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Heat Transport in Confined 2D Dust Clusters

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TR 24 complex plasmas

Abstract

Dusty plasmas are an excellent laboratory for studying many body physics [1]. Selective control over the kinetic temperature of the dust particle is required in order to investigate phenomena such as melting and heat transport. This control is achieved, e.g. by acceleration of single dust grains by the radiation pressure of moving laser spots [2].

- Simulation: Langevin Molecular Dynamics [3]
- Confine power input to the central region $\mapsto$ investigate transport properties through dust cluster [4]
- Describe radial temperature profile to analytical model: modified Bessel functions
- Determine thermal conductivity $\varkappa$ by comparison of simulation and model
- thermal conduction appears unaffected by cluster's transition from a solid-like to a liquid-like state

System of interest

The dimensionless Hamiltonian of N parabolically confined particles that interact via a Yukawa potential

$$\mathcal{H}=\sum_{i=1}^{N}\frac{\boldsymbol{p}_i^2}{2}+\sum_{i=1}^{N}\frac{r_i^2}{2}+\sum_{j<i}\frac{1}{r_{ij}}\cdot e^{-\kappa\cdot r_{ij}} \qquad (1)$$

with $r_i=|\boldsymbol{r}_i|$ and $r_{ij}=|\boldsymbol{r}_i-\boldsymbol{r}_j|$.

- Distances are in units of $l_0=\left(\frac{Q^2}{4\pi\epsilon_0 m\omega^2}\right)^{1/3}$
- The energy is given in units of $E_0=\left(\frac{m\omega^2Q^4}{16\pi^2\epsilon_0^2}\right)^{1/3}$
- The time is in units of the inverse trap frequency $t_0=\omega^{-1}$
- The screening constant κ is given by the inverse Debye length $\kappa=\lambda_D^{-1}$ in units of l_0
- The coupling parameter $\Gamma=\frac{E_{\mathrm{inter}}}{E_{\mathrm{therm}}}$ relates the typical interaction energy with the thermal energy

Simulation method

The dust component is treated exactly while neutral gas, ions and electron are treated statistically. $\rightarrow$ Langevin equation on motion

$$\dot{\boldsymbol{p}}_i=-\nabla_{\boldsymbol{r}_i}\mathcal{H}-\gamma\boldsymbol{v}_i+\boldsymbol{\eta}(t)+\sum_{l=1}^{N_{\mathrm{laser}}}\boldsymbol{f}_l(\boldsymbol{r}_i,t) \qquad (2)$$

- collisions with neutral gas background are modeled by:
  friction therm $\gamma\boldsymbol{v}$
  stochastic force $\boldsymbol{\eta}(t)$, with $\langle\eta_i(t)\eta_j(t')\rangle=2\gamma T_{\mathrm{eq}}\delta_{ij}\delta(t-t')$
- friction γ and equilibrium temperature T_{eq} are input parameters to the simulation
- l laser spots are included as time depended forces

$$\boldsymbol{f}_l(\boldsymbol{r},t)=\frac{F_0}{2\pi\sigma_x\sigma_y}\cdot\boldsymbol{e}_l\cdot\exp\left[-\frac{(x-x_l(t))^2}{2\sigma_x^2}-\frac{(y-y_l(t))^2}{2\sigma_y^2}\right]$$

F_0 : force amplitude $\quad$ $\boldsymbol{e}_l$: beam direction
σ_x,σ_y : anisotropic spot profile (Gaussian)
$\boldsymbol{r}_l(t)=(x_l(t),y_l(t))$: spot trajectory

Four laser beams are used, each one in $\pm x$ and $\pm y$ direction. In order to investigate heat transport, the laser spots' motion is constraint to a square window at the trap center, see Fig. 1. The laser trajectories are chosen randomly:

- uniform motion of the spot
- when edge of the scanned window is reached
  $\rightarrow$ dice new velocity component in this direction with opposite sign

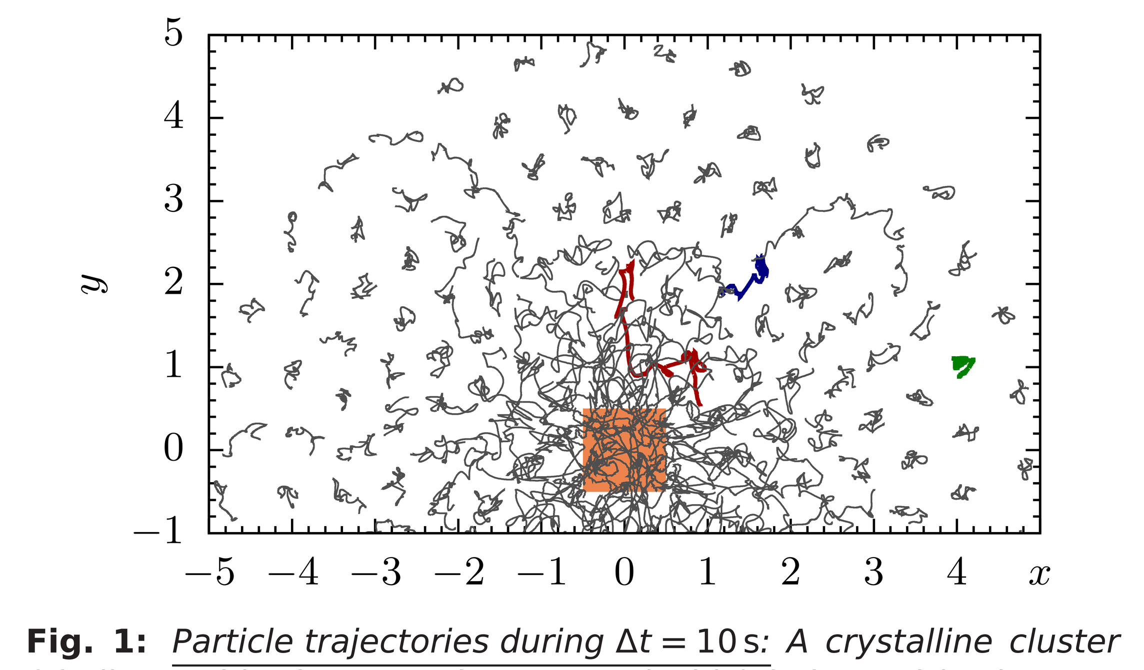

Fig. 1: Particle trajectories during $\Delta t=10\,$s: A crystalline cluster (shells outside, hexagonal structure inside) is heated in the center by lasers. The orange square in the center shows the laser heated area. Three arbitrary particles in the central, midway and outer region of the cluster are highlighted. (Simulation parameters: $N=200$ particles, screening $\kappa=1$, trap frequency $\omega=5.5\,\mathrm{s}^{-1}$, friction $\gamma=0.5\omega$, equilibrium coupling $\Gamma_{\mathrm{eq}}=200$, heating power $F_0=90$)

Simulation results

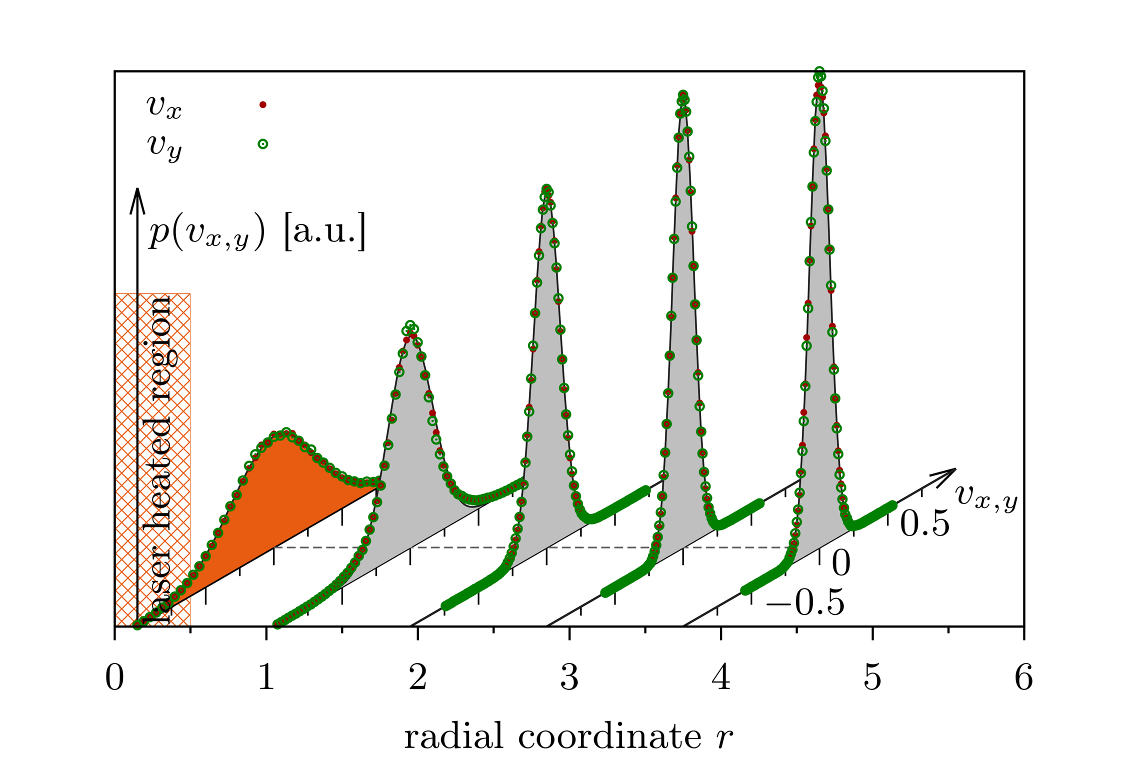

Fig. 2: Velocity distribution at different radii: Simulated Yukawa cluster with $N=200$ particles and a moderate heating power $F_0=40$. The velocity profiles are stationary during the laser heating and fit by Maxwellian distributions with decreasing width $\sigma_{v_{x,y}}(r)=\sqrt{T(r)}$. While the central velocity profile corresponds to $\Gamma=10$, the coupling of the outer particles $\Gamma=182$ is almost the equilibrium coupling $\Gamma_{\mathrm{eq}}=200$.

Temperature measurement in simulation.
Maxwellian shape of the velocity distribution at all radii $\rightarrow$ Dimensionless temperature $T=1/\Gamma$ is determined by the velocity fluctuation:

$$T=\frac{1}{2}\left(\left\langle\boldsymbol{v}^2\right\rangle-\langle\boldsymbol{v}\rangle^2\right) \qquad (3)$$

- Cluster is divided into concentric rings (bins) $\rightarrow$ radial temperature profile
- Without lasers: constant profile $T(r)=T_{\mathrm{eq}}$

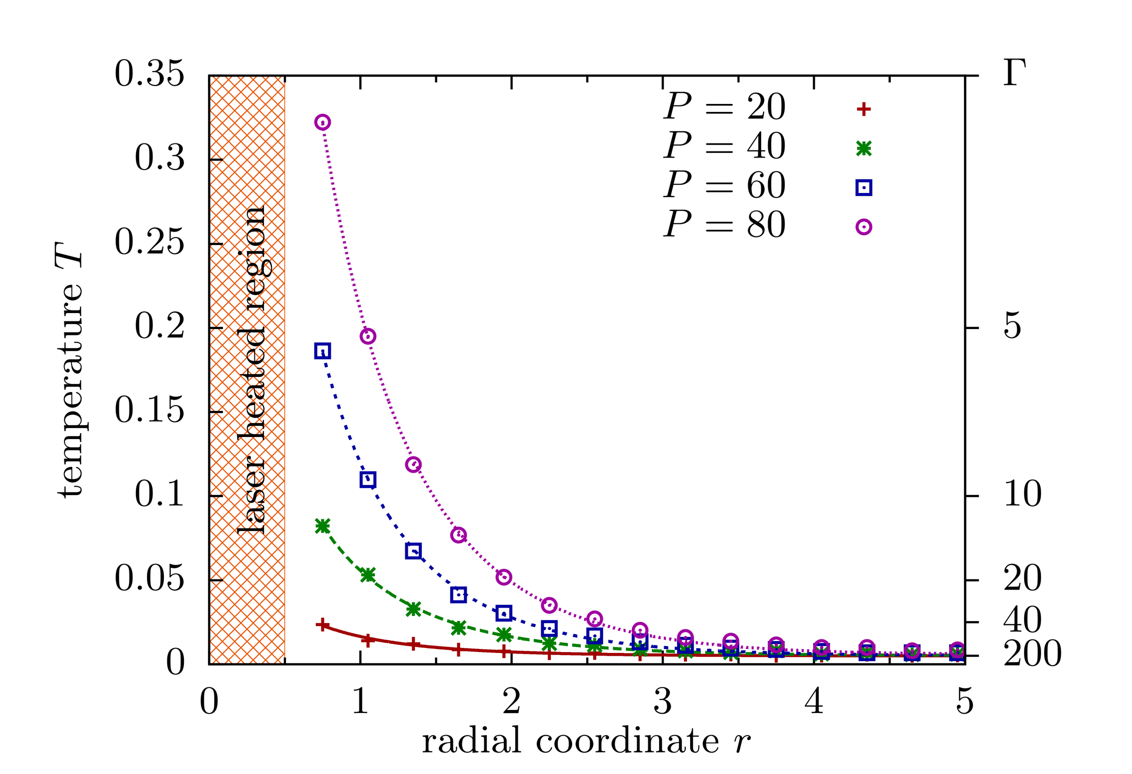

Fig. 3: Radial temperature profiles for different heating powers F_0: The equilibrium coupling is $\Gamma_{\mathrm{eq}}=200$ in all simulations. The profiles are fit by modified Bessel functions as solutions of an analytical model for the heat transport, Eq. (6).

Analytical model

Goal: determination of radial temperature profile for a strongly coupled dust cluster
Starting point: heat transport equation in fluid model [5]

$$cn\boldsymbol{v}\cdot\nabla Tk_{\mathrm{B}}=\mathrm{div}(\varkappa\nabla T)-2\gamma n(T-T_{\mathrm{eq}})k_{\mathrm{B}}+S_{\mathrm{viscous}} \qquad (4)$$

c : specific heat, n : particle number density, $\varkappa$: thermal conductivity, S_{viscous} viscous energy conversion from shear flow to heat

- laser effect: included via boundary condition
- no collective motion observed $\rightarrow$ $\boldsymbol{v}=0$
- no shear flow $\rightarrow$ source term $S_{\mathrm{viscous}}=0$
- radial symmetry $\rightarrow$ $T(\boldsymbol{r})=T(|\boldsymbol{r}|)$

Reduced radial heat transport equation:

$$\frac{1}{r}\frac{d}{dr}\left(r\varkappa\frac{dT}{dr}\right)=2\gamma nk_{\mathrm{B}}\left(T-T_{\mathrm{eq}}\right) \qquad (5)$$

Assumption: $\varkappa$ independent of r $\rightarrow$ General solution: modified Bessel functions of first (I_0) and second (K_0) kind

$$T(r)-T_{\mathrm{eq}}=AI_0\left(\sqrt{b}r\right)+BK_0\left(\sqrt{b}r\right) \qquad (6)$$

with $b=2\gamma nk_{\mathrm{B}}/\varkappa$

Boundary condition.
The cluster has a finite radius R $\rightarrow$ No heat is transported to outside R $\rightarrow$ Temperature gradient must become zero $\left.\frac{dT}{dr}\right|_{r=R}=0$

$$B=\frac{I_1\left(\sqrt{b}R\right)}{K_1\left(\sqrt{b}R\right)}A \qquad (7)$$

Solution with two free parameters A and $b\propto\varkappa^{-1}$ besides the equilibrium temperature T_{eq}.

- Amplitude A and heat conduction parameter b are determined by fitting the simulation results

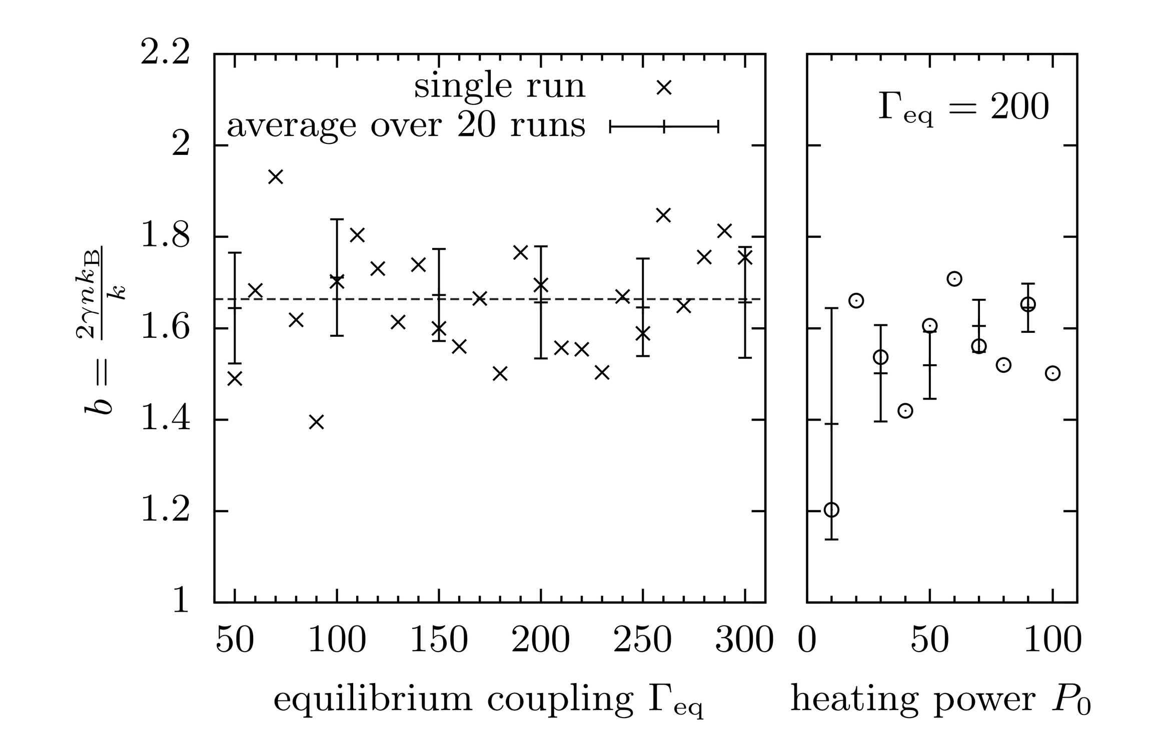

Fig. 4: Heat conduction parameter $b=2\gamma nk_{\mathrm{B}}/\varkappa$ for varied equilibrium coupling strength Γ_{eq} (left) and for varied heating power F_0 (right). The dashed line is the mean value $\bar{b}=1.66$. The error is estimated by running 20 simulations with the same parameters but different initial configurations and different random numbers.

Analysis of the results

- $b=\frac{2\gamma nk_{\mathrm{B}}}{\varkappa}=1.66$ is constant within the accuracy of temperature measurement in the simulation
- average density n has only weak temperature dependency $\rightarrow$ heat conductivity $\varkappa$ is constant
- $1/\sqrt{b}$ is the characteristic length for the heat transport
- $1/\sqrt{b}=0.78$ is close to the mean particle distance
- thermal diffusivity $\chi=\frac{\varkappa}{cn}=\frac{2\gamma k_{\mathrm{B}}}{b}\approx 4.4\,\mathrm{mm}^2\,\mathrm{s}^{-1}$ (for typical parameters: particle diameter $d_{\mathrm{p}}=6\,\mu\mathrm{m}$, $Q=10\,000\,e$, $\omega=5.5\,\mathrm{s}^{-1}$ and $\gamma=\omega/2$)
- typical range: $\chi=\mathcal{O}(1\ldots10\mathrm{mm}^2\,\mathrm{s}^{-1})$
- Results are in agreement finding $\chi=9\,\mathrm{mm}^2\,\mathrm{s}^{-1}$ with extended 2D experiments [5]

Importance of losses to the neutral gas.

- radial heat transport equation without losses:

$$\frac{d}{dr}\left(r\varkappa\frac{dT}{dr}\right)=0 \qquad (8)$$

- logarithmic temperature profile (assuming $\frac{d\varkappa}{dr}=0$)
- temperature decay much slower than in the simulation

Summary

- Heat transport in a 2D dust cluster is well described by fluid equation
- Heat conductivity is constant over the phase transition solid-like $\longleftrightarrow$ liquid-like
- Further question: connection between heat conductivity $\varkappa$ and friction γ

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