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Numerical Solution of the Quantum Landau Equation for Dense Plasmas

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Abstract

The numerical solution of the quantum Landau kinetic equation for a dense electron gas is considered. Being one of the most simple kinetic equations, it still retains essential physical features of a correlated many-particle system, such as selfconsistent static screening and Pauli blocking, and, at the same time, it is a good test case for the efficiency of numerical methods. Two schemes for the evaluation of the scattering rates and the collision integral are discussed. To illustrate our results, we present time-dependent calculations i) for the relaxation of a nonequilibrium electron distribution function and ii) for the stopping of fast electrons.

1 Introduction

Nonequilibrium phenomena on short-time scales are of topical interest in the fields of carrier kinetics in semiconductors[1, 2, 3, 4], nuclear physics[5, 6], plasma discharge physics[7], deceleration of particle beams in target plasmas[8] and non-LTE (local thermal equilibrium) population kinetics[9, 10]. The theoretical description of the relevant processes requires the calculation of the Wigner distribution function from a kinetic equation. Here, the rate of change of the momentum distribution is determined (i) by the drift in phase space (which includes the effect of spatial inhomogeneity and of external fields) and (ii) by a collision term accounting for the correlation in the system. In the case of charged particles, the electromagnetic field is coupled to the carrier relaxation via Maxwell's equation and may give rise to collective oscillations and plasma instabilities. On the other hand in a dense system, where the scattering process is strongly influenced by medium effects, by the existence of bound complexes or screening effects due to the long range Coulomb interaction in a plasma the correlation term is complicated. The complexity of the problem is further increased if (iii) the system consists of many species. Until now a fully selfconsistent treatment of this complex kinetic problem is not feasible.

Of course, there is a vast literature on kinetic theory and numerical solutions of kinetic equations. However they treat only parts of the complex problem. We mention this solution of the Vlassov equation (where collisions are completely neglected) together with Maxwell's equations for classical plasmas[11, 12] and quantum plasmas in low-dimensional semiconductors[13]. On the other hand there exist many approaches to systems in which collisions play a dominant role. In many cases a simplified classical expression for the collision integral is chosen, so complex calculations in systems consisting of many species or with a complicated geometry become feasible. This approach neglects quantum effects and avoids a selfconsistent treatment of screening. Our aim here is different. We focus on simple physical situations in dense plasmas where quantum and many-body effects due to the surrounding medium can be included into the theory and treated numerically.

A general quantum-statistical approach to this problem has to include many-particle effects, such as dynamical screening, selfenergy and Pauli blocking (phase space occupation). In general, this analysis leads to generalized non-Markovian kinetic equations[2, 4, 14, 15,

16], first solution of which have become possible in recent years[2, 3, 4, 5, 6]. We will not consider memory effects here. Other important many-body effects are related to bound states and their influence on the plasma properties, there we refer to density operator approach given in [17] and to our analysis based on the Green's functions technique[15] and to the references therein. Other important collective effects are related to plasma instabilities and particle-beam interaction (energy deposition), where first applications of quantum kinetic equations have benn given recently[18]. This latter question is of interest for the ignition of fusion pellets by heavy ion beams for the inertially confined fusion research. For such tasks, it is necessary to determine the nonequilibrium distribution functions.

The purpose of the present paper is to investigate accurately a simple kinetic equation, the Landau equation, which may serve as a model for comperative purposes and as a standard for delivering start solutions for the handling of more complicated equations. In the Landau equation, the scattering T-matrix is replaced by its Born approximation, what is justified for weakly coupled systems, including high density quantum plasmas, such as charge carrier systems in semiconductors[4].

We will show that the special techniques for the numerical solution of the Landau equation are of relevance, especially for purposes of the investigation of particle stopping and for the buil-up of instabilities. For simplicity, we consider the homogeneous and isotropic case which retains, however, essential physical features. In this paper, non-Markovian effects were neglected; see, however, [4]. The numerical solution of inhomogeneous equations is only in a preliminary stage; mainly classical work is done there using polinomial expansions[19, 20].

2 The Quantum Landau Equation

Let us start from the quantum kinetic equation for the Wigner distribution function in binary collision approximation. In the spatially homogeneous case the quantum Boltzmann equation reads

$$\begin{aligned}\frac{\partial}{\partial T} f_a(\boldsymbol{p},T) &= \sum_b \frac{1}{V}\int \frac{\mathrm{d}\boldsymbol{p}'}{(2\pi)^3}\frac{\mathrm{d}\overline{\boldsymbol{p}}}{(2\pi)^3}\frac{\mathrm{d}\overline{\boldsymbol{p}}'}{(2\pi)^3}\, 2\pi\delta(E_{ab}-\overline{E}_{ab}) \\ &\quad \times \frac{1}{2!}\left|< \boldsymbol{p}\boldsymbol{p}'|T_{ab}(E_{ab}+\imath\varepsilon,T)|\overline{\boldsymbol{p}}'\overline{\boldsymbol{p}} >^{\pm}\right|^2 \\ &\quad \times \left\{\overline{f}_a\overline{f}'_b(1\pm f_a)(1\pm f_b) - f_a f_b(1\pm\overline{f}_a)(1\pm\overline{f}'_b)\right\}. \qquad (1)\end{aligned}$$

Here E_{ab} is the two-particle energy, and the distribution functions are denoted by $f_a = f(\boldsymbol{p},T)$, $\overline{f}_a = f(\overline{\boldsymbol{p}},T)$ etc. T_{ab} is the T-matrix to be determined from the scattering theory and should include, moreover, exchange and medium effects. According to these possibilities and due to the occurrance of occupation factors $(1\pm f_a)$, eq.(1) goes far beyond the usual (classical) Boltzmann equation. Eq.(1) is appropriate especially for systems with short range forces; however, it is also applied to plasmas in which the interaction has to be screened. The consequent inclusion of dynamical screening is realized in the Lenard-Balescu kinetic equation.

The (numerical) solution of both the Boltzmann and the Lenard-Balescu equations was dealt with in numerous papers and is a time consuming task. While the quantum Lenard-Balescu equation including nonequilibrium dynamical screening was solved only recently[1], the question of the solution of the Boltzmann equation has a long history. However, in most cases there were done essential approximations to eq.(1), and especially there was chosen a simple version of the T-matrix, or a decomposition of the distribution function was performed with respect to orthogonal polynomials[19, 20]. In teh general case one has to determine the T-matrix, e.g., from the numereical solution of the Schrdinger equation; here phase space occupation (Pauli blocking) should be incorporated.

The question becomes still more complicated if memory effects and long lived correlations such as bound states are taken into account. In this paper which is mainly of methodical interest we choose the Born approximation to the Boltzmann equation, i.e, the (statically screened) Landau equation. This equation retains still several many-body features, namely screening and phase space occupation. The quantum version of the Landau equation reads for a Fermi system

$$\begin{aligned}\frac{\partial}{\partial T} f_a(\boldsymbol{p}T) &= \sum_b \int \frac{\mathrm{d}\boldsymbol{p}'}{(2\pi)^3}\frac{\mathrm{d}\overline{\boldsymbol{p}}}{(2\pi)^3}\frac{\mathrm{d}\overline{\boldsymbol{p}}'}{(2\pi)^3} V_{ab}^2(|\boldsymbol{p}-\overline{\boldsymbol{p}}|)(2\pi)^3\delta(\overline{\boldsymbol{p}}+\overline{\boldsymbol{p}}'-\boldsymbol{p}-\boldsymbol{p}') \\ &\quad \times 2\pi\delta(E_{ab}-\overline{E}_{ab})\left\{\overline{f}_a\overline{f}_b'(1-f_a)(1-f_b') - f_a f_b'(1-\overline{f}_a)(1-\overline{f}_b')\right\} \\ &= \sum_b I_{ab}^L(\boldsymbol{p}T). \end{aligned} \tag{2}$$

Here we used $\hbar = 1$. In the following, the momentum distribution is assumed to be isotropic and the two-particle energies are taken to be free particle energies, i.e. $E_{ab} = \boldsymbol{p}^2/2m_a + \boldsymbol{p}'^2/2m_b$. The potential V_{ab}^S is taken to be the statically screened Debye potential.

$$V_{ab}^S(\boldsymbol{q},0) = \frac{V_{ab}(\boldsymbol{q})}{\varepsilon(\boldsymbol{q},0)}, \qquad \varepsilon(\boldsymbol{q},0) = 1 + \frac{\kappa^2}{\boldsymbol{q}^2} \tag{3}$$

Here $V_{ab}(\boldsymbol{q}) = 4\pi e_a e_b/\boldsymbol{q}^2$ is the Fourier transform of the Coulomb potential and $\varepsilon(\boldsymbol{q},0)$ is the static dielectric function in Random Phase Approximation (RPA). For the inverse screening length we find

$$\kappa^2(T) = \sum_a \frac{16\pi^2 e_a^2 m_a}{(2\pi)^3}\int_0^\infty \mathrm{d}p\; f_a(p,T). \tag{4}$$

In this paper, we want to discuss the featurs of the Landau equation from the numerical point of view and especially consider the time evolution of the distribution function. This study is intended to serve as a standard problem for comparative purposes with systems in which additional effects such as memory, initial correlations etc. are included[4]. We want to mention that eq.(2) does not exhibit any divergencies; the long range ones are avoided by screening, and the short range ones do not occur due to quantum effects.

We introduce an abbreviation for the set of distribution functions

$$\begin{aligned}\Phi_{ab}(\overline{\boldsymbol{p}},\overline{\boldsymbol{p}}',\boldsymbol{p},\boldsymbol{p}',T) &= f_a(\overline{\boldsymbol{p}},T) f_b(\overline{\boldsymbol{p}}',T)\left[1-f_a(\boldsymbol{p},T)\right]\left[1-f_b(\boldsymbol{p}',T)\right] \\ &\quad - f_a(\boldsymbol{p},T) f_b(\boldsymbol{p}',T)\left[1-f_a(\overline{\boldsymbol{p}},T)\right]\left[1-f_b(\overline{\boldsymbol{p}}',T)\right]. \end{aligned} \tag{5}$$

With the transfer momenta $\overline{\boldsymbol{p}}-\boldsymbol{p} = \boldsymbol{p}'-\overline{\boldsymbol{p}}' = \boldsymbol{q}$, and after performing the $\overline{\boldsymbol{p}}'$ integration, the Landau collision integral can be written as

$$\begin{aligned} I_{ab}^L(p,T) &= \int \frac{\mathrm{d}\boldsymbol{p}'}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3} V_{ab}^2(q)(2\pi)\Phi_{ab}(|\boldsymbol{p}+\boldsymbol{q}|,|\boldsymbol{p}'-\boldsymbol{q}|,\boldsymbol{p},\boldsymbol{p}',t) \\ &\quad \times\delta\left(\frac{1}{2m_a}\left[(\boldsymbol{p}+\boldsymbol{q})^2-\boldsymbol{p}^2\right] + \frac{1}{2m_b}\left[(\boldsymbol{p}'-\boldsymbol{q})^2-\boldsymbol{p}'^2\right]\right). \end{aligned} \tag{6}$$

In (6), we have to carry out 4 nontrivial integrals i.e., we have the integrations over $|\boldsymbol{p}'|$, $|\boldsymbol{q}|$, $z_1 = \cos(\boldsymbol{q},\boldsymbol{p})$ and $z_2 = \cos(\boldsymbol{q},\boldsymbol{p}')$, among which one may be done with the delta distribution.

The z_2 integration is carried out using the δ distribution representing energy conservation. This leads to a condition for the integration over the modulus of p' integration, namely

$$p' = |p'| \geq \frac{1}{2}\left|\frac{m_b}{m_a}(q + 2pz_1) + q\right| = \bar{p}.$$

New variables $p' \to p' + \bar{p}$ lead to

$$\begin{aligned} I^L_{ab}(p,t) &= m_b \frac{1}{(2\pi)^3}\int_0^\infty \mathrm{d}qq \int_0^\infty \mathrm{d}p'\,(p'+\bar{p}) \int_{-1}^{1} \mathrm{d}z_1\, V^2(q) \\ &\times \Phi_{ab}\left(\sqrt{q^2+p^2+2qpz_1}, \sqrt{(p'+\bar{p})^2 - \left[\frac{m_b}{m_a}(q^2+2qpz_1)\right]}, p, p', t\right). \end{aligned} \tag{7}$$

Another possibility according to Banyai[21] is now to remove the quantities z_1 and z_2 from the combination of distribution functions. In the isotropic case, the distribution functions depend only on the absolut values of the momenta. The procedure is the introduction of two additional integrations. The price will be that the q integration will be more complicated. Equation (6) reads then

$$\begin{aligned} I^L_{ab}(p,t) &= \int \frac{\mathrm{d}\boldsymbol{p}'}{(2\pi)^3}\frac{\mathrm{d}\boldsymbol{q}}{(2\pi)^3}\int_0^\infty \mathrm{d}x\,\mathrm{d}y\, V^2_{ab}(q)(2\pi)\delta\left(\frac{1}{2m_a}\left[(x-\boldsymbol{p}^2\right] + \frac{1}{2m_b}\left[(y-\boldsymbol{p}'^2\right]\right) \\ &\times \Phi_{ab}(\sqrt{x},\sqrt{y},|\boldsymbol{p}|,|\boldsymbol{p}'|,t)\delta\left(x-|\boldsymbol{p}+\boldsymbol{q}|^2\right)\delta\left(y-|\boldsymbol{p}'-\boldsymbol{q}|^2\right). \end{aligned} \tag{8}$$

The additional δ distributions have zeros for positive arguments only. The z_1, z_2 integrations may be done using the last mentioned δ distributions, the modulus integration over $|\boldsymbol{p}'|$ is carried out with the energy conserving δ distribution, and there remain only the x, y and $|\boldsymbol{q}|$ integration. In the case of the Born approximation and if $V_{ab}(q)$ is sufficiently simple, the latter integration may be carried out, too. The variables z_1 and z_2 run, in principle, from -1 to $+1$. According to

$$\begin{aligned} \delta_x\left(x-|\boldsymbol{p}+\boldsymbol{q}|^2\right) &= \frac{1}{2pq}\delta\left[\frac{x-p^2-q^2}{2pq} - z_1\right], \\ \delta_y\left(y-|\boldsymbol{p}'-\boldsymbol{q}|^2\right) &= \frac{1}{2p'q}\delta\left[\frac{-y+p'^2+q^2}{2p'q} - z_2\right], \end{aligned} \tag{9}$$

we get for the restriction of the $|\boldsymbol{q}|$ integration ($|\boldsymbol{q}| \equiv q$)

$$\begin{aligned} q_{\min} &= \text{maximum}\left[|p-\sqrt{x}|; |p'-\sqrt{y}|\right], \\ q_{\max} &= \text{minimum}\left[p+\sqrt{x}; p'+\sqrt{y}\right], \\ q_{\min} &< q_{\max}. \end{aligned} \tag{10}$$

With these abbreviations, the q integration may be done. The conditions (10) may be controlled by a computer for given external momentum $|\boldsymbol{p}|$ and for any step of the current remaining integrations over $|\boldsymbol{p}'|$, x and y.

The integration over q is now carried out using the statically screened Coulomb potential given by (3):

$$\int_{q_{min}}^{q_{max}} \frac{\mathrm{d}q}{(q^2+\kappa^2)^2} = \frac{1}{2\kappa^2}\left[\frac{1}{\kappa}\arctan\left(\frac{q}{\kappa}\right) + \frac{q}{q^2+\kappa^2}\right]_{q_{min}}^{q_{max}}. \tag{11}$$

Then we get from (8) and (9) replacing x by x^2 and y by y^2

$$\begin{aligned} I_{ab}^L(\boldsymbol{p},t) &= \frac{(4\pi e_a e_b)^2}{2\kappa^2}\frac{m_b}{p}\frac{1}{(2\pi)^3}\int_0^\infty \mathrm{d}x\, x \int_0^\infty \mathrm{d}y\, y \left[\frac{1}{\kappa}\arctan\left(\frac{q}{\kappa}\right) + \frac{q}{q^2+\kappa^2}\right]_{q_{min}}^{q_{max}} \\ &\times \Phi_{ab}(x, y, |\boldsymbol{p}|, \sqrt{y^2 + \frac{m_b}{m_a}(x^2 - \boldsymbol{p}^2)}, t). \end{aligned} \tag{12}$$

In (12), Φ_{ab} is defined by (5), and in the isotropic case considered all distribution functions depend only on the absolute values of momenta. We want to mention again that we can take advantage of the analytical integrability of the Debye potential. Things become more complicated and cannot be reduced to a two dimensional integral in the general case of a Boltzmann equation with a T-matrix instead of the Born approximation.

3 Numerical Evaluation

3.1 Dicussion of the Collision Integral

In the following we consider a dense quantum electron gas, i.e. the relaxation of the electron distribution is calculated from a Landau kinetic equation with an electron-electron collision integral. The interaction potential is the statically screenend Coulomb potential given by (3). The momentum distribution is assumed to be isotropic.

The aim is to solve numerically the kinetic equation using two different expressions for the collision integral given by (7) and(12). Especially, we will compare the efficiency of their numerical evaluation. Three integrations must be done in (7), while there are only two integrals in equation (12).

For the further considerations, it is useful to write the Landau collision integral in terms of the scattering-in rate $\pm i\Sigma_L^<(p,T)$ and the scattering-out rate $i\Sigma_L^>(p,T)$, i.e.

$$\frac{\partial}{\partial T} f_e(p,T) = \Sigma_L^<(p,T)[1 - f_e(p,T)] - \Sigma_L^>(p,T) f_e(p,T). \tag{13}$$

The $\Sigma_L^{\gtrless}$ follow from (1) using the first Born approximation for the T-matrix and can be taken from (7) and (12) where some integrations were carried out already. We give the scattering-in and scattering-out rates explicitly for the case where the collision integral is written as the double integral expression (12) and we find

$$\Sigma_L^<(p) = \frac{8m_b}{\pi}\int_0^\infty \mathrm{d}x \int_0^\infty \mathrm{d}y G^{in}(p,x,y), \tag{14}$$

$$\Sigma_L^>(p) = \frac{8m_b}{\pi}\int_0^\infty \mathrm{d}x \int_0^\infty \mathrm{d}y G^{out}(p,x,y). \tag{15}$$

Here, we introduce the abbreviations

$$G^{in}(p,x,y) = \frac{xy}{p}\frac{1}{2\kappa^2}\left[\frac{1}{\kappa}\arctan\left(\frac{q}{\kappa}\right)+\frac{q}{q^2+\kappa^2}\right]_{q_{min}}^{q_{max}} \times\left[f_a(x)f_b(y)\left(1-f_b\left(\sqrt{y^2+\frac{m_b}{m_a}(x^2-p^2)}\right)\right)\right] \tag{16}$$

and

$$G^{out}(p,x,y) = \frac{xy}{p}\frac{1}{2\kappa^2}\left[\frac{1}{\kappa}\arctan\left(\frac{q}{\kappa}\right)+\frac{q}{q^2+\kappa^2}\right]_{q_{min}}^{q_{max}} \times\left[(1-f_a(x))(1-f_b(y))f_b\left(\sqrt{y^2+\frac{m_b}{m_a}(x^2-p^2)}\right)\right]. \tag{17}$$

As before, the mass is given in units of the reduced mass and the energy is given in Ryd units.

To study the numerical method the initial distribution function $f_e(p,0)$ was chosen to be a Gaussian distribution function of the form

$$f_e(p,0) = A\exp\left(-\frac{(p-p_0)^2}{\gamma}\right) \tag{18}$$

where γ is the width and p_0 is the maximum location.

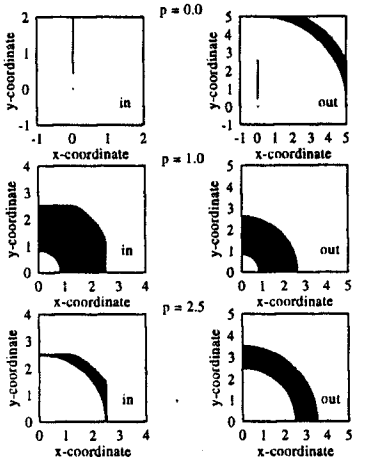

Figure 1: Integration range for the potential.The left pictures show the scattering–in term (16) (signed with in), the right pictures show the scattering–out term (17) before x, y integration for different external momenta.

A problem to treat (12) numerically is the integration range with respect to x and y. The area in which the integrands G^{in} and G^{out} give nonzero contributions to the integral is small, what could be used to reduce the integration region. But this area is different for different external momenta and it changes if the distribution function changes in time.
In Fig. 1 the regions in the x–y plane are shown where G^{in} and G^{out} give contributions to the integral. The time is kept constant and a Gaussian distribution function was used used in the Gaussian form (18) with the parameters $A = 0.15$, $\gamma = 0.05$ and $p_0 = 0.4$.

We found that a relatively high number of integration points is needed to perform the double integral. The number of points per single integration as well as the memory demand is higher compared to the triple integral if the same accuracy is required.

3.2 Relaxation of the Distribution Function

The quantum Landau equation is a simple kinetic equation which can be dealt with accurately. Therefore, it is a useful model equation to study relaxation processes in dense quantum systems. in the following the solution of the Landau equation is applied to model nonequilibrium phenomena relevant for dense plasma physics.

First let us consider the complete relaxation of a special electron distribution function in a quantum electron gas. An interesting situation is given if we have a disturbed equilibrium state due to high energetic electrons. Special problems related to such a situation are electron beams in plasmas, runaway electrons and the build-up of instabilities. Here, we will not discuss these special phenomena, but we will restrict ourselves to the relaxation of the momentum distribution which we chose in the special shape

$$f_e(p,0) = \left[\exp\left(\beta(p^2/2m_e - \mu)\right) + 1\right]^{-1} + A\exp\left(-\frac{(p-p_0)^2}{\gamma}\right). \tag{19}$$

Density and temperature of the equilibrium state described by the Fermi function are $n = 1.5 \cdot 10^{23} cm^{-3}$ and $T = 20000K$. Furthermore, the parameters of the Gaussian part of the initial distribution function are chosen to be $A = 0.1$, $p_0 = 2.5$ and $\gamma = 0.04$.

The electron number density described by (19) corresponds to that of a very dense plasma. Recently, highly ionized Carbon plasmas with electron densities exceeding $10^{23} cm^{-3}$ were produced by intensive subpicosecond laser pulses[22, 23]. the investigation of such extreme plasma conditions is of great importance for further developments in the field of inertially confined fusion experiments. Here, the theoretical modelling of the short time behaviour to give estimates about transient properties and relaxation times is of special interest.

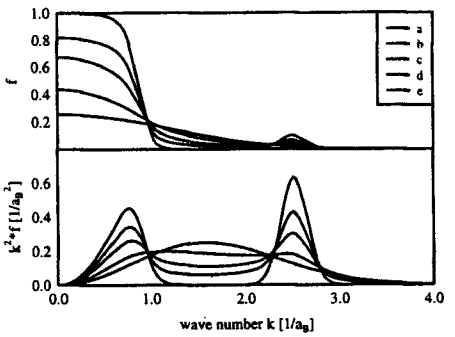

Figure 2: Relaxation of an initial distribution (19) (a: 0fs, b: 0.1fs, c: 0.2fs, d: 0.4fs, e: 2fs). The Fermi distribution corresponds to a number density $n = 1.5 \cdot 10^{23} cm^{-3}$ and a temperature $T = 20000K$. The parameters of the Gaussian part are $A = 0.1$, $k_0 = 2.5$, $\gamma = 0.04$.

The time evolution of the distribution function given by (19) is shown in Fig. 2. The behaviour of the second moment of the distribution function is presented in the lower part of the Figure. We observe a rapid thermalization with a high decay rate of the high energetic maximum of the distribution function. Due to the high density of the electron gas, the relaxation time is extremly small. In our case, equilibrium is reached already at nearly 2.0 fs.

3.3 Stopping Power

A further problem relevant in dense plasma physics is the investigation of energy deposition of a particle beam into target plasma. This problem was intensely studied in the last years, especially, in connection with inertial fusion research[8, 24, 25]. The characteristic quantity is the stopping power which is considered to be the time derivative of the mean kinetic energy of the beam particles, e.g.

$$\frac{\partial}{\partial t} < E >= \frac{1}{n_b} \int \frac{\mathrm{d}\boldsymbol{p}}{(2\pi)^3} \frac{p^2}{2m_b} f_b(\boldsymbol{q}, t) \tag{20}$$

where n_b is the density and f_b the distribution function of the beam. Usually, the stopping power calculated in such a way corresponds to the initial moment of the beam-target interaction and does not describe the complete thermalization process[26, 27, 28, 29]. In particular, the beam particle distribution function is assumed to be of delta-like shape. The result is then a time independent expression for the stopping power (stopping force) and reads, e.g., for the calculation in the framework of a Boltzmann equation[30],

$$\begin{aligned} \frac{\partial < E >}{\partial t} &= \frac{1}{(2\pi)^2 \hbar^3} \sum_c \frac{m_c^2}{m_{bc}^3} \frac{n_c \Lambda_c^3}{m_b} \frac{k_B T}{v} \int_0^\infty \mathrm{d}p p^3 Q^T(p) \\ &\quad \times \mathrm{e}^{-\frac{m_c}{2k_B T}} \left[\mathrm{e}^{(v - \frac{p}{m_{bc}})^2} p_- - \mathrm{e}^{(v + \frac{p}{m_{bc}})^2} p_+ \right]. \end{aligned} \tag{21}$$

Here $p_\pm = p \pm m_b v + (m_b m_{bc} k_B T)/(m_c p)$. Q^T is the transport cross section which is determined by the T-matrix, m_{bc} is the reduced mass.

In contrast to such procedure, the new aspect to this paper is to consider the time evolution of the deposition procedure. In general, one has to start from the coupled set of beam-target equations

$$\begin{aligned} \frac{\partial}{\partial T} f_t(p, T) &= I_{tt} + I_{tb} \\ \frac{\partial}{\partial T} f_b(p, T) &= I_{bb} + I_{bt}. \end{aligned} \tag{22}$$

In the following we will do this in a simplified manner using the Landau kinetic equation considered so far. Our model is a two-component electron gas with different initial distribution functions. The first one describes the target system which is assumed, at the beginning, to be in the thermodynamic equilibrium. The second one is given by a Gaussian distribution function. The latter corresponds to high energetic electrons and therefore it reflects some features of a particle beam. The time evolution is calculated then solving the coupled set of Landau equations (22) including the corresponding two-component collision integrals. For the collision integrals, we use the expression given by (12).

The relaxation of the system into the final equilibrium state is shown in Fig. 3. The initial distribution of the target component (right figure) is described by a Fermi distribution which corresponds to a density $n = 5 \cdot 10^{21} cm^{-3}$ and to a temperature of $T = 20000K$. The left figure shows the temporal evolution of the high energetic component (electron beam). The relaxation process is characterized by a first time interval of rapid broadening of the initial Gaussian peak. This process has a duration time of nearly 1 fs whereas the final equilibrium state is reached at 10 fs.

From the calculated distribution functions we can find the temporal evolution of the mean kinetic energies per particle. It describes the thermalization shown in Fig. 4.

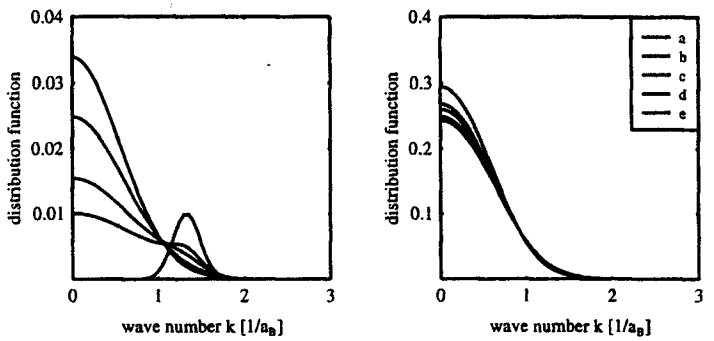

Figure 3: Relaxation of the distribution functions of a two component electron gas (a: 0fs, b: 0.1fs, c: 0.2fs, d: 0.4fs, e: 2fs). The initial state of the high energetic component (left) is given by a Gaussian distribution function with $A = 0.1$, $k_0 = 2.5$, $\gamma = 0.04$. The target system (right) is considered to be initially in thermodynamic equilibrium with $n = 5 \cdot 10^{21} cm^{-3}$ and a temperature $T = 20000K$ described by a Fermi distribution function.

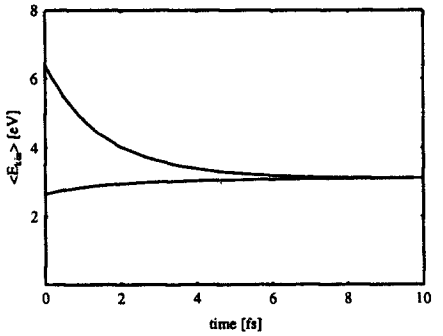

Figure 4: Temporal evolution of the mean kinetic energy of the two component electron gas considered in Figure 3 (solid line - target system, dotted line - high energetic component). Thermalization is reached nearly at 10fs.

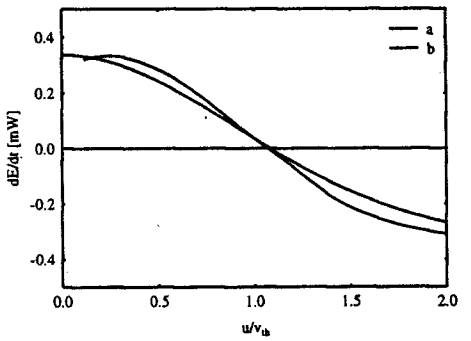

Figure 5: Energy deposition of the beam component into the target system in the initial moment (a). It is given as a function of the characteristic (beam) velocity in units of thermal velocity $v_{th} = (2k_B T/m)^{(1/2)}$. The results are compared with that obtained from an explicit stopping power expression in Landau approxiamtion (b)[30].

Finally, let us look at the energy deposition in the initial moment of relaxation. Practically it was determined considering the first time steps of numerical integration. Different Gaussian distribution functions were used for the beam component varying the location of the peak. In this way, we can model the dependence of the stopping power on the characteristic (beam) velocity. The result are shown in Fig. 5 for a target electron gas with density $n = 10^{21} cm^{-3}$ and temperature $T = 30000K$. Furthermore, a comparison is given with results obtained from explicit stopping power expressions which were derived in the usual manner from the kinetic equations[30]. Here, the collisions integral was also used in Landau approximation. The results show a relatively good agreement in the considered velocity range. Both curves describe an energy gain at low velocities and exhibit a zero approximately at a velocity equal to the thermal velocity of the target plasma. At high velocities, the expected energy loss of the beam component is observed which corresponds to a heating of the target system.

4 Conclusions

In this paper, we investigated two different possibilities to solve a special kinetic equation. The advantage of the shorter CPU time for the double integral goes along with the fact that the number of integration points has to be chosen very carefully due to the complicated structure of the integration area. The triple integral does not have this problem, and is more robust.

The example chosen – the Markovian quantum Landau equation – is the simplest one; however, the integration schemes discussed are applicable to other problems. The method outlined for the double integral may be used for the solution of the non–Markovian Landau equation, too. The triple integration may be applied to the Markovian Lenard–Balescu equation. Here we are faced with the fact that the dynamically screened potential cannot be integrated analytically.

From these examples we see that both the double and the triple integration scheme are of relevance; if applicable the double integration saves computer time. As an application of the quantum Landau equation we looked on relaxation processes relevant for strongly coupled plasmas. Plasmas with densities exceeding $10^{23} cm^{-3}$ were considered which were produced recently by high intense laser pulses. For such plasma conditions we calculated the relaxation of the electron distribution function. Finally, some features of temporal evolution of a beam target system were investigated.

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