NONPERTUBATIVE KINETICS OF COHERENT ELECTRON-HOLE EXCITATIONS IN A STRONG LASER FIELD

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Abstract

A nonperturbative kinetic description of interband tunneling under the action of a strong electric field (dynamical analogue of the Zener effect) is presented. The developed approach is based on the similarity to the Sauter-Schwinger effect and its dynamical analogue in QED. The kinetic equation for electron-hole quasiparticle excitations is derived on a dynamical basis in the framework of the oscillator representation. Numerical estimates are made for some simple cases of external fields. Detailed comparisons with the method based on the Bloch equations for electron-hole systems interacting with a time dependent electric field are performed. The proposed approach is an alternative to the Bloch theory and does not use the dipole approximation. This leads to predictions which differ from the ones based on the Bloch equations, in particular, concerning the frequency dependence of the absorption coefficient.

1 Introduction

Field ionization of atoms and molecules and inter-band transitions in solid state systems were considered initially as a peculiar tunnel effect [1]. The next important step was done by Keldysh [2], who showed that a more general approach exists, where the tunneling mechanism is realized only in the asymptotic domain of $\gamma \ll 1$ (corresponding to a high electric field strength E_0 and low characteristic frequency ν) where the adiabatic parameter (Keldysh parameter) is defined as

$$\gamma = \frac{\nu\sqrt{m\Delta}}{|e|E_0}. \tag{1}$$

Here, Δ is the energy gap in case of a semiconductor (replacing the binding energy in an atomic system), and m is the effective mass of the carriers. The other asymptotic domain $\gamma \gg 1$ (rather small field strength and high frequency) corresponds to multi-photon transitions. In the intermediate region both mechanisms are active. In the following years, Keldysh's approach was developed intensively in several different directions, we mention the Perelomov-Popov-Terentiev model (see review [3]), the Keldysh-Faisal-Reiss approximation [4], the strong field approximation [5] and so on. The general feature of these approaches is the employment of a nonperturbative energy spectrum of the system (atom, molecule or crystal) with a treatment of the interaction with an electromagnetic field in the dipole approximation on the basis of given electron Bloch wave functions, see e.g. [16, 17]. That leads to the necessity of introduction of various kinds of corrections in the basic structure of the theory (e.g. the dynamic Stark effect).

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On the other hand, at the same time, the tunneling theory of vacuum electron-positron pair creation under action of a constant electric field was constructed [6] in the framework of QED and the Dirac model of physical vacuum. The solution of this problem was obtained in Schwinger's work [7]. The transition to fast alternating fields required more refined methods based on time-dependent Bogoliubov transformations [8, 9] or its modifications (e.g., the method based on the holomorphic representation [10]). Thus, one can speak about a dynamical analogue of the Schwinger mechanism. In the case of a harmonic electric field, one can introduce here a corresponding adiabatic parameter. Its limiting values as before correspond to different mechanisms of vacuum pair creation dominated by tunneling or multiphoton effects.

The fundamental analogy between the Zener mechanism of interband tunneling [11] and Schwinger's mechanism of vacuum electron-positron pair creation in a strong electric field has been mentioned repeatedly, see, e.g., [12]. This similarity has been used recently for the investigation of the transport properties of strongly correlated quantum many-body systems [13, 14, 15].

In this work, we develop a nonperturbative kinetic theory of interband electron-hole (e - h) excitations in dielectrics under the action of a strong electric laser field using the direct analogy to the QED formalism of vacuum pair creation. The proposed method allows us to take into account on a nonperturbative basis both, tunnel and multiphoton processes, accompanying excitation by an arbitrary time-dependent electric field. Conditionally, one can speak about the dynamical Zener effect by analogy with the dynamical Schwinger effect in QED.

The present work is organized as follows. The definition of the mixed e-h states is an essential distinctive element of our approach. We are employing the rich experience of relativistic QED in describing the vacuum particle creation under the action of some external quasi-classical field (the Schwinger mechanism [7, 8, 9]) and also use some naive analogy between the Dirac picture of electron-positron vacuum and some concepts of solid state band theory as a foundation of our approach. In the framework of the two-band model this leads to a second-order (with respect to time) equation of motion for the single wave function describing both (electron and hole) states (Sect. 2). For simplicity, electron and hole dispersions are assumed to be the same. For a non-interacting e-h system we construct the Lagrange and Hamilton formalisms and define relevant decompositions of the field functions and canonical momentum with respect to the plane waves corresponding to electron and hole. Thus the formalism represents a special form of e-h representation as a coherent e-h pair state. The interaction with an external quasi-classical time-dependent homogeneous electric field is introduced in Sect. 3 as the result of the standard substitution $\mathbf{p} \to \mathbf{P} = \mathbf{p} - e\mathbf{A}(t)$. It is well known that the introduction of the external electromagnetic field in QED leads to a non-diagonal form of the commutative operators corresponding to physical observables, what makes the physical interpretation of the formalism difficult [8]. The transition to a quasiparticle (QP) representation is achieved by diagonalization of all commutative operators relevant to the complete QP characteristics of the system (e.g., energy, spin, charge). In practice, only the Hamiltonian diagonalization is often employed. Usually, the transition to the QP representation is done by a time-dependent Bogoliubov transformation (e.g., [8]). The holomorphic (oscillator) representation developed during recent years [10] is a more effective tool for this goal compared to the Bogoliubov technique because it easily allows to obtain the diagonal Hamiltonian and to derive the Heisenberg-like equations of motion for creation and annihilation operators, where it also takes into account the mixing of the states with positive and negative energies. These equations provide the basis for a

nonperturbative derivation of the kinetic equation (KE) describing the e-h pair creation and annihilation in the presence of an external electric field (Sect. 4). Excitation of e-h currents leads to generation of an internal electric field, which takes part, in its turn, in the e-h creation process. The corresponding Maxwell equation jointly with the KE put together the complete equation system of the back reaction problem. The mathematical structure of this KE is similar to the corresponding KE in QED [8, 18], on the one hand, and the Bloch equations in the solid state physics [19, 16, 17], on the other. The lines of similarity and distinction are discussed in detail in Sect. 5. The suggested formalism is alternative to the Bloch approach and is not based on the dipole model of an electron interaction with an electromagnetic field. Some features of this process and dependence on different characteristics of the non-stationary electric field are investigated on the basis of the obtained KE in Sect. 6. We show here in particular, that the e-h excitations in the case of rather strong external field acquire a behavior characteristic for the Zener breakthrough. Sect. 7 is dealing with the linear response of the electron-hole system (optical absorption spectrum and conductivity) to a weak external electric field. Here we observe certain differences with the results from the Bloch theory. The Sect. 8 contains a short summary.

The developed formalism reveals a close similarity to the corresponding QED kinetics of vacuum electron-positron plasma created under the action of a strong electromagnetic field [18]. The experimental prove of this effect is however difficult [20]. Therefore the study of similar effects in the solid state plasma could be useful for simulation and prediction of the corresponding effects in strong laser fields. We limit ourselves below to the collisionless approximation and neglect the interaction between different charge carriers.

We use the units $\hbar = c = 1$ throughout the paper.

2 Description of electron-hole pair states

Let us consider a two-band system with a completely filled lower band and the mirrored electron states in the v- and c-zones and dispersions, $\varepsilon_c = \Delta/2 + \varepsilon(\mathbf{p})$, $\varepsilon_v = -\Delta/2 - \varepsilon(\mathbf{p})$. It is assumed that the energy gap Δ is space homogeneous and stationary. The dispersion law $\epsilon(\mathbf{p})$ is fixed on the phenomenological level. We disregard all Coulomb effects and treat electrons and holes as quasi-free particles. Two Schrödinger equations can be associated with these dispersions:

$$(\hat{E} - \hat{H})\psi = 0, \tag{2}$$

where $\hat{E} = i\partial/\partial t$ and

$$\psi = \begin{Vmatrix} \psi_e \\ \psi_h \end{Vmatrix}, \qquad \hat{H} = \begin{Vmatrix} \Delta/2 + \varepsilon(\hat{\mathbf{p}}) & 0 \\ 0 & -\Delta/2 - \varepsilon(\hat{\mathbf{p}}) \end{Vmatrix} \tag{3}$$

with $\hat{\mathbf{p}} = -i\nabla$. In general case for arbitrary dispersions, this leads to independent description of electron and holes. This more complicated case is in need of separate analysis. In this section it is assumed that the external electromagnetic fields and interparticle interaction are absent.

On the other hand, we can consider a hole as an antiparticle to the electron. Their states are correlated and allow for a joint description in analogy with QED. This results in the second order dispersion

$$(E - \varepsilon_e)(E - \varepsilon_h) = [E - \Delta/2 - \varepsilon(\mathbf{p})][E + \Delta/2 + \varepsilon(\mathbf{p})] = 0 \tag{4}$$

(the analogous determinant equation is used in the theory of the stationary Zener effect [21]). The corresponding equation of motion for the total wave function which is second order with respect to time,

$$\{\hat{E}^2 - [\Delta/2 + \varepsilon(\hat{\mathbf{p}})]^2\}\Psi = 0, \tag{5}$$

where $\Psi(\mathbf{x}, t)$ is now the one-component wave function. Transition from Eq.(2) to Eq.(5) is very important for our aim.

Our next goal is to develop the Lagrange and Hamilton formalisms for this equation Eq.(5).

For simplicity, we use here and below the quadratic isotropic dispersion law, $\varepsilon(\hat{\mathbf{p}}) = \hat{\mathbf{p}}^2/2m$. We obtain then ($\Psi_{,k} = \partial\Psi/\partial x_k$, $k = 1, 2, 3$)

$$L[\Psi] = \alpha \left\{ \dot{\Psi}^*\dot{\Psi} - \frac{\Delta}{2m}\Psi^*_{,k}\Psi_{,k} - \frac{1}{4m^2}\Psi^*_{,kl}\Psi_{,kl} - \frac{\Delta^2}{4}\Psi^*\Psi \right\}, \tag{6}$$

where the dimensional constant α will be determined below. Let us take advantage of the standard definitions of the canonical momentum

$$\pi = \frac{\partial L[\Psi]}{\partial \dot{\Psi}}, \qquad \pi^* = \frac{\partial L[\Psi]}{\partial \dot{\Psi}^*} \tag{7}$$

and the Hamiltonian density

$$H = \pi\dot{\Psi} + \pi^*\dot{\Psi}^* - L[\Psi]. \tag{8}$$

Using Eq. (6), we obtain $\pi = \alpha\,\dot{\Psi}^*$ and

$$H = \frac{1}{\alpha}\pi\pi^* + \frac{\alpha}{4}\Delta^2\Psi^*\Psi + \alpha\left[\frac{\Delta}{2m}\Psi^*_{,k}\Psi_{,k} + \frac{1}{4m^2}\Psi^*_{,kl}\Psi_{,kl}\right], \tag{9}$$

so that the Hamiltonian density contains also higher-order space derivatives.

As in QFT, the wave function Ψ now loses the meaning of the state amplitude. The corresponding charge and current densities are (k=1,2,3)

$$\rho = i\alpha e(\Psi^*\dot{\Psi} - \dot{\Psi}^*\Psi) = ie(\Psi^*\pi^* - \pi\Psi), \tag{10}$$

$$j_k = ie\alpha\left\{\frac{\Delta}{2m}\left[(\Psi^*_{,k}\Psi - \Psi^*\Psi_{,k}\right] - \frac{1}{4m^2}\left[(\Psi^*\Psi_{,kll} - \Psi^*_{,kll}\Psi + \Psi^*_{,kl}\Psi_{,l} - \Psi^*_{,l}\Psi_{,kl}\right]\right\}, \tag{11}$$

where e is the electron charge. On this stage, parameter α can be fixed as $\alpha = \Delta^{-1}$ proceeding from the specific correspondence principle: at $\Delta \to \infty$ the derived relations must turn into the ordinary quantum mechanical analogies (the parameter $\Delta/2$ plays the role of the rest mass in the Einstein relation for energy and momentum; in order to neglect the state with negative energy, one can use the wave function transformation $\Psi = \Phi e^{-i\Delta t/2}$.

The transition to the momentum representation can be done in analogy with QFT. Let us carry out the decomposition of wave function $\Psi(\mathbf{x}, t)$ into a plane wave basis (below $\mathbf{p}$ is quasi-momentum, $p_n = (2\pi/L)n_k, n_k = 0, 1, 2, ...$, vector $\mathbf{p}$ belongs to the first Brilluin zone, the crystal volume is $V = L^3$)

$$\Psi(\mathbf{x}, t) = \frac{1}{\sqrt{V}}\sum_{\mathbf{p}}\int dE\,\tilde{\Psi}(E, \mathbf{p})e^{-iEt + i\mathbf{px}}, \tag{12}$$

and take into account the dispersion equation (4) in order to select the two energy bands

$$\tilde{\Psi}(E,\mathbf{p}) = \delta\{[E-\varepsilon(\mathbf{p})-\Delta/2][E+\varepsilon(\mathbf{p})+\Delta/2]\}\psi(E,\mathbf{p}). \tag{13}$$

Using the textbook relation

$$\delta[\phi(x)] = \sum_i \{|\phi'(x_i)|\}^{-1}\delta(x-x_i), \quad \phi(x_i)=0 \tag{14}$$

the decomposition (12) can be written in the form typical for a two-band model (the electron state is a superposition of c and v band contributions)

$$\Psi(\mathbf{x},t) = \frac{1}{\sqrt{V}}\sum_{\mathbf{p}}\sqrt{\frac{\Delta}{\Delta+2\varepsilon(\mathbf{p})}}\left\{a_c(\mathbf{p})e^{-i[\Delta/2+\varepsilon(\mathbf{p})]t} + a_v(-\mathbf{p})e^{i[\Delta/2+\varepsilon(\mathbf{p})]t}\right\}e^{i\mathbf{px}}, \tag{15}$$

where $a_c(\mathbf{p})$ and $a_v(-\mathbf{p})$ are the electron and hole amplitudes in momentum representation. In Eq.(15) we redefined the amplitudes $[\Delta(\Delta+2\varepsilon)]^{-1/2}a_{c,v} \to a_{c,v}$ in order to guarantee the correct physical interpretation of the observable quantities. The canonical momentum in the same representation (15) has the form

$$\pi(\mathbf{x},t) = \frac{i}{2}\frac{1}{\sqrt{V}}\sum_{\mathbf{p}}\sqrt{\frac{\Delta+2\varepsilon(\mathbf{p})}{\Delta}}\left\{a_c^{\dagger}(\mathbf{p})e^{i[\Delta/2+\varepsilon(\mathbf{p})]t} - a_v^{\dagger}(-\mathbf{p})e^{-i[\Delta/2+\varepsilon(\mathbf{p})]t}\right\}e^{-i\mathbf{px}}. \tag{16}$$

The transition to the second quantization representation is based on the standard replacement of the functions $a^{+}_{c,v}(a_{c,v})$ by the corresponding creation (annihilation) operators (below we omit the symbols $\hat{}$ everywhere).

The structures of the expansions (15) and (16) are defined by compatibility with the canonical anti-commutation relations

$$\{a_{c,v}(\mathbf{p}), a^{\dagger}_{c,v}(\mathbf{q})\} = \delta_{\mathbf{p},\mathbf{q}} \tag{17}$$

and so on.

Substituting Eqs.(15) and (16) in the Eqs.(9)-(11), we obtain the total Hamiltonian and charge in the diagonal form (the quasiparticle representation [8])

$$H_{tot} = \frac{2}{\sqrt{V}}\sum_{\mathbf{p}}[\varepsilon(\mathbf{p})+\Delta/2]\left\{a_c^{\dagger}(\mathbf{p})a_c(\mathbf{p}) + a_v^{\dagger}(-\mathbf{p})a_v(-\mathbf{p})\right\}, \tag{18}$$

$$Q = e\frac{2}{\sqrt{V}}\sum_{\mathbf{p}}\left\{a_c^{\dagger}(\mathbf{p})a_c(\mathbf{p}) - a_v^{\dagger}(-\mathbf{p})a_v(-\mathbf{p})\right\}, \tag{19}$$

where the factor 2 is included here to account for the degeneracy with respect to the spin degrees of freedom. The present here energy degeneracy of the c-v- states is eliminated in the case $\epsilon_c \neq \epsilon_v$ (e.g., $m_e \neq m_h$). Thus, the decompositions (15), (16) and the quantization rules (17) lead to the well-known operator structure of macroscopic physical quantities.

3 Quasiparticle representation in an external electric field

The presented formalism allows to straightforward include a time- dependent electric field of arbitrary frequency and amplitude. Interaction with a quasiclassical electromagnetic field in the original coordinate representation is introduced by the substitution

$\partial_\mu \to D_\mu = \partial_\mu + ieA_\mu$ (μ=0,1,2,3), where $A^{ex}_\mu + A^{in}_\mu$ is 4-potential of external and internal field and e is the electron charge with its sign. We will restrict ourselves below to the case of a nonstationary space-homogeneous electric field with 4-potential in the Hamilton gauge, $A_\mu = (0, A_1(t), A_2(t), A_3(t))$ and then $\hat{\mathbf{p}} \to \hat{\mathbf{P}} = \hat{\mathbf{p}} + e\mathbf{A}$. The usual substitution into the Hamiltonian (9) leads to a non-diagonal form of the decompositions (15), (16) in the momentum representation. The adequate interpretation of the formalism is achieved by transition to quasiparticle representation, in which all observable operators have the diagonal form. Usually, the Bogoliubov method of time-dependent canonical transformations is used [8, 9]. We will use below an economical method based on the holomorphic (oscillator) representation [23], that was developed in work [10] for the problem of the relativistic kinetics of vacuum pair creation in strong electromagnetic field.

In accordance with the method of work [10], it is necessary to make the substitution $\mathbf{p} \to \mathbf{P}$ in the dispersion law occurring in the decomposition (15) and (16) for the free field wave function and canonical momentum and also to introduce new time dependent amplitudes (or operators) $a_{c,v}(\mathbf{p}, t)$ by the replacement

$$a_{c,v}(\mathbf{p}) \exp\left[-i\varepsilon(\mathbf{p})t\right] \to a_{c,v}(\mathbf{p}, t) \tag{20}$$

and so on. The result is the following:

$$\Psi(\mathbf{x}, t) = \frac{1}{\sqrt{V}} \sum_{\mathbf{p}} \sqrt{\frac{\Delta}{\Delta + 2\varepsilon(\mathbf{P})}} \left\{a_c(\mathbf{p}, t) + a_v(-\mathbf{p}, t)\right\} e^{i\mathbf{px}}, \tag{21}$$

$$\pi(\mathbf{x}, t) = \frac{i}{2\sqrt{V}} \sum_{\mathbf{p}} \sqrt{\frac{\Delta + 2\varepsilon(\mathbf{P})}{\Delta}} \left\{a^\dagger_c(\mathbf{p}, t) - a^\dagger_v(-\mathbf{p}, t)\right\} e^{-i\mathbf{px}}. \tag{22}$$

Here it is assumed, that the dispersion law $\epsilon(\mathbf{p})$ is not changed under action of the external field besides the trivial displacement $\mathbf{p} \to \mathbf{P}$, i.e. the magnitude of the gap and the band boundaries remain invariable. Let us remark, that the Eqs. (21) and (22) allow the obvious generalization to the multiband model case. The total Hamiltonian H_{tot} can be obtained now from the free Hamiltonian (9) by the replacement $\partial \to D_k$ (k=1,2,3). The subsequent substitution Eqs. (21), (22) leads at once to the diagonal form of the Hamiltonian in QP representation

$$H_{tot}(t) = \frac{2}{\sqrt{V}} \sum_{\mathbf{p}} \left[\varepsilon(\mathbf{P}) + \Delta/2\right] \left\{a^\dagger_c(\mathbf{p}, t) a_c(\mathbf{p}, t) + a^\dagger_v(-\mathbf{p}, t) a_v(-\mathbf{p}, t)\right\}. \tag{23}$$

The new time-dependent amplitudes $a_{e,h}(\mathbf{p}, t)$ obey the exact equations of motion, which can be obtained either from the minimal action principle or using the Hamilton equations

$$\dot{\Psi} = \frac{\delta H_{tot}}{\delta \pi(\mathbf{x}, t)}, \qquad \dot{\pi} = -\frac{\delta H_{tot}}{\delta \Psi(\mathbf{x}, t)}. \tag{24}$$

Following [10], let us rewrite the action

$$S = \int d\mathbf{x} \left\{\pi(\mathbf{x}, t)\dot{\Psi}(\mathbf{x}, t) + \dot{\Psi}^*(\mathbf{x}, t)\pi^*(\mathbf{x}, t) - H_{tot}(\mathbf{x}, t)\right\} \tag{25}$$

in the QP representation with the decompositions (21), (22)

$$S = \int dt \sum_{\mathbf{p}} \Big\{ \frac{i}{2} \big([a_c^\dagger(\mathbf{p},t) - a_v^\dagger(-\mathbf{p},t)][\dot{a}_c(\mathbf{p},t) + \dot{a}_v(-\mathbf{p},t)] \\ - [a_c(\mathbf{p},t) - a_v(-\mathbf{p},t)][\dot{a}_c^\dagger(\mathbf{p},t) + \dot{a}_v^\dagger(-\mathbf{p},t)] \\ + \lambda(\mathbf{p},t)[a_v^\dagger(-\mathbf{p},t)a_c(\mathbf{p},t) - a_c^\dagger(\mathbf{p},t)a_v(-\mathbf{p},t)]\big) - H_{tot}(\mathbf{p},t)\Big\}, \quad (26)$$

where $H_{tot}(\mathbf{x},t)$ and $H_{tot}(\mathbf{p},t)$ are the Hamiltonian densities in the x- and p-representations, $\lambda(\mathbf{p},t)$ is the amplitude of interband transitions,

$$\lambda(\mathbf{p},t) = \frac{\dot{\varepsilon}(\mathbf{P})}{\Delta + 2\varepsilon(\mathbf{P})} = -\frac{e\mathbf{P}\mathbf{E}(t)}{m[\Delta + 2\varepsilon(\mathbf{P})]}, \quad (27)$$

where $E(t) = -\dot{A}(t)$ is the electric field strength. Then the operator equations of motion follow from here after variation with respect to the amplitudes and subsequent transition to the occupation number representation with the anti-commutation relations

$$\{a_{c,v}(\mathbf{p},t), a_{c,v}^\dagger(\mathbf{q},t)\} = \delta_{\mathbf{pq}} \quad (28)$$

(the remaining elementary anti-commutators equal zero). The Heisenberg-like equations of motion are the following:

$$\dot{a}_c(\mathbf{p},t) = \lambda(\mathbf{p},t)a_v(-\mathbf{p},t) + i\big[H_{tot}(t), a_c(\mathbf{p},t)\big], \\ \dot{a}_v(\mathbf{p},t) = \lambda(\mathbf{p},t)a_c(-\mathbf{p},t) + i\big[H_{tot}(t), a_v(\mathbf{p},t)\big]. \quad (29)$$

The first terms on r.h.s. of these equations describe the coupling of states related to different energy bands. Hence, the parameter (27) is the interband transition amplitude.

It is very important, that the Hamiltonian and the total charge operator have the diagonal form in this representation. Thus, the oscillator representation is simultaneously a quasiparticle one.

It is assumed that the electric field is switched off in the in - and out-states and the quasiparticle excitations become "free"and available for direct observation. In addition, it is also supposed here, that the system is found in the ground state at the initial moment $t_0 \to -\infty$ and, hence, the initial state is the vacuum state $|0>$ of electron and hole quasiparticles. This state is not equal to the out-state, where some quantity of electrons and holes can remain after switch off of the electric field.

4 Kinetic equation and observables

The basic object of the kinetic theory in the presence of an external strong field is the quasiparticle distribution function, which is defined on the in-vacuum state $|0>$ in the present situation. In the case of a space-homogeneous system, considered here, the quasiparticle distribution functions of electrons and holes are:

$$f_{c,\,v}(\mathbf{p},t) = <0|a_{c,v}^\dagger(\mathbf{p},t)a_{c,v}(\mathbf{p},t)|0>. \quad (30)$$

The general vacuum state here is product of the electron vacuum states of the valence and conduction bands. Differentiating the functions (30) with respect to time, and using the equations of motion (29) we obtain:

$$\dot{f}_{c,\,v}(\mathbf{p},t) = \lambda(\mathbf{p},t)\{f_{c,\,v}(\mathbf{p},t) + f_{v,\,c}(\mathbf{p},t)\}, \quad (31)$$

where the auxiliary correlation functions (the interband polarization functions) are introduced

$$f_{cv}(\mathbf{p},t) = <0|a_c^\dagger(\mathbf{p},t)a_v(\mathbf{p},t)|0>,$$
$$f_{vc}(\mathbf{p},t) = <0|a_v^\dagger(\mathbf{p},t)a_c(\mathbf{p},t)|0> . \tag{32}$$

The equation of motion for the functions (32) can be obtained by analogy with the equation (31). We can write it down in the integral form

$$f_{cv}^{(\pm)}(\mathbf{p},t) = \int_{-\infty}^{t} dt' \lambda(\mathbf{p},t')[1 - f_e(\mathbf{p},t') - f_h(-\mathbf{p},t')]e^{\pm 2i\theta(\mathbf{p},t,t')}, \tag{33}$$

where now $f_c = f_e$ and $f_v = 1 - f_h$, f_e and f_h are the electron and hole distribution functions. In Eq.(33) the following initial conditions have been introduced,

$$\lim_{t\to-\infty} f_{cv}^{(\pm)}(\mathbf{p},t) = 0. \tag{34}$$

The analogous requirement holds for the distribution functions (30) (absence of electrons and holes in the initial state). In Eq.(33), the dynamical phase

$$\theta(\mathbf{p},t,t') = \int_{t'}^{t} d\tau \ [\,\varepsilon(\mathbf{P}(\tau) + \Delta/2\,] \tag{35}$$

corresponds to the quantum 'beating' of the interband transition. In Eq.(33) it was taken into consideration also, that the system is electro-neutral at each moment, i.e.,

$$f_e(\mathbf{p},t) = f_h(\mathbf{p},t) = f(\mathbf{p},t). \tag{36}$$

According to that, we skip the indexes of the distribution functions below. The condition (36) leads to transformation of the band-filling factor $1 - f_e - f_h \to 1 - 2f$.

The resulting closed form of the KE now follows from Eqs. (31) and (33)

$$\dot{f}(\mathbf{p},t) = 2\lambda(\mathbf{p},t) \int_{-\infty}^{t} dt' \lambda(\mathbf{p},t')[1 - 2f(\mathbf{p},t')] \cos 2\theta(\mathbf{p},t,t'). \tag{37}$$

This equation is equivalent to an integral equation of the Volterra type. The right-hand side of the KE (37) represents the source of creation and annihilation of electron-hole pairs and has the same form as in QED [18] (with an essential difference in construction of the amplitude $\lambda(\mathbf{p},t)$), where the corresponding KE describe vacuum creation of electron-positron pairs. There is an other essential difference from QED kinetics, where m is the unique mass parameter: in the present model there are two such parameters, m and Δ. That leads to some specific behavior of the e-h-system (see Sect. 6). In more realistic models must become apparent other details of energy spectrums of electrons and holes. Thus, this non-Markovian KE is a non-perturbative result in the mean-field approximation (see below Eq.(53)), within a quasiclassical treatment of the external and internal electric fields. KE (37) can be rewritten in the evident gauge invariant form if we make the change of variables $\mathbf{p} \to \mathbf{P}$ in the distribution functions $f(\mathbf{p},t) \to f(\mathbf{P},t)$.

As can be seen from the structure of KE (37) and amplitude (27), the interband transitions are caused by carrier acceleration in an electric field on the background of

the changes of their energy (the denominator in Eq.(27)) and quantum beating with the alternating phase (35).

The KE (37) can be transformed to a system of ordinary differential equations, which is convenient for numerical analysis

$$\dot{f} = \lambda u, \qquad \dot{u} = 2\lambda(1-2f) - (2\varepsilon + \Delta)v, \qquad \dot{v} = (2\varepsilon + \Delta)u, \tag{38}$$

where $u+iv = 2f^{(+)}$ and $f^{(+)}$ is given by Eq. (33). These equations have the first integral

$$(1-2f)^2 + u^2 + v^2 = 1, \tag{39}$$

according to which the phase trajectories are located on an ellipsoid with top coordinates $f = u = v = 0$ and $f = 1, u = v = 0$. The phase (35) can be represented as the difference of the integrand at the upper and lower bounds,

$$2\theta(\mathbf{p}, t, t') = \Theta(\mathbf{p}, t) - \Theta(\mathbf{p}, t'), \tag{40}$$

where

$$\Theta(\mathbf{p}, t) = (\Delta + 2\varepsilon_0)t + \frac{2e}{m}\int^t d\tau\ \mathbf{pA}(\tau) + \frac{e^2}{m}\int^t d\tau A^2(\tau). \tag{41}$$

The relation (40) leads to the degenerate kernel in the integral part of (37), i.e.

$$\begin{aligned}\dot{f}(t) \quad = \quad & 2\lambda(t)\cos\Theta(t)\int_{t_0}^{t} dt'\lambda(t')[1-2f(t')]\cos\Theta(t') + \\ & 2\lambda(t)\sin\Theta(t)\int_{t_0}^{t} dt'\lambda(t')[1-2f(t')]\sin\Theta(t').\end{aligned} \tag{42}$$

The equivalent system of ordinary differential equations has the following form:

$$\begin{aligned}\dot{f}(t) &= 2\lambda(t)\left\{u(t)\cos\Theta(t) + v(t)\sin\Theta(t)\right\},\\ \dot{u}(t) &= \lambda(t)[1-2f(t)]\cos\Theta(t),\\ \dot{v}(t) &= \lambda(t)[1-2f(t)]\sin\Theta(t).\end{aligned} \tag{43}$$

For brevity, the momentum dependencies are omitted here.

In the low density approximation, $2f \ll 1$, the KE (37) has the closed solution

$$f(\mathbf{p}, t) = \left|\int_{-\infty}^{t} dt'\lambda(\mathbf{p}, t')e^{2i\theta(\mathbf{p},t,t')}\right|^2 = \frac{1}{4}\left\{u^2(\mathbf{p}, t) + v^2(\mathbf{p}, t)\right\}. \tag{44}$$

This result admits a simple generalization for the case of an initial quasiparticle distribution $f_0(\mathbf{p})$

$$\delta f(\mathbf{p}, t) = [1-2f_0(\mathbf{p})]\left|\int_{-\infty}^{t} dt'\lambda(\mathbf{p}, t')e^{2i\theta(\mathbf{p},t,t')}\right|^2, \tag{45}$$

where δf is the non-equilibrium correction, i.e. $f(\mathbf{p}, t) = f_0(\mathbf{p}) + \delta f(\mathbf{p}, t)$. In the same approximation the polarization function is

$$u(\mathbf{p}, t) = 2[1-2f_0(\mathbf{p})]\int_{-\infty}^{t} dt'\lambda(\mathbf{p}, t')\cos 2\theta(\mathbf{p}, t, t'). \tag{46}$$

The high density approximation corresponds to rather high field strength when $f \sim 1$. Let us introduce the deviation $F(\mathbf{p}, t)$ from the distribution function of a completely filled state,

$$f(\mathbf{p}, t) = 1 - F(\mathbf{p}, t), \qquad F(\mathbf{p}, t) \ll 1. \tag{47}$$

The substitution of this relation into KE (37) leads to the new KE for the function $F(\mathbf{p}, t)$, that has the form coinciding with KE (37). As a consequence of this surprising fact, the formula (44) preserves its view in the high density limit also.

In contrast to the vacuum case [6, 7], the critical field is not uniquely defined in the considered two- band model [21, 22]. Below we will use the following estimate for the critical field strength [22]

$$E_c = \frac{\pi \Delta^{3/2} m^{1/2}}{2\sqrt{2} e}. \tag{48}$$

After the distribution functions of electron and hole quasiparticles have been obtained, we can write down the densities of observables by averaging of the Hamilton operator (18), the electron charge (19), the total electron-hole current etc. over the vacuum state. As a result, we have the time dependent densities of total energy $\omega(t)$, carrier number and total current in the form ($[d\mathbf{p}] = (2\pi)^{-3} d^3 p$):

$$\omega(t) = 2 \int [d\mathbf{p}] \, [2\varepsilon(\mathbf{p}, t) + \Delta] \, f(\mathbf{p}, t), \tag{49}$$

$$n(t) = 4 \int [d\mathbf{p}] f(\mathbf{p}, t), \tag{50}$$

$$\mathbf{j}(t) = \frac{4e}{m} \int [d\mathbf{p}] \; \mathbf{p}\{f(\mathbf{p}, t) + u(\mathbf{p}, t)\} = \mathbf{j}_{cond}(t) + \mathbf{j}_{pol}(t), \tag{51}$$

i.e. the total current is the sum of conductivity and polarization currents. The Eqs. (50)-(52) are written in the thermodynamical limit, $V \to \infty$. The integrations here are carried out over the first Brilluin zone, $p_{max} = \pi/a$.

If the electric field is rather large ($|E(t)| \sim E_c$), it is necessary to take into account the induced electric field $\mathbf{E}_{in}(t)$ produced by the electron-hole plasma due to the external field $\mathbf{E}_{ex}(t)$. Thus, the total field is

$$\mathbf{E}_{tot}(t) = \mathbf{E}_{in}(t) + \mathbf{E}_{ex}(t). \tag{52}$$

The induced field $\mathbf{E}_{in}(t)$ obeys the Maxwell equation with the current (51)

$$\dot{\mathbf{E}}_{in}(t) = -4\pi \mathbf{j}. \tag{53}$$

This equation together with the KE (37) (or the system (38)) forms the self-consistent nonlinear system of the back reaction problem.

5 Comparison with the Bloch equations

It is interesting to note that equations (38) have the same structure as the optical Bloch equations in the theory of the coherent regime of the interband transitions for the two-band model of semiconductor systems [19]. By analogy with the Bloch equations, Eqs. (38) allow for the compact vector formulation

$$\dot{\mathbf{U}} = [\mathbf{\Omega U}], \tag{54}$$

where $U_1 = v$, $U_2 = u$, $U_3 = 2f - 1$, $\mathbf{\Omega} = 2\lambda\mathbf{e}_1 - (2\varepsilon + \Delta)\mathbf{e}_3$ and $\vec{e}_i\,(i = 1, 2, 3)$ are Cartesian unit vectors. However, the physical nature of the coefficients of the KE (37) and the Bloch equations is quite different. This is seen in the structure of the amplitude $\lambda(\mathbf{p}, t)$, Eq. (27), and the corresponding coefficients in the Bloch equations (the Rabi energy)

$$\omega_R = d_{eh}E_0, \tag{55}$$

where E_0 is the amplitude of a periodic electric field directed along axis x^3, $\mathbf{A}_{ex} = (0, 0, A_{ex}(t))$,

$$A_{ex}(t) = -(E_0/\nu)\cos\nu t, \qquad E_{ex}(t) = E_0 \sin\nu t, \tag{56}$$

and d_{eh} is the matrix element of the dipole operator calculated with the conduction and valence band wave functions.

In contrast to this mechanism of interband excitations, in the present approach we assume that electron and hole are moving as quasiparticles and the mixing mechanism is provided in this case by overlapping electron and hole states in the presence of an external field. Such mechanism is a consequence of the method of quasiparticle description and diagonalization of the Hamiltonian of the system in the presence of an external field. We underline, that the assumption about existence of coherent e-h states (Sect. 2 and 3) is the most important element of our approach.

This leads to the following important difference of the two approaches: the coefficients in the Bloch equations and one in Eq. (38) have different momentum dependences that can lead to different predictions of the observable quantities (see below Sect. 7).

The most resemblance is achieved in the framework of the rotating wave approximation for a periodic signal (56) [19]. In this case only the resonant terms are kept in the KE (37) (resonant approximation) which leads to the following system of equations (instead of Eqs. (38))

$$\begin{aligned}
\dot{f}_{RA} &= \Lambda u_{RA}, \\
\dot{u}_{RA} &= \frac{1}{2}\Lambda(1 - 2f_{RA}) - (2\varepsilon + \Delta - \nu)v_{RA}, \\
\dot{v}_{RA} &= (2\varepsilon + \Delta - \nu)u_{RA},
\end{aligned} \tag{57}$$

where

$$\Lambda(\mathbf{p}, t) = -\frac{|e|\mathbf{P}\mathbf{E_0}}{m[\Delta + 2\varepsilon(\mathbf{P})]}, \tag{58}$$

i.e. $\lambda = \Lambda \sin\nu t$. In particular, one can speak here about the analogue to Rabi flopping with a momentum and time dependent frequency $|\Lambda(\mathbf{p}, t)|$, see Fig. 1. The solution of (38) at fixed momentum has a quasi-resonant dependence on the frequency of the external field: at $\nu \approx \nu_{res} = \Delta + \mathbf{p}^2/m$, the value of $f(\mathbf{p}, t)$ changes from 0 to 1 with "Rabi frequency"(58). The unit vector $\mathbf{U}$ moves always on the surface of a sphere owing to the existence of the integral of motion (39). The "resonant" case corresponds to rotation of $\mathbf{U}$ in the plane perpendicular to the $\mathbf{e}_1$ axis. At finite detuning, $|\nu - \nu_r|$, the angular motion of the vector $\mathbf{U}$ is limited because of the fast decrease of the amplitude of the oscillations of $f(\mathbf{p}, t)$ that is accompanied by a rapid increase of its frequency. Fig. 1 shows also some nonlinear features of the distribution function evolution characteristic for the exact solution of Eqs. (38)

On the level of the system (57), the unique distinction of the two approaches is displayed in the interband amplitudes (58) and (55). The system (57) can be rewritten

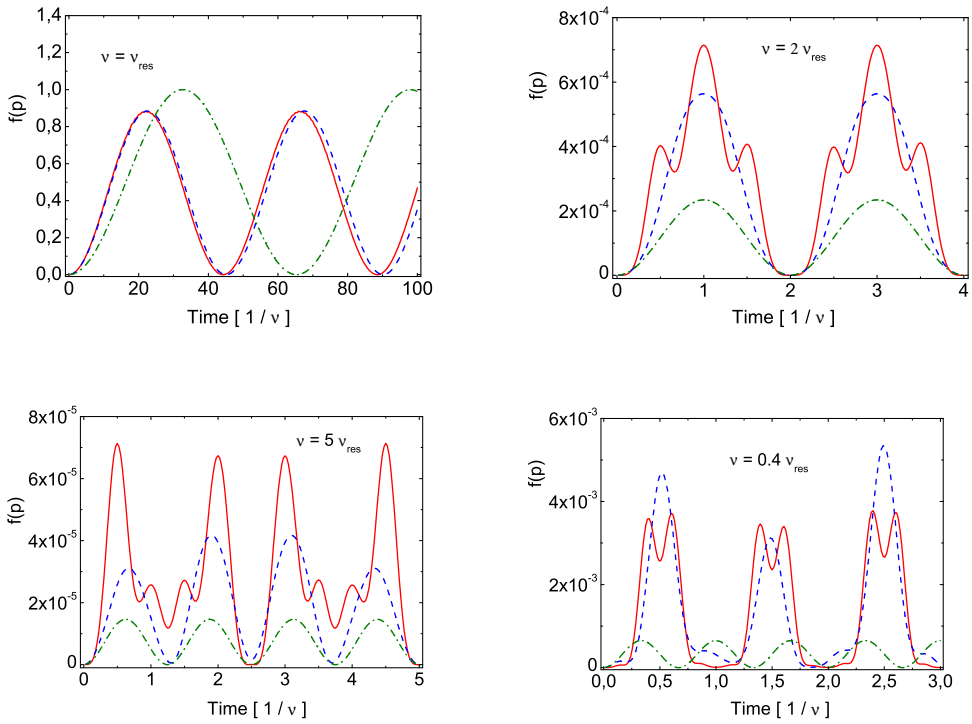

Рис. 1: Time dependence of $f(\mathbf{p},t)$ in a monochromatic field with $E = 10^7$ V/cm, energy gap $\Delta = 1$ eV and effective mass $m = m_e$ in the vicinity of the "resonance" frequency $\nu_{res} = \Delta + \mathbf{p}^2/m$ at $|p| = 8.6 \cdot 10^{-4} m_e$ ($\lambda = 600nm$): solid line – exact solution of Eqs. (38), dashed line – resonant approximation (57) and dashed-dotted line – solution of RWA equations (5.44) of Ref.[19].

in the KE form similar to Eq. (37)

$$\dot{f}_{RA}(\mathbf{p},t) = \frac{1}{2}\,\Lambda(\mathbf{p},t) \int\limits_{-\infty}^{t} dt' \Lambda(\mathbf{p},t')[1 - 2f_{RA}(\mathbf{p},t')] \cos 2\Theta(\mathbf{p};t,t'), \tag{59}$$

where

$$\Theta(\mathbf{p};t,t') = 2\theta(\mathbf{p},t,t') - \nu(t-t'). \tag{60}$$

It is obvious that the transition to the KE (59) in the resonant approximation means the selection of the slowly changing part of the distribution function in the vicinity of the resonant frequency

$$\dot{\Theta} = \Delta + 2\varepsilon(\mathbf{P}) - \nu = 0. \tag{61}$$

This condition depends on time and external field. In order to simplify the situation, one may neglect this dependence, which is achieved by the substitution $P \to p$ and $\varepsilon(\mathbf{P}) \to \varepsilon(\mathbf{p})$ in Eqs. (58) and (61). Of course, this is impossible in the case of a strong field.

For comparison of the efficiency of e-h pair creations in the framework of these two

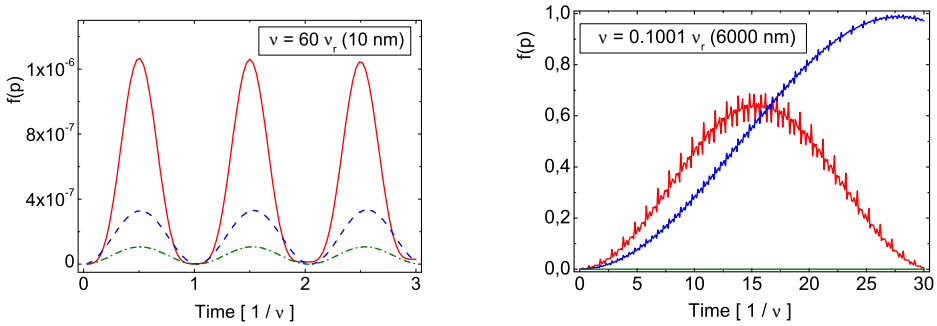

Рис. 2: The same as Fig. 1 but very far from ν_{res}.

approaches, let us find the ratio

$$\xi = \frac{\Lambda}{\omega_R} = \frac{|e||P^3|}{md_0\Delta}, \tag{62}$$

in the region of weak field (it is assumed that the vectors $\mathbf{P}$ and $\mathbf{d}$ are collinear), where the optical dipole matrix element is [19]

$$d_{eh}(\mathbf{k}) = d_0 \frac{\Delta}{\Delta + 2\varepsilon(\mathbf{k})}. \tag{63}$$

If $d_0 = |e|a_0$ and $p \lesssim \pi/a$, it follows from Eq. (62)

$$\xi \lesssim \frac{\pi}{ma_0 a\Delta} = \frac{1}{2\pi}\frac{a}{a_0}\frac{\varepsilon_0}{\Delta}, \tag{64}$$

where $\varepsilon_0 = (2\pi/a)^2/2m$ is the electron ground state energy in a one dimensional potential well with the size of an elementary cell. This is the upper bound of the parameter (62). For typical parameter values, $\xi \sim 1$, i.e. both approaches predict approximately the same level of interband excitations.

The similarity of the mathematical structure of the Bloch equations and the system (38) is not accidental. At the same time there are important differences the Bloch equation are a consequence of the approximation of a space homogeneous time-dpendent electric field (dipole approximation) with the Hamiltonian

$$H_{in}^{(d)} = -\mathbf{dE}(t) = -e\mathbf{xE}(t), \tag{65}$$

while the discussed approach is based on the more general standard replacement

$$\partial_k \to D_k \qquad or \qquad p_k \to P_k, \tag{66}$$

and accordingly

$$H_{in} = \frac{ie}{m}\vec{A}(t)\nabla - \frac{e^2}{2m}A^2(t), \tag{67}$$

which is the fundamental way of introduction of the electromagnetic field interaction with a charge. Both approaches are equivalent in a space with a continuous translational

symmetry group [26]. This equivalence is proven by the gauge transformation of the wave function

$$\Psi(\mathbf{x},t) = \exp\{-i\mathbf{A}(t)\mathbf{x}\}\tilde{\Psi}(\mathbf{x},t). \tag{68}$$

However, this transformation is impossible in a space with a discrete translational symmetry group. As a result, the theories based on the prescriptions (65) and (66) become different.

In addition, let us note, that the dipole approximation (65) violates the discrete translational invariance of the Bloch theory in contrast to Eq. (66). This circumstance is very important because the behavior of systems with broken symmetry can strongly differ from the behavior of the analogous system with unbroken symmetry (e.g., [29]). Thus, the presented approach is alternative to the Bloch one and free of the restriction $\lambda \gg d$ (the long wave length approximation) that is inherent in the Bloch equation formalism.

6 Electron-hole excitations in strong electric field

For numerical estimates, we will consider below linearly polarized electric fields of two kinds: a pulsed field with the Sauter potential $\mathbf{A} = (0, 0, A(t))$ [8],

$$A_{ex}(t) = E_0 b[\tanh(t/b) + 1], \qquad E_{ex}(t) = E_0 \cosh^{-2}(t/b), \tag{69}$$

where b is the pulse width, and periodic field (56) with the frequency ν. The characteristic field strength E must satisfy the quasiclassical field condition [24], which is necessary for application of the used formalism,

$$E \gg 1/\tau^2, \tag{70}$$

where τ is the characteristic time of the field variation: $\tau \sim b$ for the pulse field and $\tau \sim 1/\nu^2$ for the periodic one.

The parameters of the semiconductor are chosen close to the ones for silicon: the energy gap is $\Delta = 1$ eV and the effective mass $m = m_e$. Time dependence of the distribution function $f(\mathbf{p}, t)$ in a monochromatic field in the resonance and non- resonance regions are presented on Figs.1,2. The time evolution of the carrier densities calculated on the basis of the KE (37) and Eqs. (50) are presented on Figs. 3 and 4 for the pulsed field (69) with $b = 1.5 * 10^{-12}$c (left part) and a monochromatic field (56) with $\lambda = 1\mu m$ for different amplitudes $E_0 = E_m = 10^2, 10^3$ and 10^4 V/cm (right). An initial strong growth and a subsequent saturation regime are observed for a pulsed field, whereas action of a periodic field is accompanied with "accumulation" of density similar to the results for vacuum pair creation, see Ref. [18]. We note that the choice of a quadratic dispersion in the given work may lead to considerable errors for strong excitations (very strong field), and the generalization to a more realistic dispersion is an important task for the future.

Figs. 5 and 6 show the corresponding momentum distributions: the right figures are for a time-periodic field (56) with the frequency ν corresponding to the wave length $\lambda = 1000$ nm (Fig. 5) and $\lambda = 0.1$ cm (Fig. 6), where $m = m_e$ and $E_0 = 1000\ V/cm$. The left parts of figures 5 and 6 correspond to a pulsed Sauter potential (69) with $b = 2\pi/\nu$, i.e. only one period of (8) is included. Due to the choice of the prefactor, the amplitude of the vector potential in Fig. (6) is 1000 times larger than in Fig. (5). As a consequence, for a pulse, the two peaks of the distribution merge into one. Further, it is easy to see that, in the periodic case, the coherent evolution of the system is accompanied by an increase of the number of higher harmonics (growth of an instability) which eventually leads to chaotic behavior. This is due to the nonlinear dependence on the field strength.

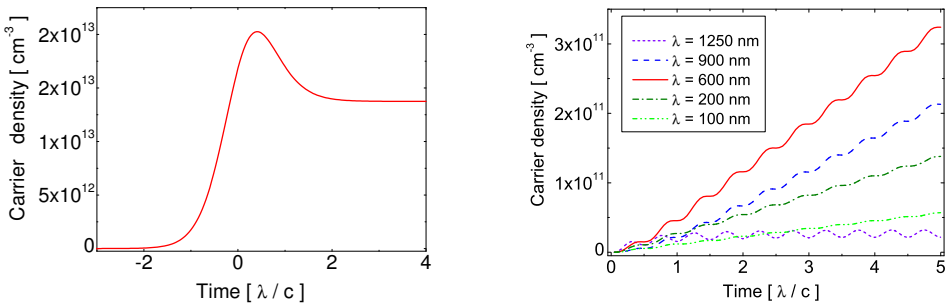

Рис. 3: Time dependence of created carrier density for the pulsed field (69) (left) and the monochromatic field (56) (right) for $E = 1000$ V/cm in the vicinity of the resonance frequency $\lambda = 600$ nm, corresponding to the maximal pair creation rate.

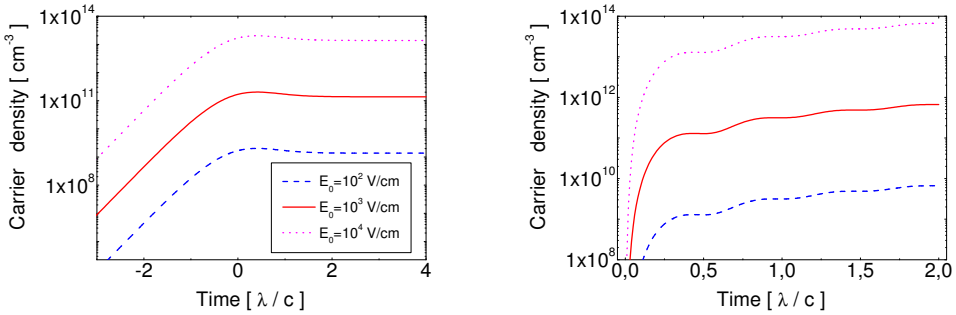

Рис. 4: The same as Fig. 3 for different field strengths.

The derivation procedure of the basic KE (37) shows (Sect. 3 and 4), that the limitation $m_e = m_h = m$ can be eliminated and one can instead understand m as the effective mass of the carriers. The presented approach is valid for arbitrary physical reasonable parameters of quasiclassical external electric field, In particular, for a harmonic field Eq. (37) is valid for the whole range of values of the Keldysh adiabatic parameter (1).

The curves of Fig. 4 demonstrate the saturation for a given field strength. This effect is not connected with the finite width of the valence and conduction bands, but is due to the dynamic equilibrium in the electron and hole subsystems. However in the case of semiconductors with narrow bands, another saturation effect is possible which is caused by the limited nature of the electron reservoir.

7 Linear response

7.1 Optical absorption

Let us show that in the leading order of perturbation theory with respect to the parameter $E_0/E_c \ll 1$ (E_0 is the amplitude of an external signal), the optical susceptibility is defined by the polarization part of the current (51) only. We consider the case that

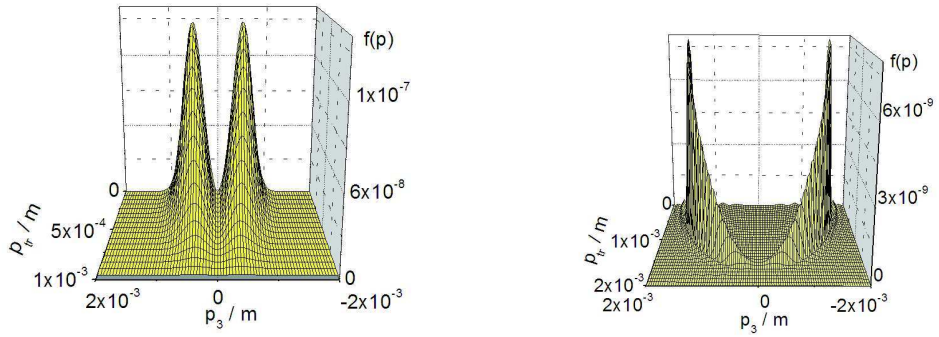

Рис. 5: The momentum spectrum of the created electron-hole pairs, left: for a pulsed field at a time long after the pulse, i.e. $t \gg b = 2\pi/\nu$ and right: for a monochromatic field with $\lambda = 1000$ nm after 5 field oscillations.

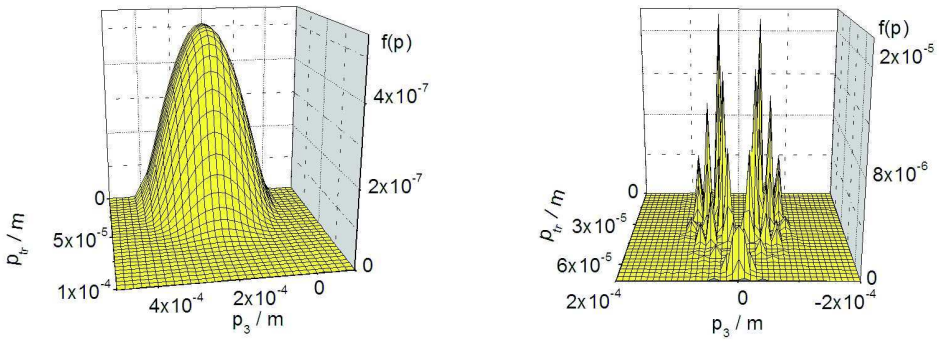

Рис. 6: The same as in Fig. 5, but for a 10^3 times larger wave length $\lambda = 0.1$ cm corresponding to a 10^3 times larger vector potential.

some free carriers are present in the initial state before switching on an external field.

In the Bloch approach, the optical susceptibility is defined by means of dipole moment density $\mathbf{P}(t)$. Below we will use a prescription which leads to connection of vector $\mathbf{P}(t)$ with the polarization function (46) in our approach. That allows to observe some close parallels and distinctions between both approaches. For a weak external field, comparison of the conductivity and polarization current densities (51) shows that we have the following estimations in the leading approximation in characteristic field strength E:

$$j_{pol} = j^{(1)}_{pol} \sim E, \qquad j_{cond} = j^{(2)}_{cond} \sim E^2, \tag{71}$$

and hence according to Eq. (51) we have

$$\mathbf{j}(t) \cong \mathbf{j}^{(1)}_{pol}(t) = \frac{4e}{m} \int [dp]\, \mathbf{p}\, u^{(1)}(\mathbf{p}, t). \tag{72}$$

On the other hand, one can use the general definition of the polarization current density

$$\mathbf{j}^{(1)}_{pol}(t) = \dot{\mathbf{P}}(t), \tag{73}$$

where, in the present approach, the vector $\mathbf{P}(t)$ is defined by the polarization current (73)

$$\mathbf{P}(t) = \frac{4e}{m} \int\limits_{-\infty}^{t} dt' \int [dp]\, \mathbf{p}\, u^{(1)}(\mathbf{p}, t'). \tag{74}$$

Using the Eq. (51) for the electric field $\mathbf{E}(0, 0, E(t))$ one can easily obtain for the Fourier component of the optical susceptibility

$$\xi(\omega) = \frac{P(\omega)}{E(\omega)} = \frac{4e^2}{m_r^2 \omega} \int [dp] \frac{(p^3)^2}{\Delta + 2\varepsilon^{(0)}} [1 - 2f_0(\mathbf{p})] \times \left\{ \frac{1}{\Delta + 2\varepsilon^{(0)} + \omega + i\delta} - \frac{1}{\Delta + 2\varepsilon^{(0)} - \omega + i\delta} \right\}, \tag{75}$$

where f_0 is the equilibrium distribution function of the free carriers and $\varepsilon^{(0)} = p^2/2m_r$ with the reduced electron-hole mass m_r.

The absorption coefficient $\alpha(\omega) = \mathrm{Im}\xi(\omega)$ is equal to

$$\alpha(\omega) = \frac{4\pi e^2}{m_r^2} \int [dp] \frac{(p^3)^2}{(\Delta + 2\varepsilon^{(0)})^2} [1 - 2f_0(\mathbf{p})] \delta(\Delta + 2\varepsilon^{(0)} - \omega). \tag{76}$$

Calculations for different dimensionalities lead to the following results:

$$D = 1: \qquad \alpha(\omega) = e^2 \sqrt{\frac{\Delta}{m}} \frac{1}{\omega^2} \left(\frac{\omega}{\Delta} - 1 \right)^{1/2} F(\omega) \theta \left(\frac{\omega}{\Delta} - 1 \right), \tag{77}$$

$$D = 2: \qquad \alpha(\omega) = \frac{e^2 \Delta}{2} \frac{1}{\omega^2} \left(\frac{\omega}{\Delta} - 1 \right) F(\omega) \theta \left(\frac{\omega}{\Delta} - 1 \right), \tag{78}$$

$$D = 3: \qquad \alpha(\omega) = \frac{e^2 m^{1/2} \Delta^{3/2}}{3\pi} \frac{1}{\omega^2} \left(\frac{\omega}{\Delta} - 1 \right)^{3/2} F(\omega) \theta \left(\frac{\omega}{\Delta} - 1 \right), \tag{79}$$

where $F(\omega)$ is the statistical factor on the energy surface $P_\omega = \sqrt{m(\omega - \Delta)}$, i.e

$$F(\omega) = 1 - f_0(P_\omega). \tag{80}$$

Thus, we obtain the characteristic frequency dependence for the different dimensions D:

$$\alpha_D(\omega) \sim \frac{1}{\omega^2} \left(\frac{\omega}{\Delta} - 1 \right)^{D/2}. \tag{81}$$

In the validity region of Eqs. (77)-(79) $\omega/\Delta \geq 1$ and $E/E_c \ll 1$, the adiabatic parameter (1) is large, $\gamma \gg 1$ corresponding to action of the multiphoton mechanism of absorption.

The standard theory of optical absorption based on the dipole approximation leads to a different frequency dependence [19]:

$$\alpha_D^{dip}(\omega) \sim \frac{1}{\omega^2} \left(\frac{\omega}{\Delta} - 1 \right)^{(D-2)/2}. \tag{82}$$

Technically, this circumstance is stipulated by the characteristic momentum dependence of the transition amplitude (27) in contrast to the Rabi frequency (55).

Let us remark, that in the case of a periodic signal (56) the phase (41) can be written as

$$\Theta(\mathbf{p}, t) = E_q t - \frac{2eE_0}{m\nu^2} \sin \nu t + \frac{1}{4m\nu} \left(\frac{eE_0}{\nu} \right)^2 \sin 2\nu t, \tag{83}$$

where

$$E_q = \Delta + 2\varepsilon(p) + \frac{1}{m}\left(\frac{eE_0}{\nu}\right)^2 \tag{84}$$

is the quasienergy. This representation can serve as a basis of an improved perturbation theory.

7.2 Conductivity

In general, the considered system is nonstationary and its properties are defined by the presence of an external field. For such kind systems, the connection of current density with the electric field strength is [28]

$$j_i(t) = \int_{-\infty}^{t} dt' \sigma_{ij}(t-t',t') E_j(t'). \tag{85}$$

The conductivity tensor $\sigma_{ij}(t-t',t')$ depends on two times: the first argument $t-t'$ takes into account the retardation effects while the second one t' corresponds to the nonstationary state of the medium caused by the influence of an external field. In the leading approximation (72), the external field does not influence the state of the medium (linear response), so the relation (85) is transformed to (for the considered polarization in $z-$direction)

$$j_3(t) = \int_{-\infty}^{t} dt' \sigma_{33}(t-t') E_3(t'). \tag{86}$$

This form is convenient for comparison with Eq. (72) and definition of conductivity tensor stipulated by the polarization current in the leading approximation. We obtain, using Eqs. (46) and (51),

$$\sigma_{33}(t) = \frac{4\sqrt{2}|e|^3}{3\pi^3} E_c I(t), \tag{87}$$

where E_c is the critical field (48) and

$$I(t) = \int_1^{z_{max}} dz (z-1)^{3/2} [1 - 2f_0(z)] \frac{\cos[z\Delta t]}{z}, \tag{88}$$

$$z_{max} = 1 + p_{max}/m\Delta = 1 + \pi/am\Delta. \tag{89}$$

In the simplest case, $f_0 = 0$, we obtain from Eq.(89) for $D = 3$

$$\sigma_{33}(\omega) = \frac{2\sqrt{2}|e|^3 E_c}{3\pi^3} \frac{1}{\omega} \left(\frac{\omega}{\Delta} - 1\right)^{3/2} \theta\left(\frac{\omega}{\Delta} - 1\right). \tag{90}$$

This result is well correlated with Eq. (79): $\alpha(\omega) = \sigma_{33}(\omega)/\omega$.

8 Summary

We have presented a quantum kinetic theory of coherent e-h pair excitations exploiting the close analogy to vacuum pair creation in a strong field (dynamical Schwinger effect). For the mathematical realization of this idea, it is natural to construct [on the

basis of Eq. (2)] an equation of motion of the second order with respect to time, which combines the dispersion properties of both electron and hole. This equation was used for derivation of the KE for the description of coherent e-h excitations under action of a nonstationary electric field (dynamical Zener effect) obtained within a non-perturbative dynamical framework. As a first step, the obtained KE was used for some simple numerical estimation of carrier density dynamics in a monochromatic and a pulsed field.

In the considered formalism, the frequency of the electric field is limited by the condition (70) (quasiclassical field). It is obvious, that in the case of low frequency $\nu \lesssim \Delta$ (in the transparency region) the process of e-h excitation is essentially of multi-photon nature and stipulated by loss of stability of electron states in the valence band under action of an external field and, as a consequence, its transition to an excited state in the conduction band [25]. The discussion of Sect. 5 shows, that there are many parallels between the proposed approach and the model based on the Bloch equations in dipole approximation.

Let us summarize the main features of the proposed theory:

1. The given approach is a consecutive kinetic theory based on a non-perturbative dynamical foundation.

2. These KE's allow to describe states of carriers far from equilibrium under action of a time-dependent electric field. In particular, they allow to calculate the non-equilibrium distribution function and all relevant physical quantities.

3. The theory allows for various modifications and improvements on the dynamical level, such as inclusion of carrier-carrier interaction in the presence of a strong field as has been demonstrated for laser plasmas [31].

4. The presented theory is valid in the whole domain of the adiabatic parameter (1) and selfconsistently covers both – the tunnel and multiphoton excitation mechanisms.

Thus, the suggested approach is an alternative to the Bloch one and leads to some different predictions (Sect. 7). A possible cause of these differences is the violation of translational invariance due to introduction of interaction with an electromagnetic field in the dipole approximation (65) in constrast to our general method (66). The peculiarity of this approach becomes apparent also in the case of multi-band models leading to the increase of degree of the time derivatives in the corresponding equation of motion (cf. Sect.2). Let us note, equations of motion with higher derivatives are considered in the modern mathematical physics (e.g., [30]).

The next important step on the way to a more adequate description of the non-stationary Zener effect is to take into account a realistic (non-parabolic) dispersion of electrons and holes.

We plane to do that in a future work.

Finally, we want to note the interesting possibility connected with the close analogy of the constructed formalism with its QED counterpart. The experimental test of vacuum tunneling in QED requires enormous field strengths which are not available in the laboratory today. It is expected that such fields will be accessible by very high power lasers in the near future but this task is very difficult and expensive. On the other hand, some qualitatively similar results can be obtained in a much simpler manner by studying interband transitions in semiconductors with present day lasers.

We thank Prof. M. Kisin, Prof. S.G. Gestrin and A.V.Prozorkevich for useful discussions.

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НЕПЕРТУРБАТИВНАЯ КИНЕТИКА КОГЕРЕНТНЫХ ЭЛЕКТРОН-ДЫРОЧНЫХ ВОЗБУЖДЕНИЙ В СИЛЬНЫХ ЛАЗЕРНЫХ ПОЛЯХ

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Аннотация

В данной работе представлено кинетическое непертурбативное описание межзонных переходов в твердых телах под действием сильного электрического внешнего поля (динамический аналог эффекта Зенера). Этот подход основан на аналогии с эффектом Заутера - Швингера и его динамического аналога в КЭД. В рамках осцилляторного представления получено кинетическое уравнение для описания эволюции квазичастичных возбуждений. Проведен численный расчет для случаев монохроматического и импульсного внешних полей. Проведено сравнение с результатами на основе уравнений Блоха. Данный подход, в отличии от теории Блоха, не использует дипольное приближение, что приводит к различным результатам, в частности, для частотной зависимости коэффициента поглощения.

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