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BBGKY Approach to Non-Markovian Semiconductor Bloch Equations

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A density operator approach to generalized semiconductor Bloch equations (SBE) for one-time observables which includes carrier–carrier scattering effects is presented. The theoretical concept is based upon the quantum generalization of the BBGKY hierarchy for the reduced density operators. The advantage of our method is its simplicity and transparence – to a large extent, derivations and physical approximations can be performed on the simplest level possible, that of a one-component system of spinless particles. We discuss various approximations for the incoherent terms in the Bloch equations which are related to carrier–carrier correlations, such as strong coupling effects and the build-up of dynamical screening as well as self-energy effects. As a result we obtain non-Markovian generalizations of the SBE.

1. Introduction

The theoretical description of fs-laser pulse excitation of electron–hole plasmas in semiconductors and the subsequent relaxation requires a kinetic treatment which is given by the semiconductor (generalized) Bloch equations (SBE). They have been derived within the framework of second quantization (creation/annihilation operators $a^\dagger$, a) using density operators [1] and Green's functions formalisms, e.g. [2]. We have shown recently [3] that the derivation of the SBE, and in particular the analysis of incoherent effects in semiconductors may be simplified considerably and performed in a very intuitive way by using the concept of the BBGKY (Bogolyubov-Born-Green-Kirkwood-Yvon) hierarchy. This approach has been successfully applied to a large variety of fields by many authors, e.g. [4]. Our work aims at bridging the gap between "conventional" kinetic theory and semiconductor optics. Here, we can only outline the main ideas of the BBGKY approach and provide some important examples, for a detailed analysis we refer to [5].

2. Summary of the BBGKY Approach to the Semiconductor Bloch Equations

The SBE are coupled equations of motion for the band populations $f^\lambda(t)$ and interband polarizations $P^{\lambda\lambda'}(t)$. Due to the Coulomb interaction between the semiconductor electrons, these equations are coupled to the equations of motion of higher order correlations giving rise to a hierarchy of equations. While the concept of second quantization derives this hierarchy starting from the "bottom", i.e. from the equation for the expectation values of two-operator products $\langle a_\lambda^\dagger a_{\lambda'}\rangle$, the BBGKY approach starts from the "top", from the well-known equation of motion for the density operator of the whole N-electron system, i.e. the Liouville-von Neumann equation. By computing the partial trace of the von Neumann equation one obtains the hierarchy [4] for N *identical spinless particles.* This is the simplest situation possible, and an important feature of our approach is that all physical approximations regarding the treatment of correlations may be performed within this simple "electron gas" picture. The fermionic properties of the electrons and the band structure of the semiconductor are accounted for at later stages of the derivation. To simplify the analysis, we will consider here only Coulomb correlations. In summary, our approach consists of the following steps: 1. Analysis of the BBGKY hierarchy and choice of the (decoupling) approximation for the carrier–carrier correlations, see Section 3; 2. explicit inclusion of spin statistics effects by anti-symmetrization of the truncated hierarchy, Section 5; 3. formal solution for the binary correlation operator $g_{12}(t)$, Section 4; 4. expansion of the first equation and of the solution $g_{12}(t)$ in terms of Bloch states (Section 6). The result is a non-Markovian generalization of the SBE.

3. BBGKY Approach to Correlations

The BBGKY hierarchy provides a straightforward approach to correlation effects by considering the correlation operators $g_{12}, g_{123}, \dots$ which are related to the density operators by $F_{12} = F_1F_2 + g_{12}$, $F_{123} = F_1F_2F_3 + F_1g_{23} + F_2g_{13} + F_3g_{12} + g_{123}$ and so on. Then the first two hierarchy equations (for Boltzmann statistics) can be rewritten as

$$i\hbar \, \frac{\partial}{\partial t} \, F_1 - [\bar{H}_1^0, \, F_1] = n \, \mathrm{Tr}_2 \, [V_{12}, \, g_{12}] \, , \tag{1}$$

$$\begin{aligned} &i\hbar \, \frac{\partial}{\partial t} \, g_{12} - [\bar{H}_{12}^0, \, g_{12}] - [V_{12}, \, F_1F_2] = [V_{12}, \, g_{12}] \\ &+ n \, \mathrm{Tr}_3 \, \{[V_{13}, \, F_1g_{23}] + [V_{23}, \, F_2g_{13}] + [V_{13} + V_{23}, \, g_{123}]\} \, , \end{aligned} \tag{2}$$

where the quasiparticle Hamiltonians and the Hartree potential are defined as $\bar{H}_1^0 = H_1 + H_1^{\mathrm{H}} + H_1^{\mathrm{f}}$, $\bar{H}_{12}^0 = \bar{H}_1^0 + \bar{H}_2^0$ and $H_1^{\mathrm{H}} = n \, \mathrm{Tr}_2 \, V_{12}F_2$. The Hamiltonian $\bar{H}_1^0$ contains the particle–field interaction part H_1^{f}, which, in the simplest case, is given by the dipole approximation $H_1^{\mathrm{f}} = -\mathbf{d}_1\mathbf{E}(t)$, where E is the total electric field and d_1 the operator of the dipole moment. These equations are exact since they still contain the full coupling to three-electron correlations.

The physical effects related to the different terms of Eqs. (1), (2) are easily understood: The right-hand side of Eq. (1) describes the influence of correlations and gives rise to the collision integrals in the SBE. In Eq. (2), the first term on the right-hand side is the ladder term, describing strong coupling effects [13] including (incoherent) excitons, while the second and third terms on the right-hand side describe polarization phenomena and screening. Correspondingly, there exist four central approximations for the correlations[1]): (i) Dynamically screened ladder approximation: it accounts for all two-electron correlations, including ladder and polarization terms, i.e. all remaining terms of Eq. (2); (ii) statically screened ladder approximation: includes ladder terms but neglects the polarization terms; (iii) polarization approximation (dynamically screened Born approxi-

[1]) In the limit of low laser intensities (which is not considered here), the decoupling can be based on perturbation expansion in the electric field, cf. [6].

mation, RPA): neglects ladders but includes polarization effects; and (iv) statically screened Born approximation: neglects ladder and polarization terms.

The last term on the right-hand side of Eq. (2) is usually neglected. However, this leads to an unphysical behavior, such as infinite lifetime of two-particle states (having real energy eigenvalues E_{12}) and of initial correlations as well as to non-Markovian scattering integrals with unlimited memory depth. This problem is cured rather naturally in the Green's functions theory by the concept of *self-energy.* It was demonstrated recently that self-energy may be introduced into the BBGKY approach also by taking into account appropriate contributions from the three-particle correlations [7, 8]. For example, in the PRA case (iii), g_{123} has to be determined from the equation [10, 5]

$$\begin{aligned} &i\hbar\,\frac{\partial}{\partial t}\,g_{123}-\{\bar{H}^0_{123}g_{123}-g_{123}\bar{H}^{0\dagger}_{123}\} \\ &=[V_{13}+V_{23},\,F_3g_{12}]+n\,\mathrm{Tr}_4\,[V_{14},\,F_1g_{234}]+n\,\mathrm{Tr}_4\,[V_{24},\,F_2g_{134}]+n\,\mathrm{Tr}_4\,[V_{34},\,F_3g_{124}]\,, \end{aligned} \tag{3}$$

where $\bar{H}^0_{123}=\bar{H}_1+\bar{H}_2+\bar{H}_3$. Solving for g_{123} and inserting the result into Eq. (2), the only change is that the Hamiltonian becomes renormalized $\bar{H}^0_{12}\to\bar{H}^0_{12}+\Sigma^+_1+\Sigma^+_2$, i.e. $\bar{H}^0_1\to\bar{H}_1=\bar{H}^0_1+\Sigma^+_1$. Σ^+_1 is the correlation part of the retarded self-energy in RPA which is familiar from Green's functions. Notice that $\bar{H}_1$ is not hermitean giving rise to complex (damped) two-particle energies ε_{12}, thus fixing all the problems mentioned above. Using this scheme, the corresponding self-energy may be derived for any hierarchy closure [7, 9, 5] leading to generalized SBE with correlations and energy renormalization effects included.

4. Formal Solution of the Equation of Motion for $\mathbf{g_{12}}$

Instead of solving the (local in time) system (1), (2) it is often preferred to deal instead with one closed kinetic equation for F_1, i.e. the SBE. This may be achieved by formally solving Eq. (2). For example, in the *screened ladder approximation*, one obtains ($Q_{12}(t) = [V_{12},\,F_1(t)\,F_2(t)]$)

$$g_{12}(t)=\mathrm{Tr}_{34}\left\{\bar{U}_{13,\,24}(tt_0)\,g_{34}(t_0)+\frac{1}{i\hbar}\int\limits_{t_0}^{t}\mathrm{d}\bar{t}\,\bar{U}_{13,\,24}(t\bar{t})\,Q_{34}(\bar{t})\right\}. \tag{4}$$

The first term is related to initial correlations and the second one describes the build-up of correlations. This expression is nonlocal both in time (it depends on Q at previous times) and in the particle indices (depending on Q_{34}, which is due to polarization effects). The propagator obeys the following equation:

$$\begin{aligned} &\mathcal{L}_{12}\bar{U}_{13,\,24}(tt')-n\,\mathrm{Tr}_5\,\{[V_{15}F_1,\,\bar{U}_{53,\,24}(tt')]+[V_{25}F_2,\,\bar{U}_{13,\,54}(tt')]\}=0\,, \\ &\bar{U}_{13,\,24}(tt)=\delta_{13}\delta_{24}\,, \end{aligned} \tag{5}$$

where $\mathcal{L}_{12}A_{12}=i\hbar(\partial/\partial t)\,A_{12}-\{(\bar{H}^0_{12}+\Sigma^+_{12})\,A_{12}-A_{12}(\bar{H}^0_{12}+\Sigma^+_{12})^\dagger\}-[V_{12},\,A_{12}]$, and the self-energy is included in $\bar{H}^0_{12}$ as discussed above. Solution (4) simplifies considerably for approximations (ii) to (iv) of Section 3. For the strong coupling approximation (ii) $\bar{U}_{13,\,24}(t\bar{t})=\bar{U}_{12}(t\bar{t})\,\delta_{13}\delta_{24}$, which allows to derive the Lippmann-Schwinger equation for the T-operator and the non-Markovian Boltzmann equation [9]. For cases (iii) and (iv), the neglect of the ladder term $[V_{12},\,A_{12}]$ in $\mathcal{L}_{12}$ allows the factorization, $\bar{U}_{13,\,24}(t\bar{t}) = \bar{U}_{13}(t\bar{t})\bar{U}_{24}(t\bar{t})$. Case (iv) is trivial: $\bar{U}_{13,\,24}(t\bar{t})=\bar{U}_1(t\bar{t})\,\bar{U}_2(t\bar{t})\,\delta_{13}\delta_{24}$, [7]. Here, we illustrate the concept on the case of *dynamical screening*, (iii). Then the factorized propagators $\bar{U}_{13}(t\bar{t})$ and $\bar{U}_{24}(t\bar{t})$ satisfy the linear quantum Vlasov equation

$$i\hbar \frac{\partial}{\partial t} \bar{U}_{13}(t\bar{t}) - (H_1^{\mathrm{eff}} \bar{U}_{13}(t\bar{t}) - \bar{U}_{13}(t\bar{t}) H_1^{\mathrm{eff}\,\dagger}) - n \,\mathrm{Tr}_5 \,[V_{15},\, F_1(t)\, \bar{U}_{53}(t\bar{t})] = 0 \tag{6}$$

with $H_1^{\mathrm{eff}} = H_1 + H_1^{\mathrm{HF}} + \Sigma_1^+ + H_1^{\mathrm{f}}$ and the initial condition $\bar{U}_{ab}(tt) = \delta_{ab}$. Thus, the problem is reduced to an effective single-particle dynamics. The dielectric propagator $\bar{U}_{13}$ defines the correlation operator g_{12}, Eq. (4), which, inserted into Eq. (1), yields the collision integrals (including initial correlations) in the non-Markovian SBE

$$I_1(t) = \frac{n^3}{i\hbar} \,\mathrm{Tr}_{234} \int_{t_0}^{t} \mathrm{d}\bar{t} [V_{12},\, \bar{U}_{13}(t\bar{t})\, \bar{U}_{24}(t\bar{t})\, \{i\hbar\delta(t_0 - \bar{t})\, g_{34}(\bar{t}) + Q_{34}(\bar{t})\}] \tag{7}$$

and the retarded RPA self-energy $\Sigma_1^+(t\bar{t}) = (n^3/i\hbar) \,\mathrm{Tr}_{234}\, V_{12} \bar{U}_{13}(t\bar{t})\, \bar{U}_{24}(t\bar{t})\, V_{34} F_4(\bar{t})$.

On the other hand, to establish the correspondence to the Green's functions results, it is convenient to transform Eq. (6) into an integral equation

$$\bar{U}_{13}(tt') = \bar{U}^0_{13}(tt') + n^2 \,\mathrm{Tr}_{57} \int_{t'}^{t} \mathrm{d}\bar{t}\, \bar{U}^0_{17}(t\bar{t})\, [V_{75},\, F_7(\bar{t})]\, \bar{U}_{53}(\bar{t}t)\,, \tag{8}$$

where the free (quasiparticle) propagator $\bar{U}^0$ is the solution of Eq. (6) with the trace term set to zero. The momentum representation of Eq. (8) is (homogeneous case)

$$\begin{aligned} \langle \mathbf{p}_1,\, \mathbf{p}_3 |\, U_{13}(tt')\, | \mathbf{p}_1 + \mathbf{q},\, \mathbf{p}_3' \rangle &= \delta(\mathbf{p}_1 - \mathbf{p}_3)\, \delta(\mathbf{p}_3 + \mathbf{q} - \mathbf{p}_3')\, K(\mathbf{p}_3,\, \mathbf{p}_3 + \mathbf{q}; tt') \\ &\quad + \int_{t'}^{t} \mathrm{d}\bar{t}\, K(\mathbf{p}_1,\, \mathbf{p}_1 + \mathbf{q}; t\bar{t})\, \{f_{\mathbf{p}_1}(\bar{t}) - f_{\mathbf{p}_1 + \mathbf{q}}(\bar{t})\} \\ &\quad \times V(\mathbf{q})\, I(\mathbf{p}_3,\, \mathbf{p}_3',\, \mathbf{q}; \bar{t}t')\,, \end{aligned} \tag{9}$$

where the first term on the right-hand side is the matrix element of $\bar{U}^0$, $V(q)$ is the Coulomb matrix element and $\langle \mathbf{p}_1 |\, nF_1\, | \mathbf{p}_1' \rangle = \delta(\mathbf{p}_1 - \mathbf{p}_1')\, f_{\mathbf{p}_1}$. An equation for the unknown integral

$$I(\mathbf{p}_3,\, \mathbf{q}; tt_0) \equiv \frac{1}{i\hbar} \int \mathrm{d}\bar{\mathbf{p}} \langle \bar{\mathbf{p}},\, \mathbf{p}_3 |\, U_{13}(tt_0)\, | \bar{\mathbf{p}} + \mathbf{q},\, \mathbf{p}_3' \rangle \tag{10}$$

follows directly from integrating Eq. (9) over $\mathbf{p}_1$,

$$I(\mathbf{p}_3,\, \mathbf{p}_3',\, \mathbf{q}; tt') = \delta(\mathbf{p}_3 + \mathbf{q} - \mathbf{p}_3') \int_{t'}^{t} \mathrm{d}\tau\, \varepsilon^{-1}(\mathbf{q}; t\tau)\, K(\mathbf{p}_3,\, \mathbf{p}_3 + \mathbf{q}; \tau t')\,. \tag{11}$$

Here we defined ε^{-1} which obeys a Dyson equation

$$\varepsilon^{-1}(\mathbf{q},\, tt') = \delta(t - t') + V(\mathbf{q}) \int_{t'}^{t} \mathrm{d}\bar{t}\, \Pi(\mathbf{q},\, t\bar{t})\, \varepsilon^{-1}(\mathbf{q},\, \bar{t}t') \tag{12}$$

with

$$\Pi(\mathbf{q},\, t\bar{t}) = \frac{1}{i\hbar} \int \mathrm{d}\mathbf{p}_1\, K(\mathbf{p}_1,\, \mathbf{p}_1 + \mathbf{q},\, t\bar{t})\, \{f_{\mathbf{p}_1 + \mathbf{q}}(\bar{t}) - f_{\mathbf{p}_1}(\bar{t})\}\,, \tag{13}$$

being the nonequilibrium RPA polarization function with self-energy included, which has previously been derived from Green's functions using the generalized Kadanoff-

Baym ansatz [11]. Here, it is not an ansatz, but an exact result of the BBGKY hierarchy with the RPA closure (iii) and g_{123} given by Eq. (3). ε^{-1} is the nonequilibrium generalization of the inverse dielectric function which is related to the screened potential by $\varepsilon^{-1} = V^{\mathrm{s}}/V$, and Eq. (12) fully describes the fs build-up of screening and of the plasmon spectrum. For more details and the inclusion of Fermi statistics see [10, 5].

5. Anti-Symmetrization of the Equations of Motion

The effects of the Fermi statistics of the electrons can be made explicit in the hierarchy by performing an antisymmetrization of the density operators, $F_1 \to F_1$, $F_{12} \to F_{12}(1 - P_{12})$ etc. [12]. Here, P_{12} is a binary permutation operator acting on two-particle states according to $P_{12}\,|21\rangle = |12\rangle$. Following this procedure, the first hierarchy equation (1) transforms into

$$i\hbar\,\frac{\partial}{\partial t}\,F_1 - [\bar{H}_1,\,F_1] = n\,\mathrm{Tr}_2\,[V_{12}^{\pm},\,g_{12}] \tag{14}$$

which contains all possible exchange effects via $V_{12}^{\pm} = V_{12}(1 - P_{12})$, appearing in the collision integral and in the mean field (Hartree-Fock) Hamiltonian $H_1^{\mathrm{H}} \to H_1^{\mathrm{HF}} = n\,\mathrm{Tr}_2\,V_{12}^{\pm}F_2$. Analogously, the antisymmetrization of the second hierarchy equation [3] and of the self-energy [9, 5] is performed.

6. Bloch Representation

Equation (14) together with the solution (4) for g_{12} constitute in fact the SBE in general operator notation. The familiar form of coupled equations for band populations and interband transitions, is obtained by expanding this operator equation in terms of appropriate Bloch states [3] $|s\rangle = |\lambda_s k_s\rangle$, where λ is the band index and $\langle s \mid s'\rangle = \delta_{\lambda_s,\lambda_{s'}}\delta_{k_s,k_{s'}}$ and $\sum_{s=1}^{N} |s\rangle\,\langle s| = 1$.

6.1 Bloch representation of the kinetic equation

Denoting the Bloch matrix elements of F_1 and g_{12}, by $f^{\lambda_1\lambda_1'}(k_1)$ and $g^{\lambda_1\lambda_{1'}}_{\lambda_2\lambda_{2'}}(k_1k_2k_1'k_2')$, we obtain the general matrix form of the spatially homogeneous SBE

$$\begin{aligned}&\left\{i\hbar\,\frac{\partial}{\partial t} - (E^{\lambda_1}_{k_1} - E^{\lambda_1'}_{k_1})\right\} f^{\lambda_1\lambda_1'}(k_1)\\ &- \sum_{\bar{\lambda}_1}\{\hbar\bar{\Omega}^{\lambda_1\bar{\lambda}_1}(k_1)\,f^{\bar{\lambda}_1\lambda_1'}(k_1) - f^{\lambda_1\bar{\lambda}_1}(k_1)\,\hbar\bar{\Omega}^{\bar{\lambda}_1\lambda_1'}(k_1)\} = I^{\lambda_1\lambda_1'}(k_1)\,,\end{aligned} \tag{15}$$

where $E^{\lambda_s}_{k_s}$ is the (unrenormalized) one-particle energy and the effective Rabi energy includes the matrix element of H^{f} and of the Hartree-Fock energy $\hbar\bar{\Omega}^{\lambda\lambda'}(k) = \hbar\Omega^{\lambda\lambda'}(k) + E^{\mathrm{HF}}_{\lambda\lambda'}(k)$. The collision term contains direct and exchange contributions

$$\begin{aligned}I^{\lambda_1\lambda_1'}(k_1) = &\sum_{k_2\lambda_2\bar{k}_1\bar{k}_2} V(k_1 - \bar{k}_1)\,\delta_{k_1+k_2,\bar{k}_1+\bar{k}_2}\{g^{\lambda_1\lambda_1'}_{\lambda_2\lambda_2}(\bar{k}_1\bar{k}_2k_1k_2) - g^{\lambda_1\lambda_1'}_{\lambda_2\lambda_2}(k_1k_2,\,\bar{k}_1\bar{k}_2)\}\\ &- \sum_{k_2\lambda_2\bar{k}_1\bar{k}_2} V(k_1 - \bar{k}_2)\,\delta_{k_1+k_2,\bar{k}_1+\bar{k}_2}\{g^{\lambda_2\lambda_1'}_{\lambda_1\lambda_2}(\bar{k}_1\bar{k}_2k_1k_2) - g^{\lambda_1\lambda_2}_{\lambda_2\lambda_1'}(k_1k_2,\,\bar{k}_1\bar{k}_2)\}.\end{aligned} \tag{16}$$

6.2 Bloch representation of the solution $g_{12}(t)$

What is left now is to calculate the correlation matrix elements by expanding the solution $g_{12}(t)$ in the Bloch basis. The necessary steps are best demonstrated on the static Born approximation (iv), where under the $\bar{t}$ integral in Eq. (4) we have $\bar{U}_1(t\bar{t})\,\bar{U}_2(t\bar{t})\,Q_{12}(\bar{t})\,\bar{U}_1^{\dagger}(t\bar{t})\,\bar{U}_2^{\dagger}(t\bar{t})$. To obtain $g^{\lambda_1\lambda_1}_{\lambda_2\lambda_2}$, we need to compute the matrix elements of Q_{12} and $\bar{U}_1$. For the first one we use the antisymmetrized expression [3] $Q_{12} = \hat{V}_{12}F_1F_2 - F_1F_2\hat{V}_{12}^{\dagger}$ with the shielded potential $\hat{V}_{12} = (1 - nF_1 - nF_2)\,V_{12}$ which has the Bloch matrix element

$$\langle 12|\,\hat{V}_{12}\,|2'1'\rangle = \left\{\delta_{\lambda_1\lambda_1'}\delta_{\lambda_2\lambda_2'} - \delta_{\lambda_2\lambda_2'}f^{\lambda_1\lambda_1'}_{k_1k_1'} - \delta_{\lambda_1\lambda_1'}f^{\lambda_2\lambda_2'}_{k_2k_2'}\right\} V(k_1 - k_1')\,\delta_{k_1+k_2,k_1'+k_2'}\,.$$

The quasiparticle propagator under the influence of the laser field obeys the equation

$$\left\{i\hbar\,\frac{\partial}{\partial t} - \bar{H}_1\right\}\bar{U}_1(tt') = 0\,, \qquad \bar{U}_1(tt) = 1\,, \qquad \bar{H}_1 = H_1 + H_1^{\mathrm{HF}} + \Sigma_1^+ + H_1^{\mathrm{f}} \tag{17}$$

which has the Bloch matrix representation,

$$i\hbar\,(\partial/\partial t)\,\bar{U}_k^{\lambda\lambda'}(tt') - \sum_{\bar{\lambda}}\bar{H}_k^{\lambda\bar{\lambda}}(t)\,\bar{U}_k^{\bar{\lambda}\lambda'}(tt') = 0\,.$$

These equations contain the full field–matter interaction scenario including multi–photon absorption, field ionization etc. and are well suited for numerical integration. Analytical solutions are possible in limiting cases, e.g. by expanding $U_k^{\lambda\lambda'}$ into a Fourier series in terms of the field harmonics. Here, we restrict ourselves to a two-band (c, v) model under the influence of a low intensity optical pulse $E_0(t)\cos\omega_o t$, $\mathrm{d}E_0 \ll E_{\mathrm{gap}}$, which allows to apply the rotating wave approximation, e.g. [2]. If we further use the local approximation for the selfenergy, i.e. $\Sigma^+(tt') \to \Sigma^+(t-t')$, we obtain $\bar{U}(tt') = \bar{U}(t-t')$ and $\bar{U}_k^{\lambda\lambda'}(0) = \delta_{\lambda\lambda'}$. The solution of Eq. (17) is then

$$\begin{aligned}
\bar{U}_k^{\mathrm{cc}}(t) &= \exp\left\{-\frac{i}{\hbar}\int_0^t \mathrm{d}\bar{t}\left[\varepsilon_k^{\mathrm{c}}(\bar{t}) - \frac{|d_{\mathrm{cv}}E_0|^2}{\varepsilon_k^{\mathrm{v}}(\bar{t}) + \hbar\omega_0}\right]\right\}, \\
\bar{U}_k^{\mathrm{vc}}(t) &= -\frac{d_{\mathrm{cv}}E_0}{\varepsilon_k^{\mathrm{v}}(\bar{t}) + \hbar\omega_0}\,\mathrm{e}^{-i\omega_0 t}\,,
\end{aligned} \tag{18}$$

and similar expressions for $\bar{U}^{\mathrm{vv}}$ and $\bar{U}^{\mathrm{cv}}$ [5]. In the weak intensity limit, in the exponent only the renormalized (complex) band energies $\varepsilon_k^{\mathrm{c,v}}$ remain, leading to four exponential factors in $g^{\lambda_1\lambda_1'}_{\lambda_2\lambda_2}$, i.e. the familiar oscillations (with the real parts of $\varepsilon^{\mathrm{c,v}}$) which are damped (by the imaginary parts of $\varepsilon^{\mathrm{c,v}}$).

7. Discussion

With the coupled equations (15), the result for the relevant matrix elements of (4) and the solution of Eq. (17) we have obtained very general non-Markovian SBE which contain correlations and self-energy and a general coupling to the electromagnetic field. While these equations can of course be obtained using various theoretical concepts, the BBGKY approach allows for a simple and straightforward derivation. In particular, the treatment of carrier correlations and of the laser–matter interaction can be performed within a very compact and transparent operator notation, cf. Eqs. (2) and (17), respectively. Thus, the present approach is well suited to extend the theory to more complex approximations.

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Note added in proof: A similar density operator analysis of the dynamical screening problem (without self-energy) has been performed independently by U. Hohenester and W. Pötz (Phys. Rev. B **56**, 13177 (1997).