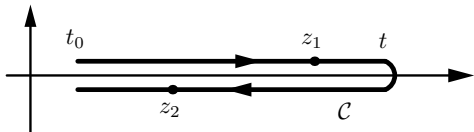
two times $z, z' \in \mathcal{C}$ ("Keldysh contour"), arbitrary one-particle basis $|\phi_i\rangle$

$$G_{ij}(z, z') = \frac{i}{\hbar} \langle \hat{T}_{\mathcal{C}} \hat{c}_i(z) \hat{c}_j^\dagger(z') \rangle \quad \text{average with } \hat{\rho}_N$$

pure or mixed state

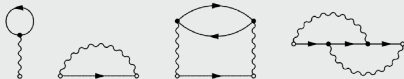
Keldysh–Kadanoff–Baym equations (KBE) on \mathcal{C} (2×2 matrix):

$$\sum_k \left\{ i\hbar \frac{\partial}{\partial z} \delta_{ik} - h_{ik}(z) \right\} G_{kj}(z, z') = \delta_{\mathcal{C}}(z, z') \delta_{ij} - i\hbar \sum_{klm} \int_{\mathcal{C}} d\bar{z} w_{iklm}(z^+, \bar{z}) G_{lmjk}^{(2)}(z, \bar{z}; z', \bar{z}^+)$$



KBE: first equation of Martin–Schwinger hierarchy for $G, G^{(2)} \dots G^{(n)}$

- $\int_{\mathcal{C}} w G^{(2)} \rightarrow \int_{\mathcal{C}} \Sigma G$, Selfenergy
- Nonequilibrium Diagram technique
 Example: Hartree–Fock + Second Born selfenergy



- Correlation functions G^{\lessgtr} obey real-time KBE

$$\sum_l \left[i\hbar \frac{d}{dt} \delta_{i,l} - h_{il}^{\text{eff}}(t) \right] G_{lj}^>(t, t') = I_{ij}^{(1),>}(t, t'),$$

$$\sum_l G_{il}^<(t, t') \left[-i\hbar \frac{d}{dt'} \delta_{l,j} - h_{lj}^{\text{eff}}(t') \right] = I_{ij}^{(2),<}(t, t'),$$

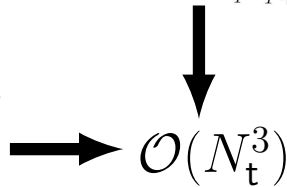
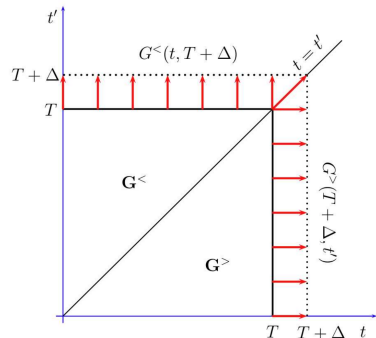
with the effective single-particle **Hartree–Fock Hamiltonian**

$$h_{ij}^{\text{eff}}(t) = h_{ij}^0 \pm i\hbar \sum_{kl} w_{ikjl}^{\pm} G_{lk}^<(t)$$

and the collision integrals

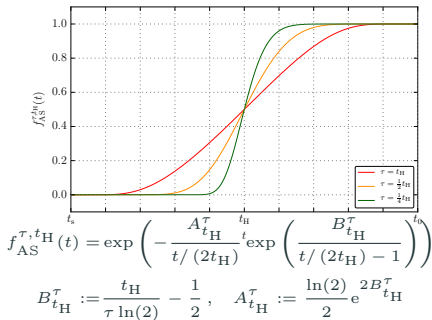
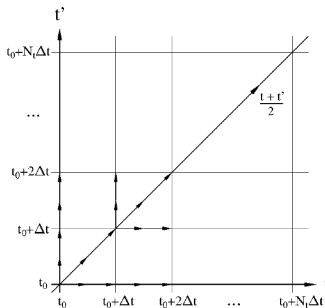
$$I_{ij}^{(1),>}(t, t') := \sum_l \int_{t_s}^{\infty} d\bar{t} \left\{ \Sigma_{il}^R(t, \bar{t}) G_{lj}^>(\bar{t}, t') + \Sigma_{il}^>(t, \bar{t}) G_{lj}^A(\bar{t}, t') \right\},$$

$$I_{ij}^{(2),<}(t, t') := \sum_l \int_{t_s}^{\infty} d\bar{t} \left\{ G_{il}^R(t, \bar{t}) \Sigma_{lj}^<(\bar{t}, t') + G_{il}^<(t, \bar{t}) \Sigma_{lj}^A(\bar{t}, t') \right\}.$$



- numerically demanding due to N_t^3 scaling (most competing methods time linear)

Full two-time solutions: Danielewicz, Schäfer, Köhler/Kwong, Bonitz/Semkat, Haug, Jahnke, van Leeuwen, Stefanucci, Verdozzi, Berges, Garny ...



1. adiabatically slow switch-on of interaction for $t, t' \leq t_0$ [1-3]
2. solve KBE in $t - t'$ plane for $G^{\geq}(t, t')$ with same code

[1] A. Rios et al., Ann. Phys. **326**, 1274 (2011), [2] S. Hermanns et al., Phys. Scr. **T151**, 014036 (2012)

[3] M. Watanabe and W. P. Reinhardt, Phys. Rev. Lett. **65**, 3301 (1990)

Nonequilibrium selfenergy approximations²

Choice depends on coupling strength, density (filling)

Hartree-Fock (HF, mean field): $\sim w^1$

Second Born (2B): $\sim w^2$

GW: ∞ bubble summation,
 dynamical screening effects

particle-particle T-matrix (TPP):

∞ ladder sum in pp channel

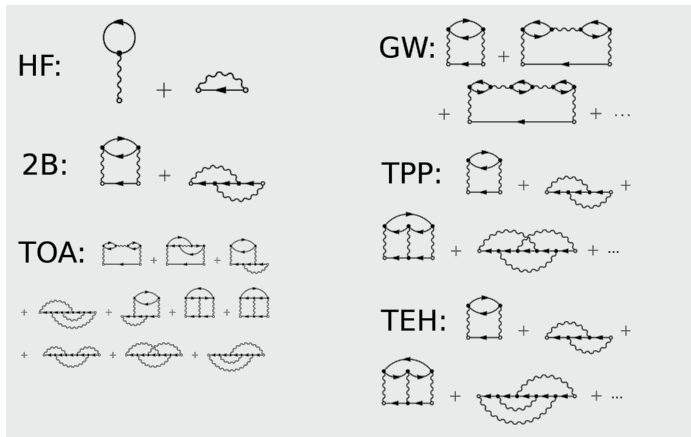
particle-hole T-matrix (TPH/TEH):

∞ ladder sum in ph channel

3rd order approx. (TOA): $\sim w^3$

dynamically screened ladder (DSL):

includes GW and TPP + TPH diagrams



²Conserving approximations, two-time functions $\Sigma(t, t')$, applies for ultra-short to long times

Review: Schlünzen *et al.*, J. Phys. Cond. Matt. **32**, 103001 (2020)

DSL – details and benchmarks: Joost *et al.*, submitted for publication, arXiv:2202.10061

- **full propagation** on the time diagonal ($I := I^{(1),<}$):

$$i\hbar \frac{d}{dt} G_{ij}^{<}(t) = [h^{\text{HF}}, G^{<}]_{ij}(t) + [I + I^\dagger]_{ij}(t)$$

- **reconstruct off-diagonal NEGF** from time diagonal:

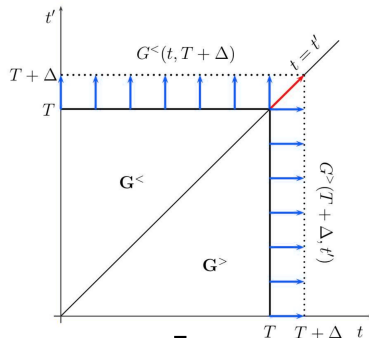
$$G_{ij}^{\geq}(t, t') = \pm \left[G_{ik}^{\text{R}}(t, t') \rho_{kj}^{\geq}(t') - \rho_{ik}^{\geq}(t) G_{kj}^{\text{A}}(t, t') \right]$$

$$\text{with } \rho_{ij}^{\geq}(t) = \pm i\hbar G_{ij}^{\geq}(t, t)$$

- HF-GKBA: use Hartree–Fock propagators for $G_{ij}^{\text{R/A}}$

$$G_{ij}^{\text{R/A}}(t, t') = \mp i\Theta_{\text{C}}(\pm[t - t']) \exp\left(-\frac{i}{\hbar} \int_{t'}^t d\bar{t} h_{\text{HF}}(\bar{t})\right) \Big|_{ij}$$

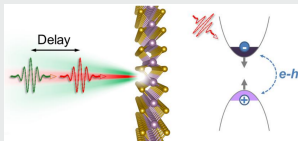
- conserves total energy



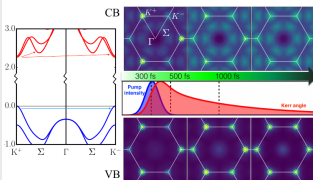
$\mathcal{O}(N_t^2)$

⁶ P. Lipavský, V. Špička, and B. Velický, Phys. Rev. B **34**, 6933 (1986);
K. Balzer and M. Bonitz, Lecture Notes in Physics **867** (2013)

2D Layered Materials

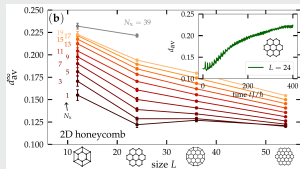


E. A. Pogna *et al.*,
 ACS Nano **10**, 1182 (2016)



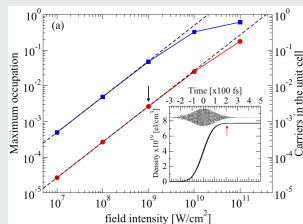
A. Molina-Sánchez *et al.*,
 Nano Lett. **17**, 4549 (2017)

Ion Stopping in Hexagonal Lattices



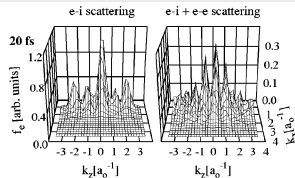
K. Balzer *et al.*,
 PRL **121**, 267602 (2018)

Semiconductors



D. Sangalli *et al.*,
 PRB **93**, 195205 (2016)

Laser-Induced Heating of Dense Plasmas



H. Haberland *et al.*,
 PRE **64**, 026405 (2001)
 gauge-invariant
 multi-photon absorption
 inv bremsstrahlung heating

Further results: Hermanns *et al.*, PRB 2014
 Bonitz *et al.*, phys. stat. sol. (b) 2019
 atoms, molecules: Stefanucci *et al.*

- **full propagation** on the time diagonal as for ordinary HF-GKBA:

$$i\hbar \frac{d}{dt} G_{ij}^<(t) = [h^{\text{HF}}, G^<]_{ij}(t) + [I + I^\dagger]_{ij}(t)$$

- but collision integral defined by correlated two-particle Green function

$$I_{ij}(t) = \pm i\hbar \sum_{klp} w_{iklp}(t) \mathcal{G}_{lpjk}(t)$$

- which obeys an ordinary differential equation⁴

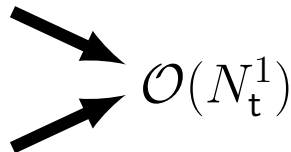
$$i\hbar \frac{d}{dt} \mathcal{G}_{ijkl}(t) = [h^{(2),\text{HF}}, \mathcal{G}]_{ijkl}(t) + \Psi_{ijkl}^\pm(t)$$

- the initial values

$$G_{ij}^{0,<} = \pm \frac{1}{i\hbar} n_{ij}(t_0) =: \pm \frac{1}{i\hbar} n_{ij}^0,$$

$$\mathcal{G}_{ijkl}^0 = \frac{1}{(i\hbar)^2} \{ n_{ijkl}^0 - n_{ik}^0 n_{jl}^0 \mp n_{il}^0 n_{jk}^0 \},$$

determine the density and the pair correlations existing in the system at the initial time $t = t_0$



⁴two-particle commutator: $[A, B]_{ijkl}(t) = \sum_{pq} [A_{ijpq}(t)B_{pqkl}(t) - B_{ijpq}(t)A_{pqkl}(t)]$

- other selfenergy approximations can be reformulated in the G1–G2 scheme in similar fashion:⁵

$$i\hbar \frac{d}{dt} \mathcal{G}_{ijkl}(t) = \left[h^{(2),\text{HF}}(t), \mathcal{G}(t) \right]_{ijkl} + \Psi_{ijkl}^{\pm}(t) + \underbrace{L_{ijkl}(t)}_{\text{TPP}} + \underbrace{P_{ijkl}(t)}_{\text{GW}} \pm \underbrace{P_{jikl}(t)}_{\text{TPH}}$$

with (times dropped)

$$L_{ijkl} := \sum_{pq} \left\{ \mathfrak{h}_{ijpq}^L \mathcal{G}_{pqkl} - \mathcal{G}_{ijpq} \left[\mathfrak{h}_{klpq}^L \right]^* \right\}, \quad \mathfrak{h}_{ijkl}^L := (i\hbar)^2 \sum_{pq} \left[\mathcal{G}_{ijpq}^{\text{H},>} - \mathcal{G}_{ijpq}^{\text{H},<} \right] w_{pqkl},$$

$$P_{ijkl} := \sum_{pq} \left\{ \mathfrak{h}_{qjpl}^{\Pi} \mathcal{G}_{piqk} - \mathcal{G}_{qjpl} \left[\mathfrak{h}_{qkpi}^{\Pi} \right]^* \right\}, \quad \mathfrak{h}_{ijkl}^{\Pi} := \pm (i\hbar)^2 \sum_{pq} w_{qipk}^{\pm} \left[\mathcal{G}_{jplq}^{\text{F},>} - \mathcal{G}_{jplq}^{\text{F},<} \right]$$

and the Hartree/Fock (H/F) two-particle Green functions

$$\mathcal{G}_{ijkl}^{\text{H},\gtrless}(t) := G_{ik}^{\gtrless}(t, t) G_{jl}^{\gtrless}(t, t), \quad \mathcal{G}_{ijkl}^{\text{F},\gtrless}(t) := G_{il}^{\gtrless}(t, t) G_{jk}^{\lesseqgtr}(t, t)$$

- including all terms results in the dynamically-screened-ladder (DSL) approximation

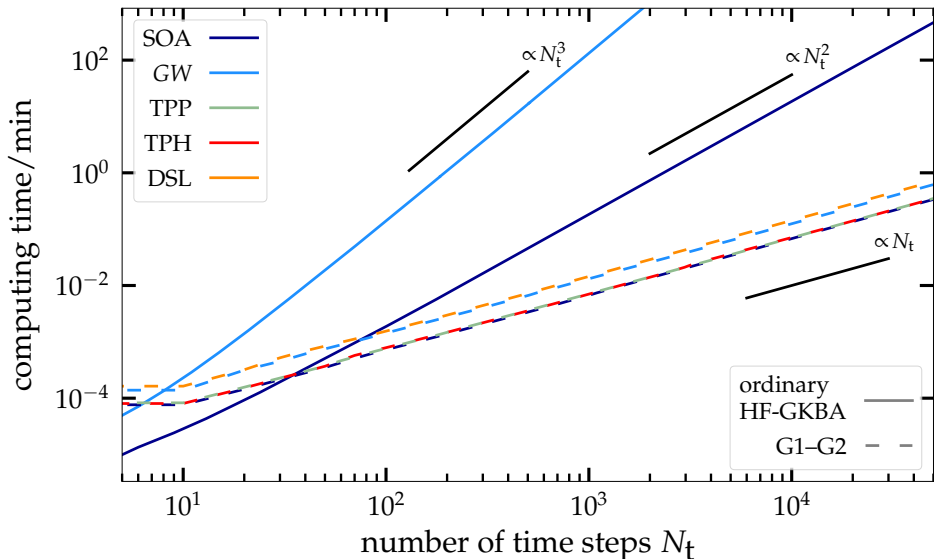
⁵ J.-P. Joost, N. Schlünzen, and M. Bonitz, Phys. Rev. B **101**, 245101 (2020)

- linear time scaling outweighs introduction of 4-dimensional two-particle Green function
 → new scheme an improvement in most cases of practical relevance

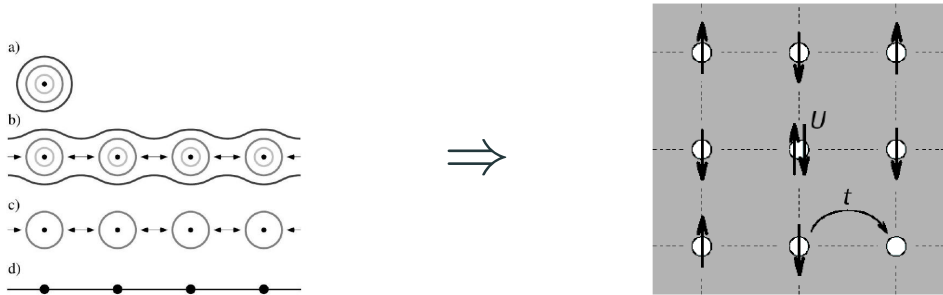
Σ

Basis	HF-GKBA	2B	GW	TPP	TPH	DSL
general	standard	$\mathcal{O}(N_b^5 N_t^2)$	$\mathcal{O}(N_b^6 N_t^3)$	$\mathcal{O}(N_b^6 N_t^3)$	$\mathcal{O}(N_b^6 N_t^3)$	–
	G1–G2	$\mathcal{O}(N_b^5 N_t^1)$	$\mathcal{O}(N_b^6 N_t^1)$	$\mathcal{O}(N_b^6 N_t^1)$	$\mathcal{O}(N_b^6 N_t^1)$	$\mathcal{O}(N_b^6 N_t^1)$
	speedup ratio	$\mathcal{O}(N_t)$	$\mathcal{O}(N_t^2)$	$\mathcal{O}(N_t^2)$	$\mathcal{O}(N_t^2)$	–
Hubbard	standard	$\mathcal{O}(N_b^3 N_t^2)$	$\mathcal{O}(N_b^3 N_t^3)$	$\mathcal{O}(N_b^3 N_t^3)$	$\mathcal{O}(N_b^3 N_t^3)$	–
	G1–G2	$\mathcal{O}(N_b^4 N_t^1)$	$\mathcal{O}(N_b^4 N_t^1)$	$\mathcal{O}(N_b^4 N_t^1)$	$\mathcal{O}(N_b^4 N_t^1)$	$\mathcal{O}(N_b^4 N_t^1)$
	speedup ratio	$\mathcal{O}(N_t/N_b)$	$\mathcal{O}(N_t^2/N_b)$	$\mathcal{O}(N_t^2/N_b)$	$\mathcal{O}(N_t^2/N_b)$	–
HEG	standard	$\mathcal{O}(N_b^3 N_t^2)$	$\mathcal{O}(N_b^3 N_t^3)$	$\mathcal{O}(N_b^3 N_t^3)$	$\mathcal{O}(N_b^3 N_t^3)$	–
	G1–G2	$\mathcal{O}(N_b^3 N_t^1)$	$\mathcal{O}(N_b^3 N_t^1)$	$\mathcal{O}(N_b^4 N_t^1)$	$\mathcal{O}(N_b^4 N_t^1)$	$\mathcal{O}(N_b^4 N_t^1)$
	speedup ratio	$\mathcal{O}(N_t)$	$\mathcal{O}(N_t^2)$	$\mathcal{O}(N_t^2/N_b)$	$\mathcal{O}(N_t^2/N_b)$	–

- time-linear scaling illustrated for the 10-site Hubbard chain



- Single band, small bandwidth, input from electronic structure calculations
- straightforward extension: multiple bands, long range interaction etc.

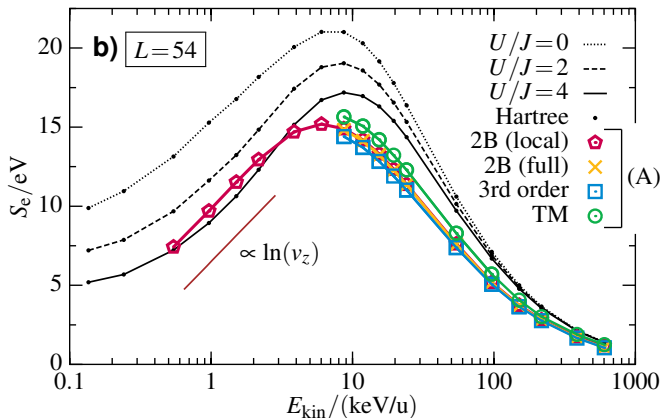
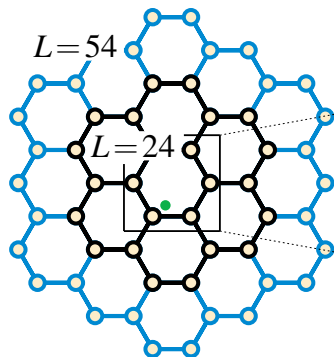


$$\hat{H}(t) = J \sum_{ij, \alpha} h_{ij} \hat{c}_{i\alpha}^\dagger \hat{c}_{j\alpha} + U \sum_i \hat{c}_{i\uparrow}^\dagger \hat{c}_{i\uparrow} \hat{c}_{i\downarrow}^\dagger \hat{c}_{i\downarrow} + \sum_{ij, \alpha\beta} f_{ij, \alpha\beta}(t) \hat{c}_{i\alpha}^\dagger \hat{c}_{j\beta}$$

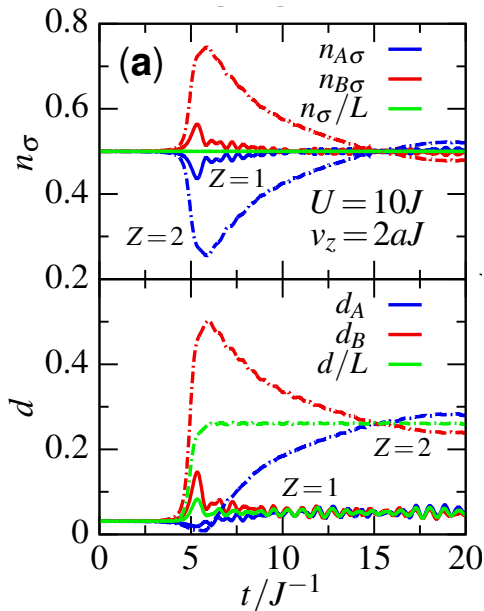
$h_{ij} = -\delta_{\langle i, j \rangle}$ and $\delta_{\langle i, j \rangle} = 1$, if (i, j) is nearest neighbor, $\delta_{\langle i, j \rangle} = 0$ otherwise
 use $J = 1$, on-site repulsion ($U > 0$) or attraction ($U < 0$)

f : external time-dependent excitation

Example ion impact: $f(t) \sim \sum_i -\frac{Z(t)e_0^2}{|\mathbf{R}(t) - \mathbf{r}_i|}$, NEGF-Ehrenfest approach



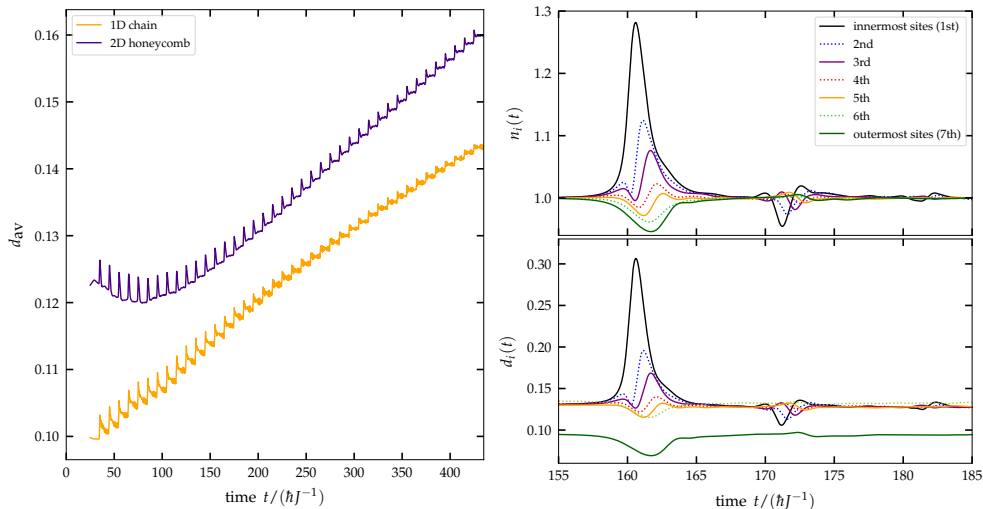
Left: finite honeycomb cluster and impact point of ion (green). Right: Ion energy loss S_e vs. impact energy. Black lines: Hartree approximation for e-e interaction. Colors: different approximations for the correlation selfenergy. From: Balzer *et al.* PRB (2016)



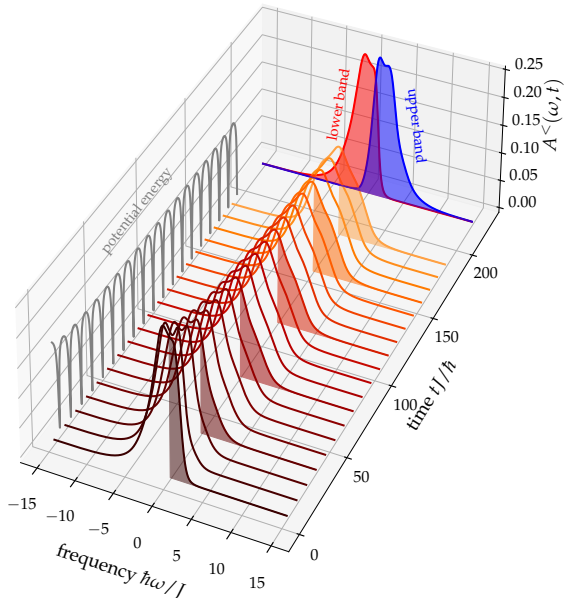
Top: electron density dynamics during ion impact ($t = 6$). Full (dash-dotted) lines: proton (α -particle). n_A : honeycomb ring closest to impact point. n_B second nearest ring. Green lines: mean density

Bottom: d_A (d_B): doublon number at nearest (second nearest) honeycomb ring.
 \Rightarrow increased doublon number remains in cluster after impact. Spreads uniformly across cluster.

From: Balzer *et al.* PRL (2018)



Left: dynamics of total doublon number upon up to 40 ion impacts for a 1D chain and 2D cluster with 96 sites. Right: site resolved density dynamics. From: Borkowski *et al.* *phys. stats. sol. (b)* (2022), <https://doi.org/10.1002/pssb.202100511>. Follow up of Balzer *et al.*, *PRL* **121**, 267602 (2018)



Time evolution of the photoemission spectrum (spectral function $A^<(\omega, t)$) during 20 ion impacts, honeycomb, $L = 12$ with $U/J = 4$.

Occupation of upper Hubbard band (positive energies) increases with time. Red (blue): (un)occupied states of un-excited system.

$$t_0 = \hbar/J \approx 0.17\text{fs}, \quad a_0 = 1.42, \\ v_0 = a_0/t_0 \approx 0.8 \cdot 10^6 \text{m/s}$$

from Bonitz *et al.* *phys. stats. sol.* (b) **257**, 1800490 (2019)

Summary

- Results for ion stopping in 2D graphene or TMDC monolayers
- potentially highly sensitive diagnostic of electronic correlation effects
- Nonequilibrium Green functions–Ehrenfest approach
- efficient time propagation using the G1–G2 scheme⁶
- generation and propagation of doublons via multiple ion impacts⁷
- ion neutralization treated via embedding selfenergy approach, see appendix⁸

Outlook

- Extension to bilayers and TMDCs
- Quantum treatment of electron dynamics in ion
- Impact of highly charged ions gives rise to electron emission (Auger, ICD).
Electron number highly sensitive to electron mobility⁹

detailed version of talk at: www.theo-physik.uni-kiel.de/bonitz/talks.html

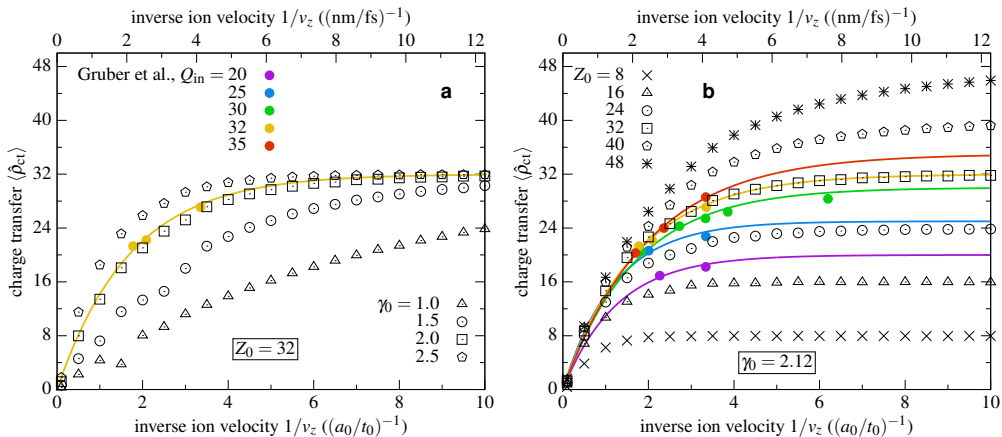
⁶Schlünzen *et al.*, Phys. Rev. Lett. 2020

⁷Balzer *et al.*, Phys. Rev. Lett. 2018; Borkowski *et al.*, phys. stat. sol. (b) (2022)

⁸Balzer and Bonitz, Contrib. Plasma Phys. **62** (2) e202100041 (2022)

⁹A. Niggas *et al.*, submitted for publication

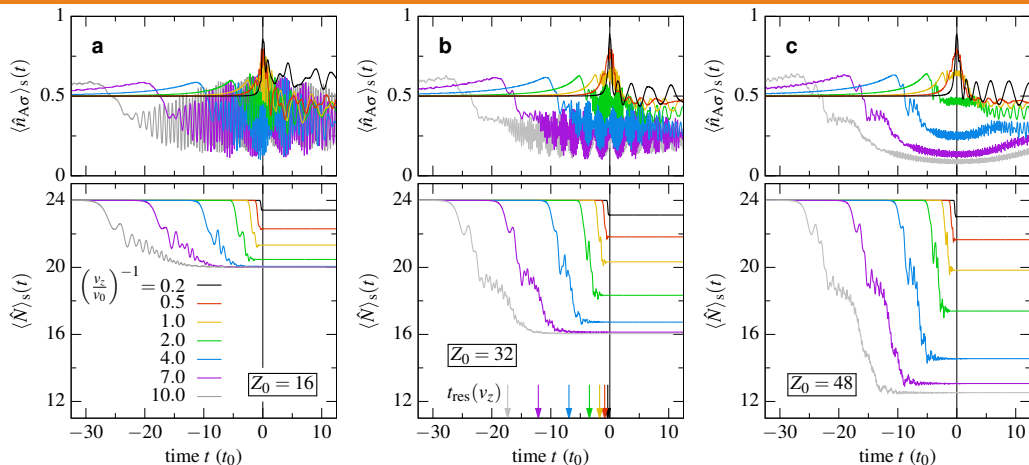
Appendix: NEGF-embedding scheme for charge transfer from graphene nanoflake to impacting high Z ion



NEGF-embedding selfenergy scheme for resonant charge transfer, $L = 24$. Exp. data from Gruber et al. Nat. Commun. 2016, **7**(1), 13948. Single adjustable parameter γ_0 works for all charges.

Figure from Balzer and Bonitz, Contrib. Plasma Phys. **62** (2) e202100041 (2022)

Charge transfer from graphene nanoflake to impacting high Z ion



Dynamics of total particle number $\langle \hat{N} \rangle_s$ (in one band) and occupation $\langle \hat{n}_{0\sigma} \rangle_s$ of innermost honeycomb (graphene, $L = 24$ sites) during ion impact. Colors: NEGF results for different ion velocities. Arrows: moment when ion passes through the resonance point $z_{\text{res}} = \sqrt{3}a_0$, in front of the plane.

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