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Plasmons and Instabilities in Quantum Plasmas

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Abstract

Collective excitations in degenerate one-, two- and three-dimensional plasmas including quantum confinement effects are discussed within the random phase approximation (RPA). The results of the analytic continuation of the retarded dielectric function for these systems are presented.

1. Introduction

The fabrication of low-dimensional semiconductor microstructures (quantum wells, quantum wires, quantum dots, superlattices etc.), renewed the interest in the study of collective excitations in quantum plasmas. Measurements of plasmon spectra can be done with high accuracy, allowing quantitative comparison with the theoretical results. Moreover, compared to plasmas in ionized gases, these semiconductor systems exhibit a number of additional new features which are based on quantum confinement effects. Confining the wavefunctions basically, to a plane (quantum well) or line (quantum wire), both the single-particle as well as the many-particle properties of the plasma are significantly changed [1]. The reduction of dimensionality leads to

- drastic changes of the density of states: the energy dependence changes from $E^{1/2}$ (3D), to E^0 (2D) to $E^{-1/2}$ (1D);
- reduction of the pair continuum;
- essential lowering of carrier scattering rates;
- strongly reduced screening;
- increased binding energies of bound states (excitons etc.);

- weakening of the wavevector dependence of the Coulomb potential from q^{-2} (3D), to q^{-1} (2D) to $ln\, q$ (1D,) [2].

All these modifications provide a strong motivation to investigate how the reduction of the dimensionality and the increased quantum confinement affect the properties of collective excitations. In this contribution we analyze collective plasma excitations in low dimensional systems. For better comparison with other quantum plasmas, we neglect band structure effects and we consider only longitudinal intrasubband excitations. The dielectric function is calculated within the random phase approximation.

2. Analytic Continuation of the Dielectric Function

The longitudinal plasmon spectrum follows from the zeroes of the dielectric function. It is common to solve this dispersion relation approximately, using a small damping expansion. According to this scheme one solves $Re\, \epsilon(q,\omega) = 0$ with ω being the real part of the complex frequency for zero imaginary part. This is correct for undamped plasmons (e.g. optical plasmon at $T = 0$). However, this approximate solution scheme can lead to essential errors, e.g. for the damping rates in a plasma at elevated temperatures. In many cases damped plasmons exist although $Re\, \epsilon(q,\omega)$ has no zeroes with $Im\omega = 0$. Moreover, in the nonequilibrium situation, the generation of undamped or unstable modes and the splitting of modes cannot be described correctly.

In order to obtain a more general analysis we consider the analytic continuation of the dielectric function into the complex frequency plane:

$$\epsilon(q,\omega,\gamma) = 1 - \sum_a V_{aa}(q)\Pi_a(q,\omega,\gamma) \tag{1}$$

where Π is the polarization function and the plasmon damping is $\gamma = -Im\, \omega$. The analytic continuation of the dielectric function for (effectively) one-dimensional plasmas was accomplished by Landau by appropriate deformation of the integration contour in the expression for the Vlasov dielectric function. This method was applied to 1D quantum plasmas in [5,7]. However, in order to find the analytic continuation for cylindrically and spherically symmetric plasmas as well as for other symmetries, it is more convenient to use the general spectral representation for the polarization function ($z = \omega - i\,\gamma$) [3]

$$\Pi(q,z) = \int \frac{d\omega}{2\pi} \frac{\hat{\Pi}(q,\omega)}{z-\omega}. \tag{2}$$

Here, $\Pi(q,z)$ defines two functions $\Pi^{\gtrless}$ which are analytic in the upper and lower z−half-plane, respectively. For $\Pi^{\gtrless}$ exist well-known diagrammatic expansions. The spectral function $\hat{\Pi}(q,\omega)$ is given by the discontinuity at the branch cut along the axis $Im\, \omega = 0$:

$$\hat{\Pi}(q,\omega) = \Pi^{>}(q,\omega + i\,\epsilon) - \Pi^{<}(q,\omega - i\,\epsilon). \tag{3}$$

In the RPA and the quasiparticle picture Eq.(2) yields the analytic continuation of the Lindhard polarization [3], and Eq.(3) gives [4] ($\hbar = 1$)

$$\hat{\Pi}(q,\omega) = 2\int \frac{dp}{(2\pi)^3} \quad \{f[E(p)] - f[E(p+\omega]\} \\ \times \quad 2\pi\delta[\omega + E(p) - E(p+q)] \tag{4}$$

The analytic continuation of Eq.(4) allows to continue the dielectric function.

3. Results for 1D, 2D and 3D quantum plasmas

The differences between 1D, 2D and 3D plasmas arise from the different angle dependencies in the Lindhard polarization and in the integral of Eq. (4). The result for the analytic continuation of the polarization is

$$\Pi(q,\omega,\gamma) = \begin{cases} \frac{m}{2q}\int_0^\infty \frac{dk}{\pi} f(k) \left\{ \frac{1}{p_- - k} - \frac{1}{p_+ - k} + \frac{1}{p_- + k} - \frac{1}{p_+ + k} \right\}, & 1D, \\ \frac{m}{2q}\int_0^\infty \frac{dk}{\pi} k\, f(k) \left\{ \frac{sgn[\omega - E(k)]}{\sqrt{p_-^2 - k^2}} - \frac{sgn[\omega + E(k)]}{\sqrt{p_+^2 - k^2}} \right\} , & 2D, \\ \frac{m}{2q}\int_0^\infty \frac{dk}{2\pi^2} k\, f(k) \left\{ ln\frac{p_- + k}{p_- - k} - ln\frac{p_+ + k}{p_+ - k} \right\} , & 3D, \end{cases} \tag{5}$$

where we defined $p_\pm = \frac{m}{2q}[\omega - i\gamma \pm E(k)]$ and $E(k) = \frac{k^2}{m}$, (m is given in units of the reduced mass).

The corresponding result for the spectral function is

$$\hat{\Pi}(q,\omega) = \begin{cases} 2i\frac{m}{q}\left\{ f(p_-^{(0)}) - f(p_+^{(0)}) \right\} & , 1D, \\ 2i\frac{m}{q}\int_{p_-^{(0)}}^\infty dp\, p \frac{f(p^2/m) - f(p^2/m + \omega)}{\sqrt{p^2 - p_-^{(0)2}}} & , 2D, \\ 2i\frac{m}{2q}\int_{p_-^{(0)}}^\infty dp\, p \left\{ f(p^2/m) - f(p^2/m + \omega) \right\}, & 3D, \end{cases} \tag{6}$$

with $p_\pm^0 = \frac{m}{2q}|\omega - E(k)|$.

Together with Eq.(1) Eqs.(5 and 6) define the retarded dielectric function on the whole complex frequency plane. The results are valid for arbitrary distribution functions, including those for nonequilibrium states. For several classes of distribution functions (including the case $T = 0$) analytic results can be given. Furthermore, known results can be generalized to the case of arbitrary finite damping. A detailed discussion of the properties of the retarded dielectric function in the complex frequency plane and the corresponding plasmon dispersion and damping/growth rates is presented elsewhere [7,8].

Here, we only summarize only the basic properties of collective excitations in 1D, 2D and 3D quantum plasmas.

- The optical plasmon in two and three-dimensional plasmas vanishes when it enters the pair continuum. In 1D it exists for arbitrary wavenumbers.
- In contradiction to 3D the frequency of the optical plasmon in one- and two-dimensional systems has a vanishing long wavelength limit.
- The analytic continuation of the retarded dielectric function has a second complex zero at the upper edge of the pair continuum [5,7,8].
- The second (acoustic) mode in a two-component plasma at $T = 0$ is always damped in 2D and 3D, but can be undamped in 1D (in a certain interval of wavenumbers) [5,7].
- Plasmon instabilities are not possible in 3D sytems (like in classical plasmas), but they are possible in 2D [6] and, even stronger, in 1D systems [5,6].
- Quantum confinement leads to a decrease of plasmon damping or growth, reducing the energy transfer between fast carriers and plasma oscillations.
- Variation of the sample size modifies the Coulomb potential and thus allows to change the growth rates and the instability conditions [5].

References

[1] H. Haug and S.W. Koch, *"Quantum Theory of the Optical and Electronic Properties of Semiconductors"* (2nd Ed.), World Scientific Publishing Co. 1993

[2] This is only the small q limit, a better approximation is $V(q) \sim K_0(qd)$, where K_0 is the Bessel function of second kind and d the wire width.

[3] W.D. Kraeft, D. Kremp, W. Ebeling and G. Röpke, *"Quantum Statistics of charged particle systems"*, Akademie-Verlag, Berlin, 1986

[4] Th. Bornath, thesis, Universität Rostock 1987

[5] M. Bonitz, R. Binder, and S.W. Koch, Phys. Rev. Lett. **70** (1993) 3788

[6] P. Bakshi, J. Cen and K. Kempa, J. Appl. Phys. **64** (1988) 2243, Sol. State Comm. **76** (1990) 835

[7] M. Bonitz, R. Binder, D.C. Scott, S.W. Koch, and D. Kremp, submitted to Phys. Rev. E

[8] M. Bonitz, R.Binder, D.C. Scott, and S.W. Koch, Intern. J. Mod. Phys. (invited review), 1994