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Quantum Kinetic Theory of Laser Plasmas

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Abstract

A quantum kinetic equation for plasmas in laser fields is presented. Nonequilibrium properties like the electron–ion collision frequency and collisional absorption are calculated on the basis of a numerical solution of this equation as well as using perturbation theory.

1 Generalized kinetic equation

The classical kinetic theory of plasmas in time dependent electric fields was developed in important papers of Silin [1] and later by Klimontovich [2]. In these papers kinetic equations for ultrafast processes where derived and applied to the determination of transport properties of plasmas in strong high–frequency electric fields. At the same time Oberman and Dawson [3] developed a mean field theory for these systems.

The recent developments in the field of short-pulse laser technology make this problem again very acute. The aim of the present paper is to give a generalization to a quantum kinetic description.

An important problem is the energy transfer between the plasma and the field described by the well–known eqs.

$$\frac{dW_{\rm field}}{dt} = -\langle\, \mathbf{j}\cdot\mathbf{E}\,\rangle\,, \qquad \mathbf{j} = \sum_a e_a \int \frac{d^3p}{(2\pi\hbar)^3}\,\frac{\mathbf{p}}{m_a}\, f_a(\mathbf{p},\mathbf{R},t)\,,$$

where the electric current density $\mathbf{j}$ is determined by the Wigner distribution $f(\mathbf{p},\mathbf{R},t)$. The Wigner distribution follows usually from a quantum mechanical Boltzmann–like kinetic equation. In a strong high-frequency field, however, such an equation is not adequat (for a detailed discussion see [4]), and more general kinetic equations have to be derived.

In order to find a more general kinetic equation we started from the time–diagonal Kadanoff-Baym equation and used the self–energy in Second Born approximation for a statically screened potential [4] which results in

$$\left\{\frac{\partial}{\partial t} + e_a\mathbf{E}(t)\cdot\nabla_{\mathbf{k}}\right\} f_a(\mathbf{k}_a,t) = 2\,\mathrm{Re}\sum_b \int \frac{d^3k_b d^3\bar{k}_a d^3\bar{k}_b}{(2\pi\hbar)^6} |V_{ab}(\mathbf{k}_a - \bar{\mathbf{k}}_a)|^2 \tag{1}$$
$$\times\delta(\mathbf{k}_a+\mathbf{k}_b-\bar{\mathbf{k}}_a-\bar{\mathbf{k}}_b)\int_{t_0}^{t} d\bar{t}\ \exp\left\{\frac{i}{\hbar}\left[(\epsilon_{ab}-\bar{\epsilon}_{ab})(t-\bar{t}) - (k_a-\bar{\mathbf{k}}_a)\cdot\mathbf{R}_{ab}(t,\bar{t})\right]\right\}$$
$$\times\left\{\bar{f}_a\bar{f}_b\,(1-f_a)(1-f_b) - f_af_b(1-\bar{f}_a)(1-\bar{f}_b)\right\}|_{\bar{t}}$$

with the distribution functions having arguments $\mathbf{k}_a + \mathbf{Q}_a(t, \bar{t})$.

This kinetic equation is more general than Boltzmann–like kinetic equations. It is a non–Markovian equation which conserves the full energy. The time dependent field modifies the collision integral in several ways. (i) There are shifts of the momenta, $\mathbf{Q}_a(t,t') = -e_a \int_{t'}^{t} dt_1 \, \mathbf{E}(t_1)$, produced by the field during the collision time (intra-collisional field contribution). (ii) In addition to the usual collisional energy broadening there appears a field dependent broadening given by

$$R_{ab}(t,\bar{t}) = \left(\frac{e_a}{m_a} - \frac{e_b}{m_b}\right) \int_{\bar{t}}^{t} dt' \int_{t'}^{t} dt'' E(t''). \tag{2}$$

(iii) An important feature is the nonlinear dependence of the collision integral on the field strength which leads to interesting effects. We find by Fourier expansion of the collision integral that Coulomb collisions in a strong electric field
– are accompanied by emission and absorption of multiple photons, and
– give rise to generation of higher harmonics in the time dependence of the distribution function.

Finally let us remark that a generalization of the collision integral to dynamical screening leads to a more complicated expression which we considered in Ref. [5].

2 Transport properties. Electron–ion collision frequency

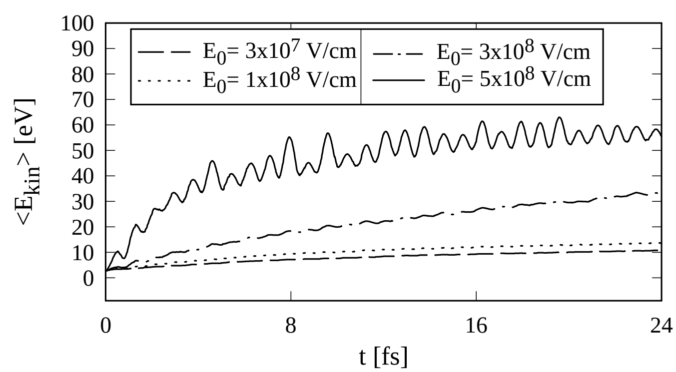

Fig. 1: Time evolution of the kinetic energy for different field strengths

In order to study the transport properties of the laser plasma we have to find solutions of the kinetic equation by numerical or perturbation methods. Let us first consider the behavior of the energy of the plasma. Using numerical solution of the non–Markovian kinetic equation (1), we find the time evolution of the kinetic energy which is given in Fig. 1. In the initial time an uncorrelated plasma was adopted, i.e., we have only kinetic energy. Then there is an increase of the kinetic energy because of the build–up of correlation energy. Without electric field the kinetic energy would reach a stationary value, but in a laser plasma we observe the well–known collisional absorption.

The collisional absorption is usually determined by the electron–ion collision frequency which is for the case of high field frequencies, ω_0, defined by $\nu_{ei}(E) = 4\pi \langle \mathbf{j} \cdot \mathbf{E} \rangle \omega_0^2 / \langle \mathbf{E}^2 \rangle \omega_{\rm pl}^2$. In order to discuss this quantity in detail we consider the special situation $\omega_0 >> \omega_{pl}$, $v_0 >> v_{th}$ with v_0 and $v_{\rm th}$ being quiver velocity and thermal velocity, respectively [4, 5]. In this case the influence of the collisions is small and, following an idea of Silin, we can now determine the distribution function and the electrical current density by perturbation theory with respect to the collision integral [6]. Using the collision integral including dynamical screening we find for the electron–ion collision frequency [7]

$$\nu_{ei}(\mathbf{E}) = 4\pi \frac{8\sqrt{2\pi} Z^2 e^4 n_e n_i \sqrt{m_e}}{\langle \mathbf{E}^2 \rangle (k_B T)^{3/2}} \frac{\omega_0^4}{\omega_{\rm pl}^2} \sum_{n=1}^{\infty} n^2 \int_0^\infty dq \, \frac{1}{q^3} \frac{1}{|\varepsilon_0^R(\mathbf{q}, n\omega_0)|^2} \tag{3}$$

$$\times \sinh\left(\frac{n\hbar\omega_0}{2k_BT}\right) / \frac{n\hbar\omega_0}{2k_BT} \exp\left\{ -\frac{n^2 m_e \omega_0^2}{2k_BT\, q^2} - \frac{\hbar^2 q^2}{8 m_e k_B T} \right\} \int_0^1 dz \; J_n^2\left(\frac{eE_0 q}{m_e \omega_0^2}\, z\right)$$

The classical limit of this expression is well known. It was given first by Klimontovich [2] and later re–derived by Decker et al. [8]. The classical limit is connected with the usually divergency difficulties and therefore cut off procedures are necessary. In our quantum mechanical expression, however, the divergencies of the classical theory are avoided without any arbitrary cut-off procedures. Divergencies at small wave numbers q do not occur on behalf of the application of a screened potential and the factor $\exp[-n^2 m_e \omega_0^2 / 2k_BTq^2]$. Divergencies at large q cannot exist because quantum mechanics provides for a convergence factor $\exp[-\hbar^2 q^2/8m_e k_B T]$. To compare with the classical relations it is obvious to introduce by the relations $n^2 m_e \omega_0^2/2k_BTq^2 = 1$ and $\hbar^2 q^2/8 m_e k_B T = 1$ effective cutoffs $q_{\max} = \omega_o/v_{th}$ and $q_{\min} = \Lambda_e$. These parameters are just the cutoffs in the Silin theory.

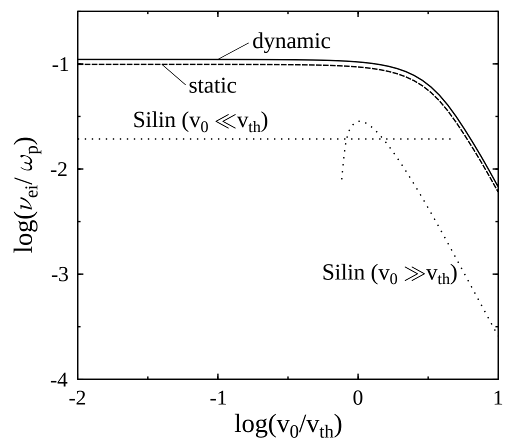

Fig. 2: Electron ion collision frequency in a hydrogen plasma as a function of v_0/v_{th} at $T = 7 \cdot 10^4$K.

The behavior of the collision frequency (3) and the modifications by quantum mechanics are represented in Figs. 2 and 3. Here the solid curve corresponds to the relation (3). The dashed line gives the result for the case of static screening. The dotted lines are high field and low field asymptotic results of the classical theory of Silin. We observe remarkable deviations of the classical results; however, the deviations are decreasing with increasing temperatures. The importance of quantum effects in dependence of the temperature is demonstrated in Fig. 3. All classical theories [1, 2, 8, 9] break down at lower temperatures due to the cut off procedures. Semiclassical theories with Kelbg's quantum potential like molecular dynamic simulation in [10] and HNC calculation in [11] are in reasonable agreement with our calculations in a much wider range of temperatures.

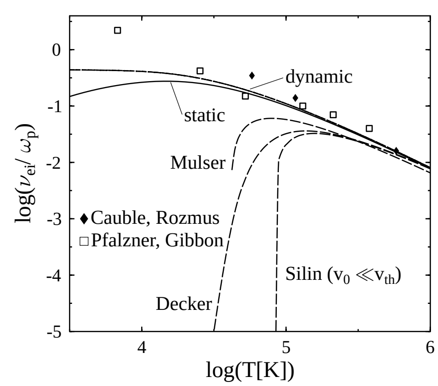

Fig. 3: Electron ion collision frequency in a nonequilibrium hydrogen plasma ($\omega_0/\omega_{\rm pl} = 3$, $v_0/v_{\rm th} = 0.2$, $n_e = 10^{21}{\rm cm}^{-3}$).

The perturbation theory presented above involves strong approximations (e.g. use of an oscillating Maxwell distribution function for the electrons) applied to the general kinetic equation.

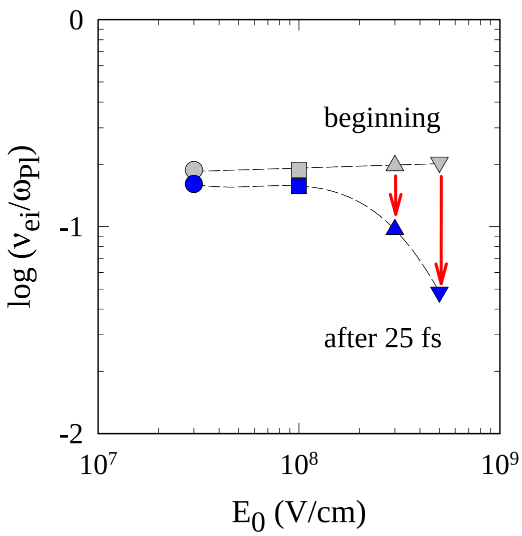

Fig. 4: Electron ion collision frequency vs. field strength at the initial time and after 25 fs.

The numerical solution of the kinetic equation shows a strong anisotropy and non-Maxwellian behavior of the distribution function [12]. Therefore it is interesting to determine the collision frequency with the time dependent distribution function from the numerical solutions. The result is given in Fig. 4 and shows a rapid decrease of ν_{ei} during the first few laser periods. The nonequilibrium plasma exhibits strongly increased inverse bremsstrahlung absorption and emission of higher field harmonics [12]. In the long–time limit, $t > 100fs$, a Maxwellian distribution function is reached and ν_{ei} approaches the analytical curves given above.

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